## CAT 1999 Answer Key

|  | 3 | $3$ | 2 | $8 \mathrm{Cl}$ | 2 | 12\% | 4 | Obe | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 42 | 1 | 82 | 3 | 122 | 1 | 162 | 3 |
| 3 | 3 | 43 | 2 | 86 | 1 | 123 | 4 | 163 | 3 |
| 4 | 2 | 44 | 2 | 84 | 3 | 124 | 4 | 164 | 3 |
| 5 | 1 | 45 | 3 | 86 | 2 | 125 | 2 | 165 | 4 |
| 6 | 2 | 46 | 1 | 86 | 4 | 126 | 4 |  |  |
| 7 | 3 | 47 | 1 | 88 | 1 | 127 | 1 |  |  |
| 8 | 2 | 48. | 4 | 88 | 2 | 128 | 2 |  |  |
| 9 | 1 | 49 | 3 | 89 | 1 | 129 | 1 |  |  |
| 10 | 1 | 50 | 3 | 90 | 4 | 1830 | 4 |  |  |
| ¢ | 4 | 5 | 2 | $9{ }^{1}$ | 4 | 13 | 2 |  |  |
| 12 | 3 | 52 | 4 | 92 | 2 | 132 | 1 |  |  |
| \%3 | 3 | 53. | 2 | 93 | 3 | 133 | 4 |  |  |
| 14 | 1 | 44 | 2 | 04 | 3 | 134 | 1 |  |  |
| 15 | 2 | 55 | 3 | 95 | 2 | 135 | 3 |  |  |
| 16 | 2 | 66 | 1 | 96 | 4 | 136 | 2 |  |  |
| 17 | 1 | 57 | 4 | 97 | 1 | 138 | 1 |  |  |
| 18 | 4 | 58. | 2 | 98. | 1 | 138 | 1 |  |  |
| 19 | 1 | 59 | 1 | 99 | 4 | 139 | 2 |  |  |
| 20 | 4 | 60 | 3 | 100 | 2 | $1{ }^{1}$ | 2 |  |  |
| 24 | 3 | 6 | 3 | 10 | 2 | 1, ${ }_{4}$ | 1 |  |  |
| 22 | 3 | 62 | 1 | 102 | 1 | 142 | 3 |  |  |
| 23 | 4 | 63 | 3 | 108 | 4 | 143 | 4 |  |  |
| 24 | 1 | 64 | 2 | 104 | 3 | 124 | 2 |  |  |
| 25 | 3 | 65 | 3 | 105 | 4 | 185 | 4 |  |  |
| 26 | 1 | 66 | 2 | 106 | 3 | 116 | 4 |  |  |
| 27 | 1 | 67 | 1 | 107 | 2 | 18 | 2 |  |  |
| 28 | 2 | 68. | 4 | 108 | 4 | 1480 | 4 |  |  |
| 29 | 4 | 69 | 1 | 109 | 1 | 149 | 3 |  |  |
| 30 | 4 | 70 | 3 | \%10 | 1 | 150 | 2 |  |  |
| 3 31 | 1 | 7 | 3 | $1 \mathrm{Cl}^{\text {a }}$ | 3 | 151 | 3 |  |  |
| 32 | 4 | 72 | 4 | 12 | 1 | 152 | 1 |  |  |
| 33 | 2 | 73 | 2 | 113 | 2 | 153 | 4 |  |  |
| 34 | 1 | 74 | 2 | 114 | 3 | 154 | 1 |  |  |
| 35 | 2 | 75 | 3 | 115 | 2 | 155 | 3 |  |  |
| 36 | 3 | 16 | 2 | 16 | 1 | 156 | 4 |  |  |
| 3 H | 1 | 77 | 1 |  | 3 | 137 | 4 |  |  |
| 38 | 4 | 78 | 4 | 188 | 3 | 158 | 2 |  |  |
| 39 | 2 | 79 | 4 | 119 | 2 | 159 | 1 |  |  |
| 40 | 4 | 80 | 3 | 1200 | 4 | 160 | 4 |  |  |

