Chp 10: Wave Optics

* Wave theory of light:

- Newton's corpuscular theory:

① Every source of light emits large number of tiny particles known as "corpuscular" in a medium surrounding the source.
② Corpuscules are perfectly elastic, rigid but weightless.
③ Corpuscles travel with very high speed (3×10^8), but in different medium, their speed decreases.
④ When the corpuscules hit our retina in the eye, we get sensation called light.
⑤ Different colours of light are due to different size of corpuscules.

- Corpuscular theory could not explain:

① Reflection and refraction on surface of liquid.
② Interference, diffraction, polarization.
③ Speed of light in denser medium as per corpuscular theory should increase, but by experiment its found to be denser.
④ The mass of source must decrease after long emission but it never happens.
⑤ Therefore, corpuscular theory failed.

- Huygens theory:

① Tells us that a flow of new wavefronts are formed and how they propagate.
② Light travels in form of longitudinal waves with uniform velocity in homogeneous medium.
③ Different colours are due to different wavelength of waves.

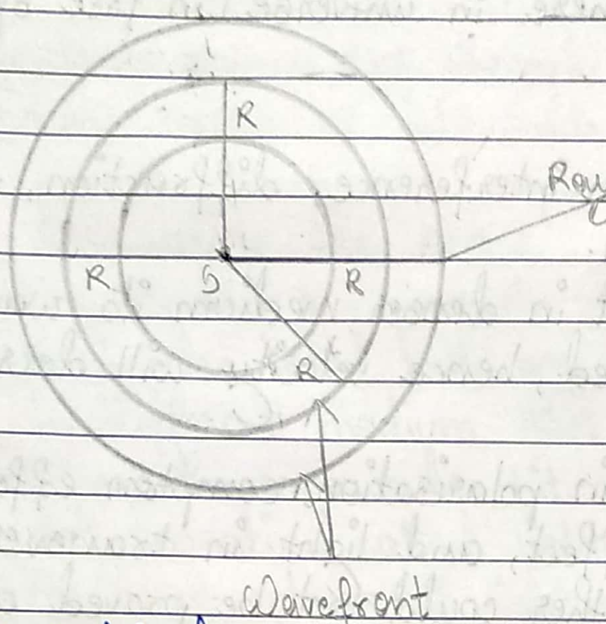
(4) When wave enters our eyes and gives energy we have sensation of light.
(5) The waves are formed in hypothetical medium "ether", which is present everywhere in universe (in free space and vaccum).
(6) Success:
(i) It could explain interference, diffraction, reflection and refraction.
(ii) It predicts that in denser medium its wavelength will be decreased, hence velocity will decrease.
(7) Failure:
(i) Could not explain polarisation, compton effect, photoelectric effect, and light in transverse.
(ii) Existence of ether could not be proved as there are traces of drag by Michelson Morley experiment.
(iii) Energy of waves does not depend on its amplitude but depends on frequency.
(iv) Colour does not depend on wavelength.

* Construction and propagation of wave form (Huygens principle):

- Every point of wavefront becomes secondary source of light.
- This secondary source gives their own light waves. Within a small time they produce their own waves called secondary wavelets. These secondary wavelets have same speed and wavelength as waves by primary source. At any instant a common tangential surface on all these wavelets give new wavefront.

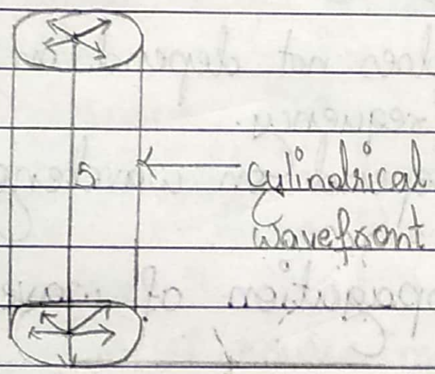
* Shape of wavefront:

- Spherical wavefront:



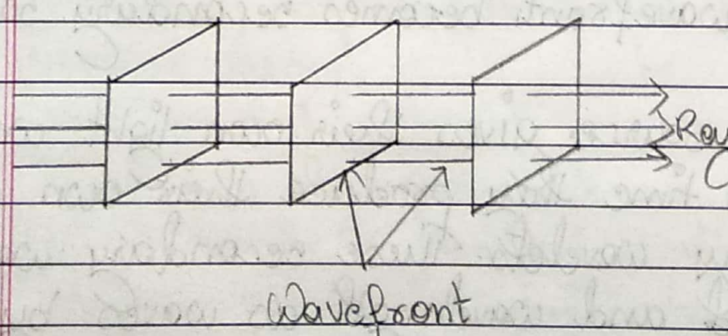
Source-
Point source.
Wavefront-
spherical.

- Cylindrical wavefront:



Source-
Line source
Wavefront-
Cylindrical

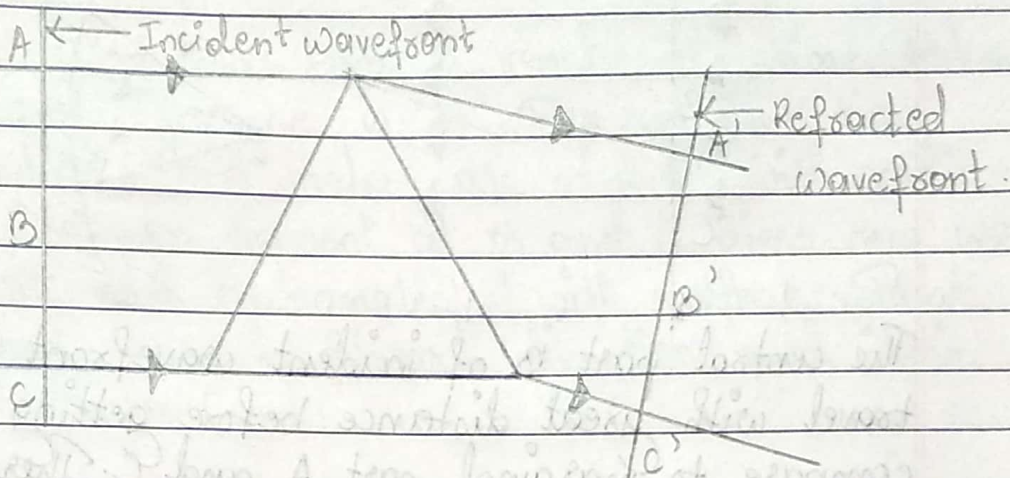
- Plane wavefront:



Source:
(i) Plane surface
(ii) Point source (far away)
Wavefront-
Plane

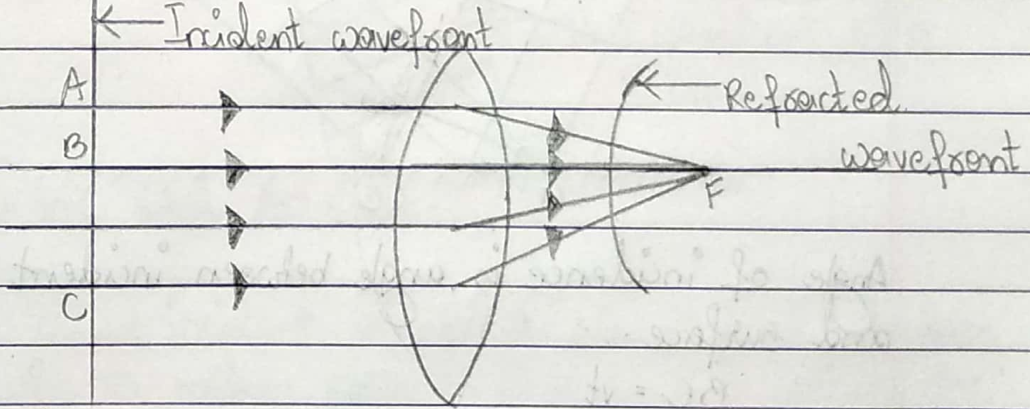
* Behaviour of prism, lens and mirror:

- Prism:



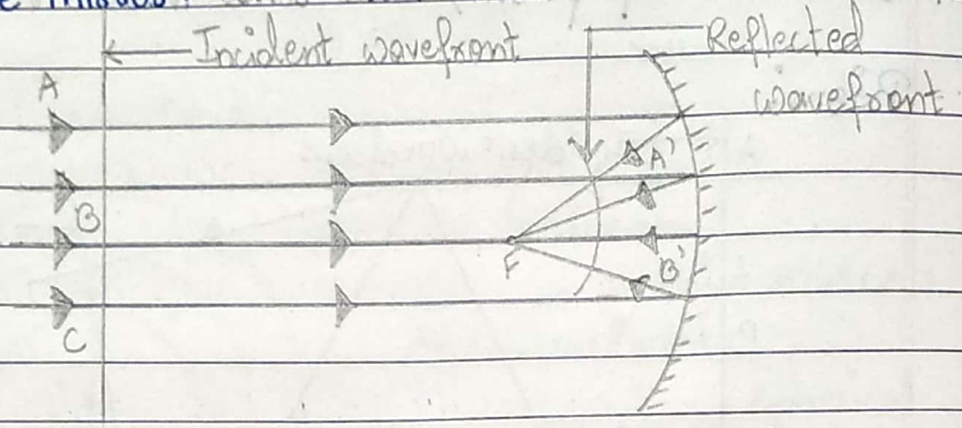
Since the speed of light in glass is smaller than in air, therefore lower part C of plane wavefront which travels great thickness of glass prism is slowed down most, whereas the upper part A travels with minimum thickness slowed down the most. This explains the tilting of wavefront in prism after refraction.

- Convex lens:



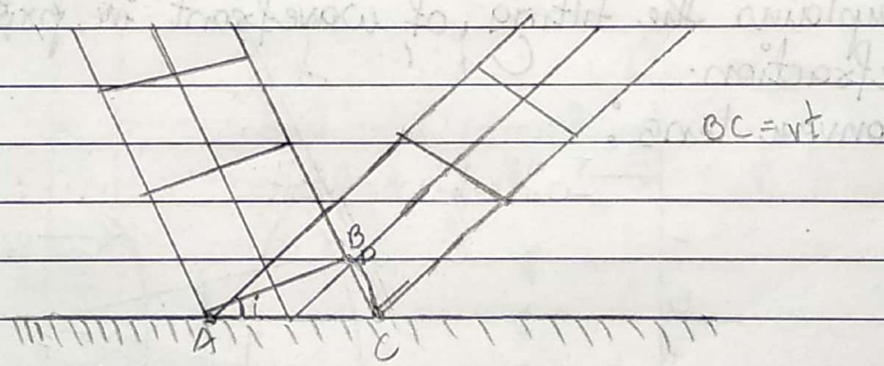
The central part B of plane wavefront travels with great thickness of lens and is therefore slowed down the most. The marginal part A and C of wavefront travels with minimum thickness and therefore slows down the less.

- Concave mirror:



The central part B of incident wavefront has to travel with great distance before getting reflected, compare to marginal part A and C. Therefore, central portion B' of reflected wave-front is closer to mirror than marginal part A' and C'

* Reflection explained by wave theory:



Angle of incidence is angle between incident wavefront and surface.

$BC = vt$

In ΔABC and ΔADC,

$AC = AC$

$BC = AD$

$\angle B = \angle D = 90^\circ$

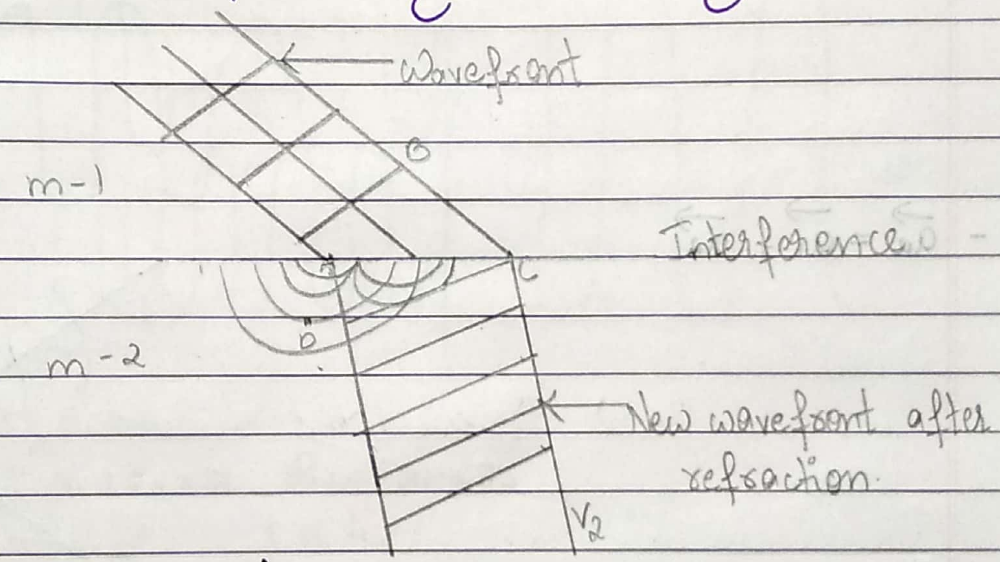
$\therefore$ Due to RHS congruent $\Delta ABC \cong \Delta ADC$

Since, BC = AD,
Opposite angle $\angle i = \angle r$ (This is law of reflection).

- Process -

When wave from 'B' reaches 'C', time taken is 't' and distance is 'vt'. During this time distribution of A makes arc with D with radius "vt". Common tangent at B and C gives new wave-front DC with an angle 'r' with surface. This wavefront moves in forward direction and it is reflected wavefront.

* Refraction explained by wave theory:



BC = V_1t and AD = V_2t.