## CAT 1999 Solutions

1. Consider the case when we choose the box with label red or white.
Even if we know what is actually in the box, we cannot predict contents of the other boxes. If we choose the red and white box, and say the box actually contains white - then you know that the box labelled white contains the red ball; and the last one the white ball.
2. We know that the lower limiting perimeter of any polygon $S_{1}$ is the circumference of the inscribed circle $(2 * \pi)$
The upper limiting perimeter of any polygon $S_{2}$ is the circumference of the circumscribed circle: $2 * \pi$
This difference of perimeter reduces as the number of sides increase. Breaking up the expression into L1(13)/L2(17) + $2 * \pi / \mathrm{L} 2(17)$. Both the individual terms will be very close to 1, but greater than one.
3. A triangle can be formed by choosing 3 points, 2 from one line and the third from the other. This can be done in ${ }^{11} \mathrm{C}_{2}$ and ${ }^{10} \mathrm{C}_{1}$ ways OR ${ }^{11} \mathrm{C}_{1}$ and ${ }^{10} \mathrm{C}_{2}$ ways. So required number of ways $=55 \times 10+11 \times 45$ $=550+495=1045$.
4. $40 \times 0.75=30 \%$ of men earn $>25,000$ a year. So \% ge of women earning $>25,000$ $=45-30=15 \%$.
Total $\%$ ge of women $=60 \%$.
So fraction earning $>25000=15 / 60=1 / 4$
5. Area will be maximized with a right isosceles triangle, whose diagonal is equal to fence length $=100 \mathrm{~m}$. So sides will be
$100 / \sqrt{ } 2$. So area $=1 / 2 \times(100 / \sqrt{ } 2) \times(100 / \sqrt{ } 2)=$ $10,000 / 4=2500$.
6. The algorithm will be to check a pair of numbers for GCD, and then use this GCD along with the next number to find out the new GCD. This will require one less iteration than the total numbers in the set.
So $n-1$ is the right answer.
7. This is an interesting property of squares of 111. The original number has 15 digits - so the square root should have 8 digits.
8. $342=7^{3}-1$. When we divide $7^{84}$ by this number, at the end of all the divisions, we will be left with a remainder of 1 .
9. The only number that fits in is $21^{2}=441$. So value of $b$ is 1 .
10. Equation will be of the form: $700 \times 25=\mathrm{F}+25 \mathrm{~V}$ and $600 \times 50=\mathrm{F}+50 \mathrm{~V}$. Solving we get $\mathrm{V}=500, \mathrm{~F}=5000$.
For 100 students total cost
$=5000+50000=55000$.
Average $/$ student $=55000 / 100=550$.
11. $17 y=4 x-1$.

So $17 y \leq 4000 ; \quad y \leq 235$.
$x$ can have $235 / 4=58.75$ values.
Since it is an integer it will be 58 values.
12. Use a Venn diagram - $\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}=78$; What is asked is: $(A \cap B+B \cap C+C \cap A-2 \times A \cap B \cap C)$, which is simply $27-10=17$.
13. Let side $\mathrm{AB}=1$, perimeter $=4$. Then $B D=P Q=\sqrt{ } 2$. Diagonal $\mathrm{PR}=$ diameter $\mathrm{PQ}=2$. Circumference
of outside circle $=2 \pi$. Ratio $=2 \pi / 4=\pi / 2$.
14. Number of ways in which we can select at least one student out of $x$ is
${ }^{2 n+1} C_{n}+{ }^{2 n+1} C_{n-1}+{ }^{2 n+1} C_{n-3}+\ldots+{ }^{2 n+1} C_{1}$, which is given as 63 .
Plug in the options; it works only for $n=3$.
15. At 7:30 am, the distance between the 2 trains is $100-(50+20)=30 \mathrm{~km}$.
Relative speed is $60+30=90 \mathrm{~km} / \mathrm{h}$.
So time remaining is $30 / 90=1 / 3 \mathrm{hr}=20 \mathrm{~min}$.
16. The equation is $42-v=k \times \sqrt{ } n$.

Using data in the question, we get $42-24=$
$k \sqrt{ } 9$. Solving we get $k=6$.
For the wagon to just move, speed $=0$.
So $42=3 \sqrt{ } n$. Solving we get $n=49$.
With 49 compartments the train will not move, so we need to reduce by 1 . Hence the answer is 48.
17. Substitute values of $x$, as $x=2 \times 3 \times 4 \times 5=120$ and check the options. Both A and C are correct.
18. For solving modulus questions, use $r-6=+/-$ 11 , so $r=-5$ or 17 .
Similarly $2 q-12=+/-8$. So $q=10$ or 2 .
Min. value of $q / r=10 /-5=-2$. i.e $4^{\text {th }}$ option.
19. Going by options, we see that $2^{\text {nd }}$ option is not possible. Also since Mrs. B is two places to the left of Mrs. E, so it cannot be to the right of Mr. A. So $3^{\text {rd }}$ and $4^{\text {th }}$ options are out.