- Process -

When incident wavefront 'AB' strikes interface at 'A', the B makes wave medium 1, with velocity 'V_1'. In time 't' it reaches 'B' to 'BC'. During this, time waves from 'A' makes an arc of radius 'V_2t'. Since velocity in medium 2 is 'V_2', the common tangent is 'CD' which is refracted wave form. It moves in a direction normal to itself. This is how direction changes because $BC > AD$.

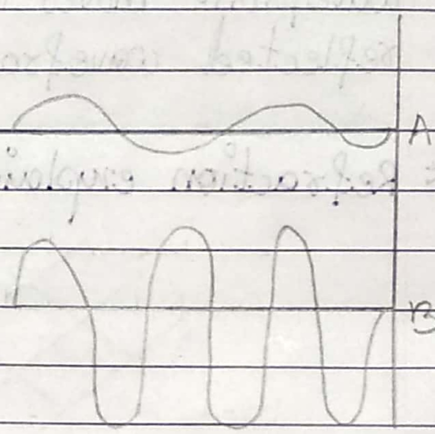
$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$, $\sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}$

Dividing the two,

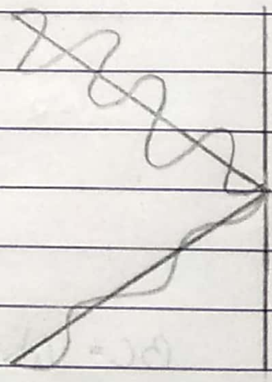
$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = e_1$,

* Interference of light waves:

- $Intensity_B > Intensity_A$
  $I \propto a^2$ (amplitude)

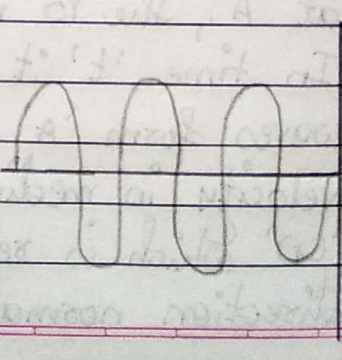


- $\vec{a_R} = \vec{a_1} + \vec{a_2}$

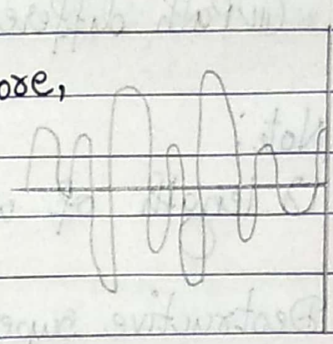


* Types of waves:

- Consistent:
  This wave is consistent wave and continuously same, therefore continuous brightness.

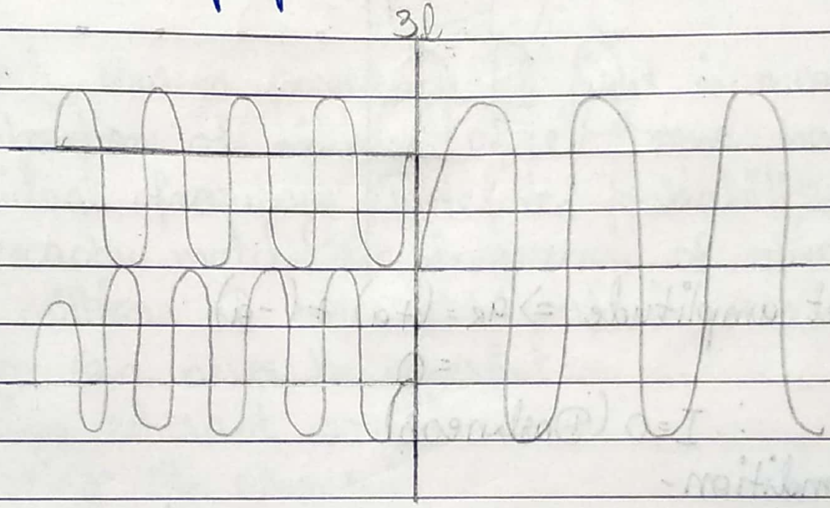


- **Inconsistent:**
This wave is inconsistent. Therefore, brightness is not continuous



* Superposition:

- **Constructive superposition:**



(1) $a_R = a_1 + a_2$
If $a_1 = a_2 = a$ then $a_R = 2a$
$I \propto a^2 \quad \therefore I \propto 4a^2$

(2) **Result** -
Amplitude will be double '2a', intensity will be 4 times.

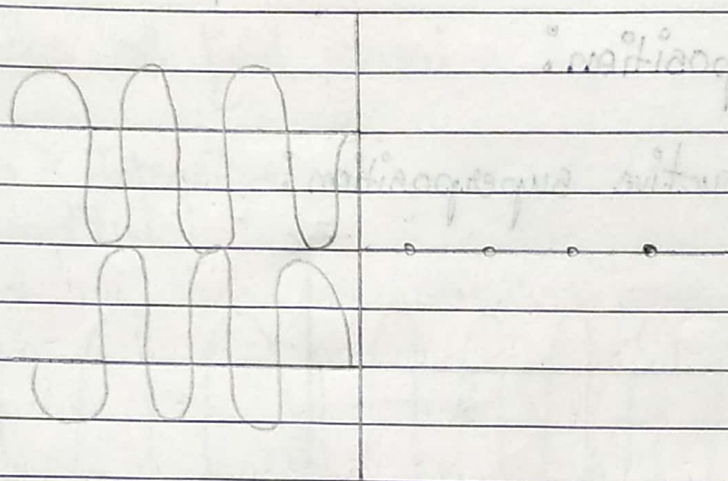
(3) **Condition** -
Crests and troughs of both waves should reach at same instant.

(4) **Requirements:**
(i) Coherent ~~sources~~ waves (eg. frequency, wavelength)
(ii) Phase difference $(2n\pi) = 0, 2\pi, 4\pi \; (n = 0, 1, 2, \ldots)$

(iii) Path difference $= 0, 1\lambda, 2\lambda$ ($n\lambda$; $n = 0, 1, 2, 3 \dots$).

Note:
1λ length of wave $= 2\pi$ change in phase.

- Destructive superposition:



① Net amplitude $\Rightarrow a_R = (+a) + (-a)$
$= 0$

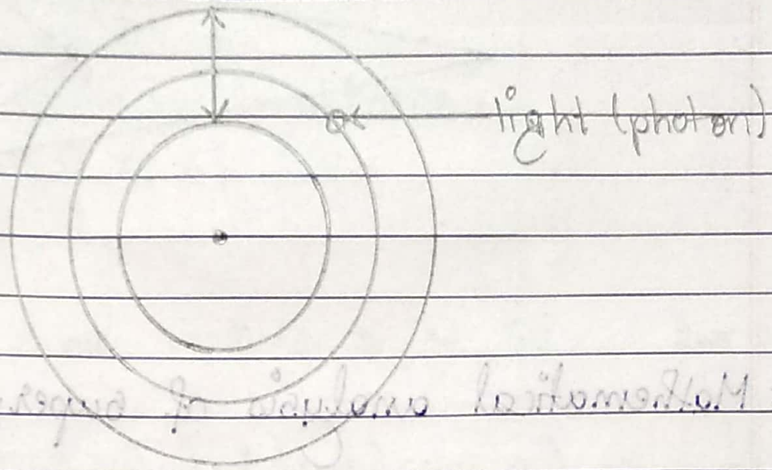
$\therefore I = 0$ (Darkness)

② Condition -
Crest of one wave and trough of other wave.

③ Requirement:
(i) Coherent wave.
(ii) Phase difference $= \pi, 3\pi, 5\pi \; ((2n+1)\pi;\; n = 0, 1, 2 \dots)$.
(iii) Path difference $= \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \; \left((2n+1)\frac{\pi}{2},\; n = 0, 1, 2 \dots\right)$

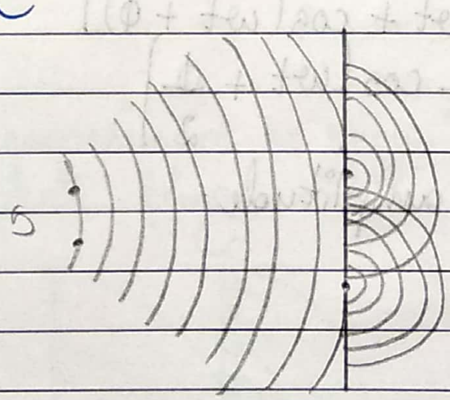
Scanned by CamScanner

* Coherent sources (Any two sources of light are never coherent):

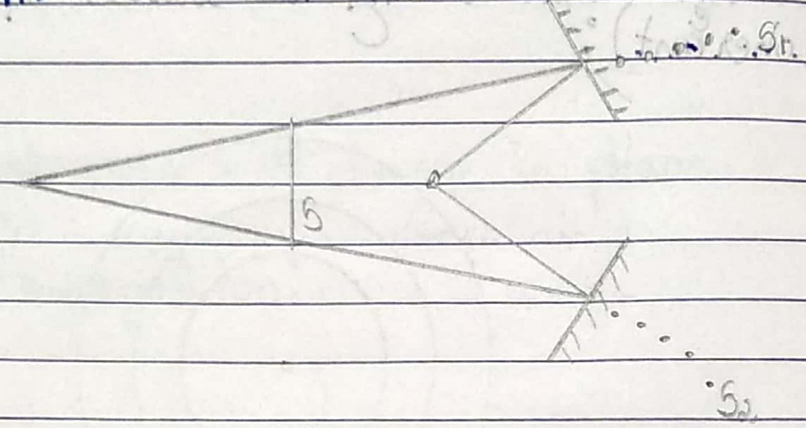


- In each sources frequency of light is mean frequency of vibration of millions of electrons having individual frequency. Therefore, probability that it will exactly match the frequency of other source with millions of other electrons is zero. Hence two sources can never be coherent.
- Creating coherant waves:

(1) Splitting the sources:



② Reflection:



* Mathematical analysis of superposition:

- There are two ~~ways~~ waves of same frequency 'ω' but at constant phase difference 'ϕ', reaches screen with displacement y_1 and other with y_2 at same instant. Then on superposition, resultant displacement will be $y = y_1 + y_2$.

$$y_1 = a\cos\omega t$$
$$y_2 = a\cos(\omega t + \phi)$$

$$y = y_1 + y_2 = a[\cos\omega t + \cos(\omega t + \phi)]$$
$$= 2a\cos\frac{\phi}{2}\cos\left(\omega t + \frac{\phi}{2}\right)$$

Where, $2a\cos\frac{\phi}{2}$ is amplitude.

$$\therefore I = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

Maximum value of intensity, $I_0 = 4a^2$.

$$\therefore I = I_0\cos^2\left(\frac{\phi}{2}\right)$$

① I_0 maximum intensity $= 4a^2 = 4I$
② It has no term of 't' hence coherent and consistent.
③ Depends only on ϕ.

- **Conditions for maximum brightness:**

$\cos^2 \frac{\Phi}{2} = 1$, $\cos \frac{\Phi}{2} = \pm 1$.

$\therefore \frac{\Phi}{2} = 0, \pi, 2\pi \ldots n\pi$ $\quad \therefore \phi = 0, 2\pi, 4\pi, 6\pi \ldots 2n\pi$

Path difference : $0, \lambda, 2\lambda, 3\lambda \ldots n\lambda$.

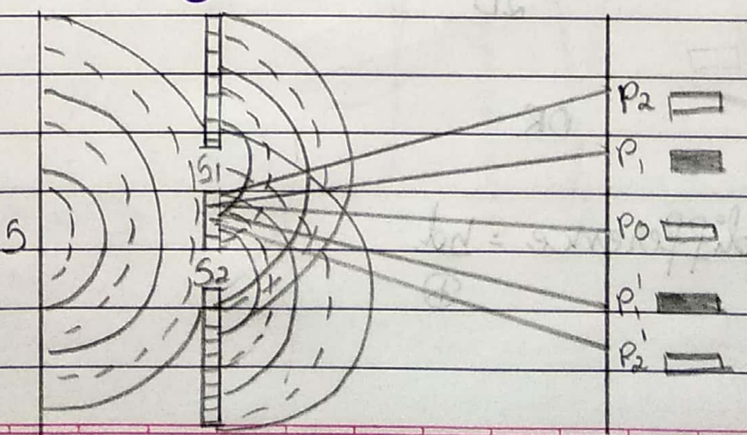
- **Condition for minimum brightness:**

$\cos^2 \frac{\Phi}{2} = 0 \therefore \cos \frac{\Phi}{2} = 0$

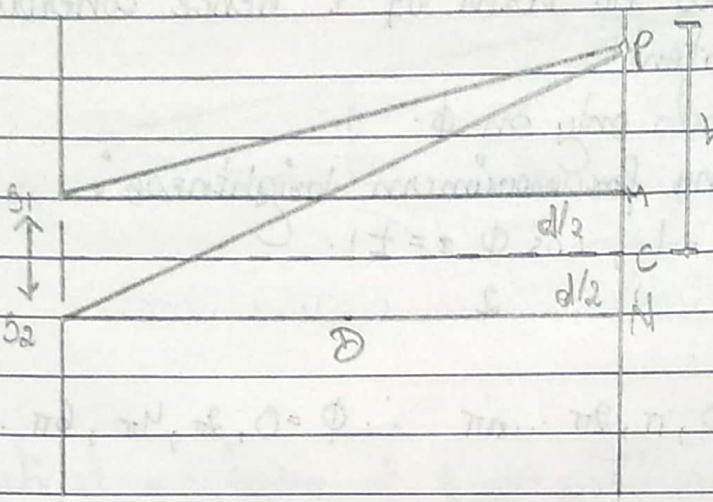
$\frac{\Phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ $\quad \therefore \phi = \pi, 3\pi, 5\pi \ldots (2n-1)\pi$.