Hence answer is $1^{\text {st }}$ option.
20. $F(f(x, y))=-|x+y| G(f(x, y))=|x+y|$. Substitute this in all options and check. Option 4 will read as $|x+y|-|x+y|+|x+y|=\mid x+y$ $\mathrm{I}=|-x-y|$.
21. $f(\mathrm{G}(f(1,0)), f(\mathrm{~F}(f(1,2)), \mathrm{G}(f(1,2)))$
$=f(\mathrm{G}(f(1,0)), f(3,-3))$
$=f(G(f(1,0)), 0)$
$=f(-1,0)=1$.
22. Substitute in option 3 and check.

We get $2 x \times 2 x / 4=x^{2}$
23- If the positions from left to right are 1 to 7
$24 \quad$ A and $G$ take up positions 6 and 7
$B$ takes up position 4
C and D are in places 1 and 5 , so that they are as far
as possible.
$E$ and $F$ are in positions 2 and 3 .

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $c$ | $e$ | $f$ | $b$ | $d$ | $a$ | $g$ |

25. Total distance covered $=35 \times 2+45 \times 2=160$ km . Petrol consumption by Aditi
$=(160 / 3) / 16+(160 / 3) / 24+(160 / 3) / 30=$ $(160 / 3) / 6=80 / 9=8.9$ litres.
26. Min. petrol $=160 / 24=6.67$ litres.

27- In the first round, G gets 8 and pays 16 . -ve 29. balance of 8

In the second round he gets Rs. 20. Balance $=$
12. In the $3^{\text {rd }}$ round he gets $6 \&$ pays 6 . Balance $=12$.
In the $4^{\text {th }}$ round he gets $8 \&$ pays 16 . Balance $=$ 4.

His gain is maximum at the end of round 2 or 3 ,
in which case his gain would be 12 .
28. The min G has to have is the lowest negative balance $=8$.
29. Since he made a net gain of 4 at the end of the game, he must have started of with $100-4=96$.
30. If all of $S 1$ and $S 2$ are +ve , then the greatest number will be in $S 2$. If all are -ve, then the greatest number will continue to be in $S 1$. No definite conclusion possible.
31. All elements of $S 1$ are smaller than the smallest element of $S 2$. In the given situation, the smallest element of $S 2$ is $A 25$. Even by exchanging it with the greatest element of $S 1$, the ascending order will still remain.
32. The addition will be maximum for the lowest element of $S 1$ - which is $L$. The sum that will have to be added will be such that it becomes equal to the greatest number of $S 2-$ which is $G$. The number to be added will hence be $G-L$.
33. The average speed will be $2 \times 45 \times 55 / 100$ $=49.5 \mathrm{~km} / \mathrm{h}$.
34. Let dist $\mathrm{BC}=x$. Since both reach at the same time, $(100+x) / 61.875=\sqrt{ }\left(100^{2}+x^{2}\right) / 49.5$. Solving this quadratic for $x$, we get $x=40$ or $x=$ 300.

We know that $x<100$. So $x=40$.
So distance $A C=\sqrt{ }\left(100^{2}+40^{2}\right)=105$ (approx.)
35. BD will be having the same length as AD (which is $105 / 2=52.5$ ) as D is the diameter of the circumcircle - and B will lie on the circumference of this circle.
36. Glucose on being sweetened by 100 times will have a sweetness of 74 .
Sweetness of a saccharin-sucrose mixture of ratio 1: $x$ is
$(1 \times 675+x \times 1) /(1+x)=74$.
Solving for $x$ we get, $x=9$.
37. 1 g of glucose, 2 g of sucrose and 3 g of fructose will have a sweetness of
$(1 * 0.74+2 * 1+3 * 1.7) / 6=1.3$.
38. In the best case scenario, all $m$ containers are nearly having the same volume $v_{l}$. So each time we fill a white container, volume $1-v_{l}$ will remain.
Since there are $m$ such containers having volume $v_{1}$, empty space will be $m\left(1-v_{l}\right)$.
39. Let $\mathrm{m}=1$ and $\mathrm{n}=1$. Option (a) gives the answer as 4
and option (d) gives the answer as 'greatest integer
less than or equal to $2^{1}$.
less than or equal to $2^{\prime}$. So, both of these cannot be
the answer. Option (b) gives the answer as 'smallest
integer greater than or equal to $2^{\prime}$ and option (c)
gives the answer as 1 . But the actual answer can be
greater than 1 as the volume of the vessel is 21.

Hence, (b) is the answer.
$\mathrm{Q}=2 \mathrm{P} .2$ chances $\mathrm{Q}=2, \mathrm{P}=1 ; \mathrm{Q}=4, \mathrm{P}=2$.


| 106 | Tougher in B obviously is in conjunction with tricky in line 1. DC too gel well. Hence the option. |
| :---: | :---: |
| 107 | AC talk of upsetting and restoring a particular balance. C goes very well with line 6 . Did you notice the words queen in D and her in line 6? |
| 108 | Note the word they in line B. C explains what has been disc used in $D$. |
| 109 | These in A refers to motors in D only. C obviously contrasts very well with A. |
| 110 | They of A links up with revolutions of B. Besides line C tends to tone down the point made by line D by using but in relation with unexceptionable. |
| 135 | Trade of OPEC $=33 \%$ of imports Plus $10 \%$ of exports, For US the figures are 9 and $19 \%$. So even without calculating we can say that OPEC is bigger. |
| 136 | Lowest total trade was with Others, Export was $1 \%$ of $34 \mathrm{~b}=340 \mathrm{~m}$ |
| 137 | Highest trade deficit is OPEC $=23 \%$ of $41-10 \%$ of $34=6.0$ |
| 138 | By visual inspection it has to be USA or Asia but A imports less |
| 139 | Here we need to only see market shares for a relative judgement - the share of Other east Europe decreased from 3 to 2. <br> US increased from 19 to 23 ; Increase of $4 / 19$. Asia increased from 15 to 18 ; Increase of $3 / 15$. 4/19 is bigger than $3 / 15$ so the answer is USA. |
| 140 | Trade deficit in $97-98=40779-33979=6800$. <br> Trade deficit in $98-99=(28126-21436) \times 12 / 8$ $=6690+3345 .$ <br> Increase in deficit $=3345-110 / 6800=47 \%$ |
| 141 | By visual inspection. |
| 142 | Values are for Arhar 800/1900, Pepper 2000/18000, Sugar 90/1460 and Gold 500/4000. Lowest for sugar. |
| 143 | Average all the percentage changes to get the answer as $4.3 \%$ increase |
| 144 | This is the highest for Arhar $=8 / 19=40 \%$ |
| 145 | By visual inspection |
| 146 | By visual inspection |
| 147 | The profitability for the 4 years are $2.2 / 100$, $4 / 250,6 / 300$ and $8 / 280$. So highest in 1998 |
| 148 | Profitability is down in $95-96$, up in $96-97$, so we cannot make any firm conclusions. |
| 149 | For drinking it is Bangladesh, for Sanitation it is Philippines |
| 150 | Check coverage data for both |
| 151 | $\begin{aligned} & 70 \%(1-x)=14 \% x=29 \% \\ & 70-29=70 x-14 x=56 x . \\ & \text { So } x=41 / 56=73 \% \end{aligned}$ |
| 152 | Philippines is about $50 \%$, since average of 66 and 88 is 77 . For Indonesia it is more than $50 \%$ rural - and China is more like India in Rural ( $>$ 70\%) |
| 153 | India is not on coverage frontier because (i) it is below Bangladesh and Philippines for drinking water. <br> (ii) for sanitation facilities it is below Philippines, Sri Lanka, Indonesia and Pakistan. |
| 154 | . For questions 154 and 155: <br> The disparity for the coverage of rural sector is as follows. |

$164 \quad 2^{\sqrt{x}}=x$.
$x=4,16$ satisfy this equation.
So both statements are required.
165 Statement I gives us the number of white flowers.
But
we know that a white seed gives both red or white
flowers. Thus, proving statement II, gives the number
of red flowers. But both black and white seeds give
red flowers, again providing no solutions