Path difference $= \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots (2n-1)\frac{\lambda}{2}$.

* **Young's experiment to show interference fringes and to formulate fringe width:**



- Fringe width:



'd' is the distance between slits (source S_1 and S_2). 'D' is distance of source to screen. 'C' is a point equidistant from S_1 and S_2 such that wave reaches at the same phase. Therefore, it is a bright point called central maxima.

Path difference between S_2P and $S_1P = (S_2P - S_1P)$.

Now, $S_2P^2 - S_1P^2 = \left[D^2 + \left(n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(n - \frac{d}{2}\right)^2\right]$

$(S_2P - S_1P)(S_2P + S_1P) = 2hd$.

Since $D \gg d$ $\therefore$ $S_2P \simeq S_1P \simeq D$ for addition

$\therefore (S_2P - S_1P) = \frac{2hd}{2D}$

OR

Path difference $= \frac{hd}{D}$

① Condition that P is a bright point:

$$\frac{hd}{D} = 0, 1\lambda, 2\lambda \ldots n\lambda \; (n=1,2,3\ldots)$$

∴ For n^{th} bright fringe where n is number of fringe of central fringe n^{th} fringe $= \frac{nD\lambda}{d}$

② Condition for P is a dark point:

$$\frac{hd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots (2n-1)\frac{\lambda}{2}.$$

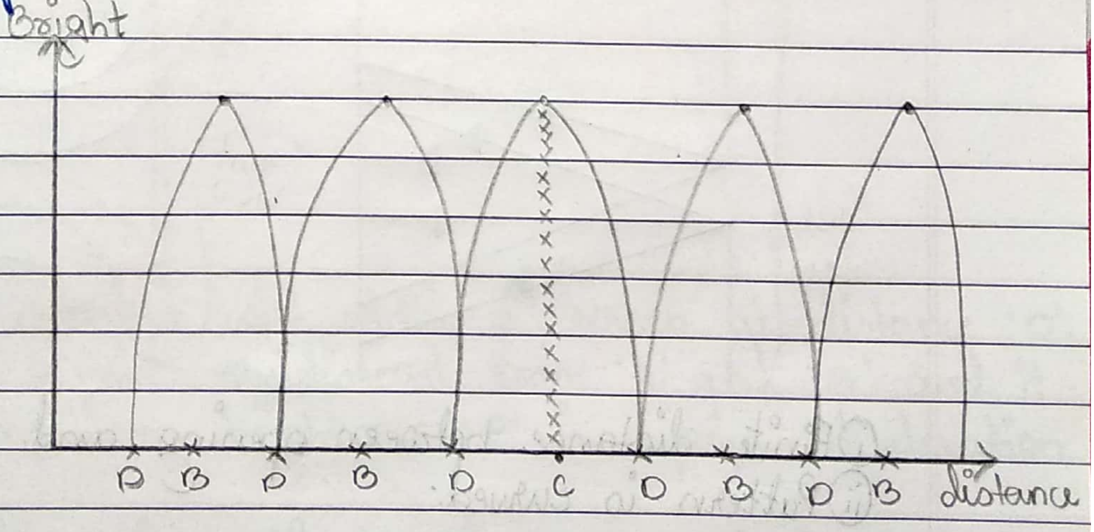
$$\therefore h_{nth\,dark} = \frac{(2n-1)}{2}\frac{D}{d}\lambda.$$

③ Width for dark and bright fringe:

$$W_B = h(n)_D - h(n-1) = \frac{D\lambda}{d} \qquad W_c = \frac{D\lambda}{d}$$

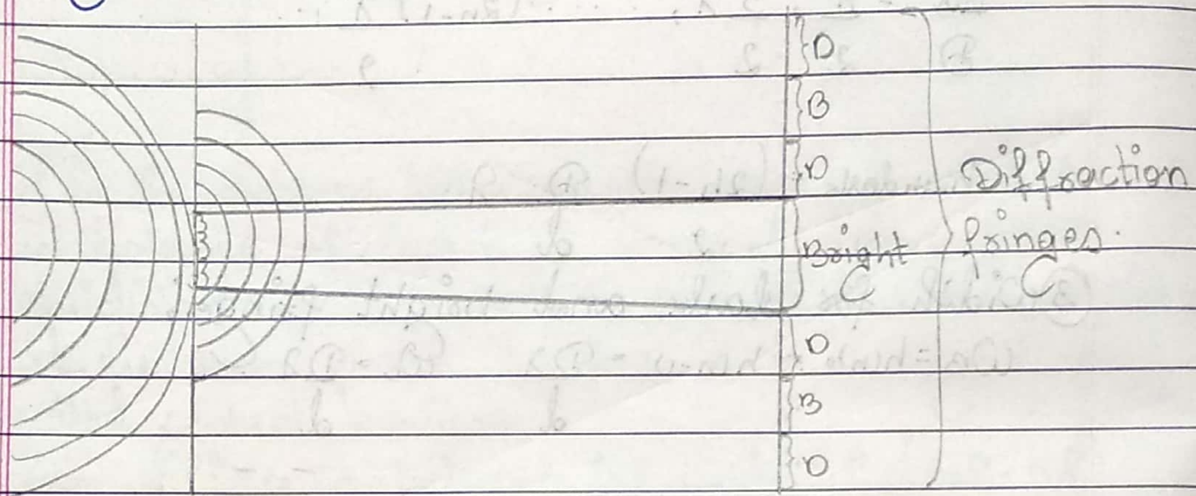
$$W_D = h_{nB} - h(n-1)B = \frac{D}{d}\lambda$$

④ Graph:



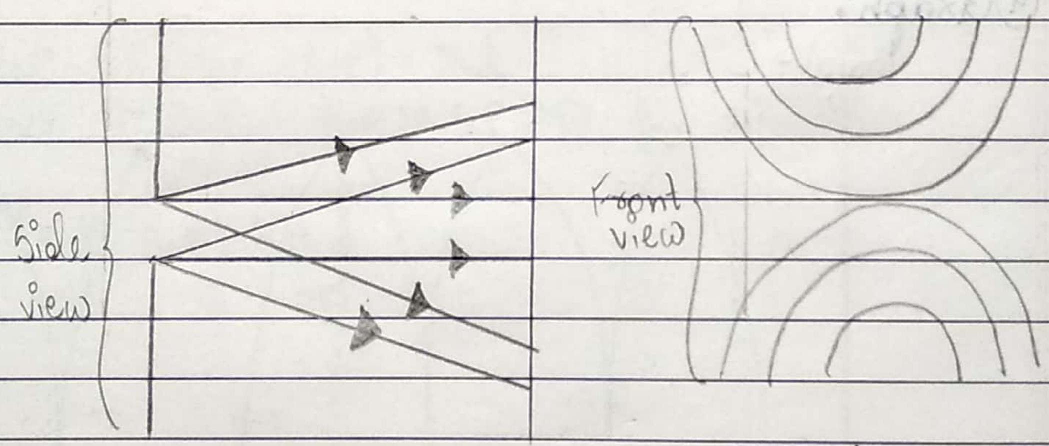
* Diffraction of light:

- The phenomena of bending of light around the corners of small obstacle or aperture and its consequent spreading into the region of geometrical showed is called diffraction of light.
- Single slit experiment:



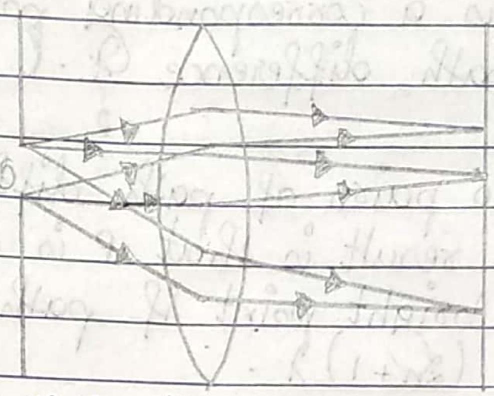
- Types of diffraction:

① Fresnel diffraction:



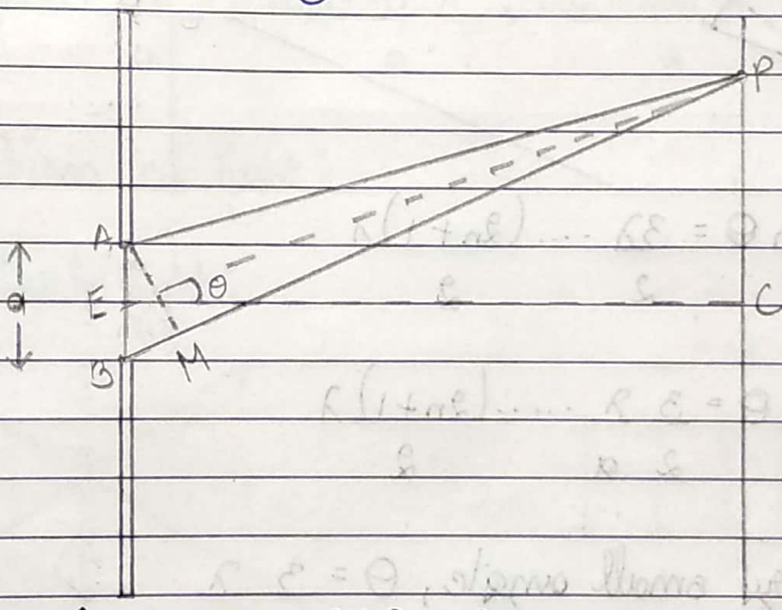
(i) Finite distance between opening and screen.
(ii) Pattern is curved.

② Fraunhofer diffraction:



(i) Virtual infinite distance between opening and screen.
(ii) Pattern is sharp and straight.
(iii) Infinite due to lens.

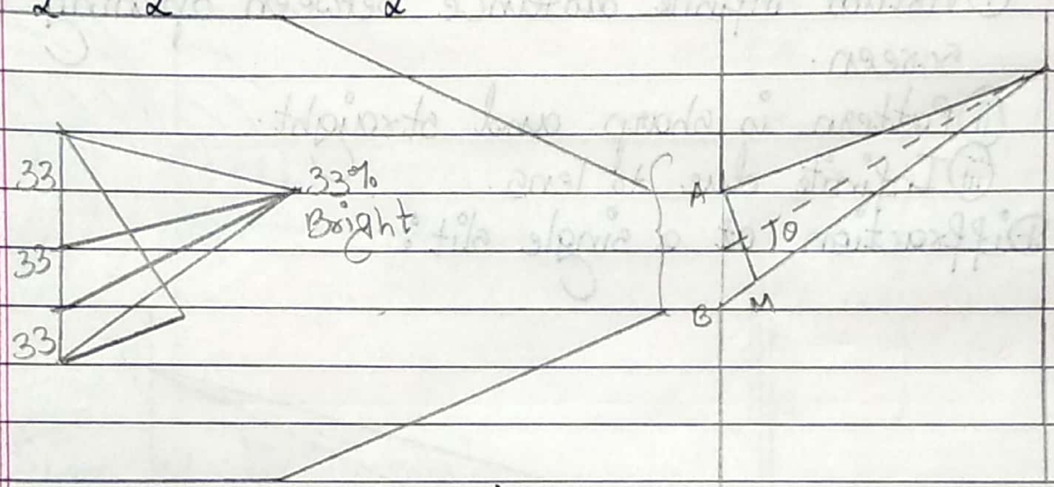
- Diffraction at a single slit:



AB is opening with width 'a', screen at distance 'D', 'C' is a point equidistant from 'A' and 'B' and it has maximum brightness 'P' is a point at elevation 'θ'.
Difference in path length is $BM = a\sin\theta$.
It is observed that when $a\sin\theta = \lambda, 2\lambda \ldots n\lambda$, then 'P' is a dark point.

'E' is mid-point between A, B. For all point from A to E, there is a corresponding point between E to B with a path difference $\frac{\lambda}{2}$.

(There are no pairs of path difference λ).
Hence, total result is that P is a dark point.
P will be a bright point if path difference is $\frac{3\lambda}{2}, \frac{5\lambda}{2} \dots \frac{(2n+1)\lambda}{2}$.

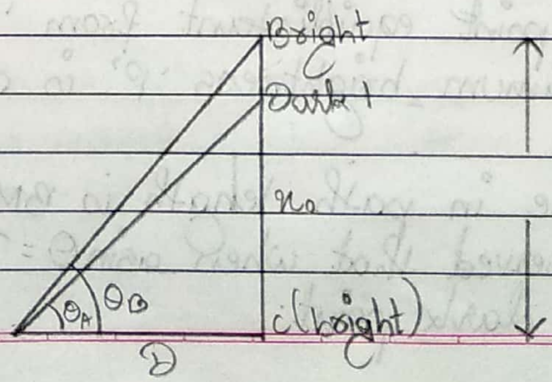


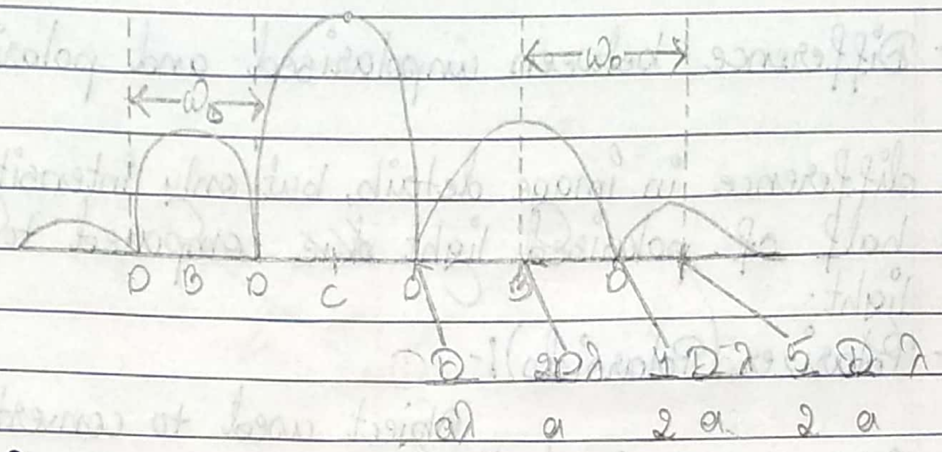
$$a \sin\theta = \frac{3\lambda}{2} \dots \frac{(2n+1)\lambda}{2}$$

$$\sin\theta = \frac{3}{2}\frac{\lambda}{a} \dots \frac{(2n+1)\lambda}{2}$$

For very small angle, $\theta = \frac{3}{2}\frac{\lambda}{a}$

- Graph:





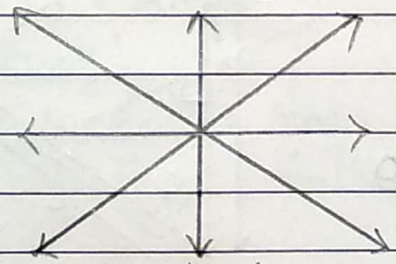
$$\theta = \frac{\lambda}{a} = \frac{x}{D}$$

$$\therefore x = \frac{D}{a}\lambda$$

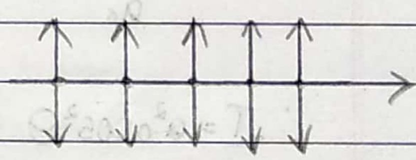
Width of central max = $\frac{D}{a}\lambda$, $\omega_B = \frac{D}{a}\lambda$, $\omega_0 = \frac{2D}{a}\lambda$

* Polarisation of light:

- Unpolarised light:

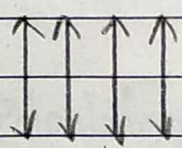


Front view

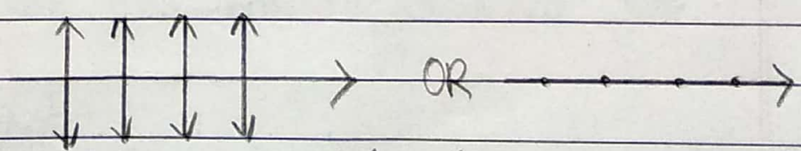


Side view.

- Polarised light:



Front view

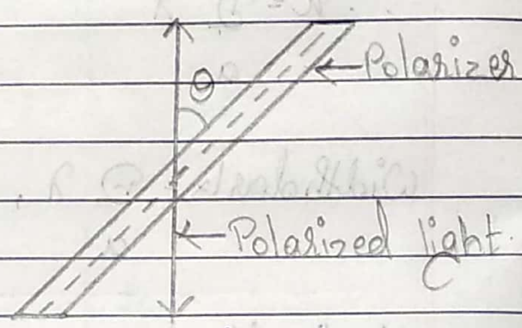


OR

Side view

- Difference between unpolarised and polarised light- No difference in image detail, but only intensity will be half of polarised light ~~due~~ compared to unpolarised light.
- Polarizer (Polaroids):- Object used to convert unpolarised light to polarized light.
- Intensity of a polarised light when it is allowed to pass through polarizer (Malus Law):

$I \propto \cos^2\theta$

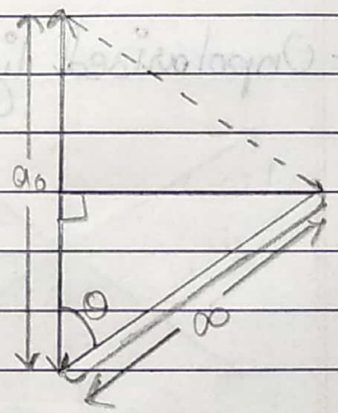


① Proof:

$I \propto a^2$

$I_0 \propto a_0^2$

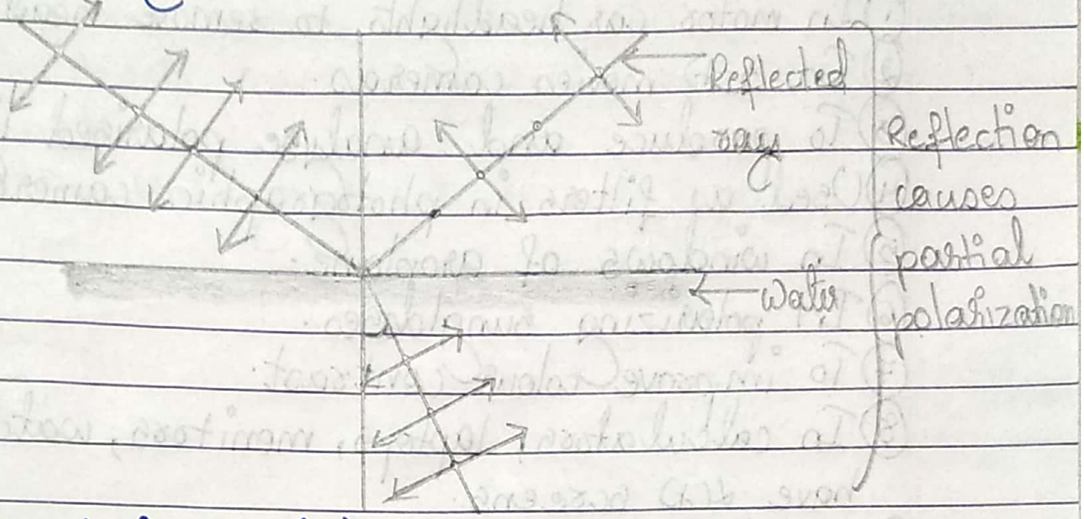
$\cos\theta = \frac{a}{a_0}$ $\quad \therefore a = a_0\cos\theta$



$\therefore I = a^2{}_0\cos^2\theta \Rightarrow I = I_0\cos^2\theta$

Hence, $I \propto \cos^2\theta$.

- Polarization by reflection:



- Angle of polarization (i_p) -
It is that angle in which reflected light is fully polarised.

Note:
In this situation the reflected light and refracted light are at 90° to each other.

- Brewster's law:

① ${}_1\mu_2 = \tan i_p$

② Proof:

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90 - r)} = \frac{\sin i}{\cos i} = \tan i$$

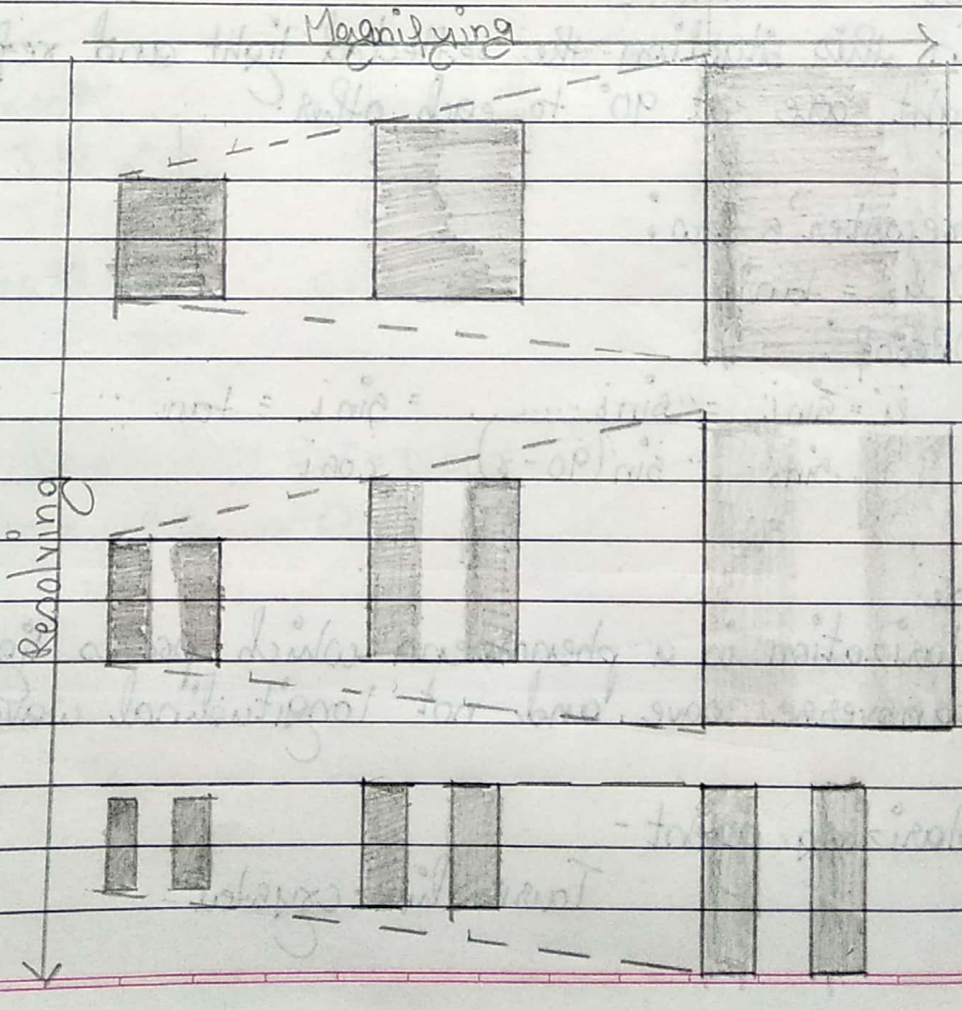
Note:
Polarization is a phenomena which proves light is a transverse wave and not longitudinal wave.

- Polarizing agent -
Tarmaline crystal

- Applications:

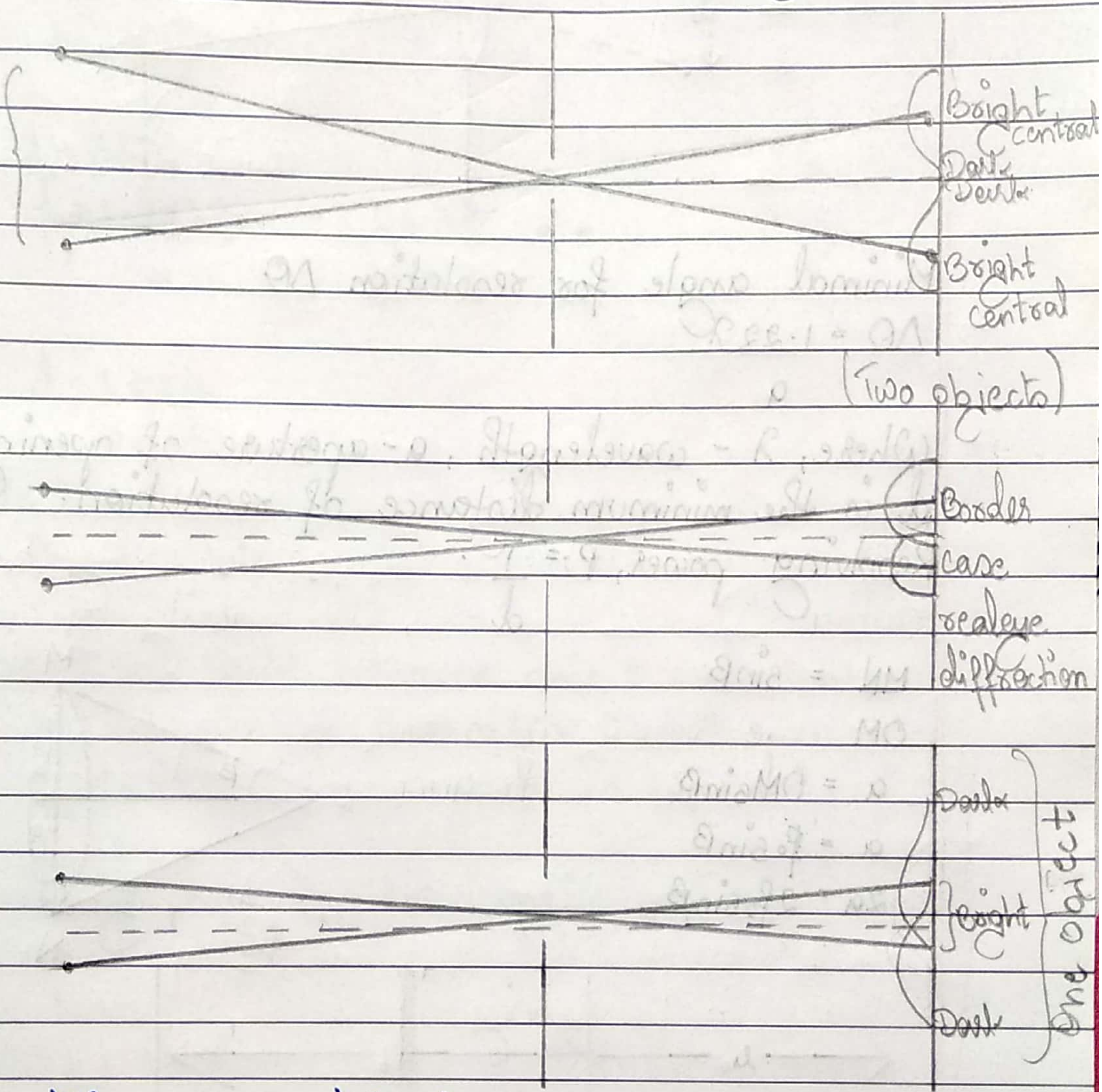
1. In motor car headlights to remove headlight glare.
2. In 3-D movies cameras.
3. To produce and analyse polarised light.
4. Used as filters in photographic cameras.
5. In windows of aroplane.
6. In polarizing sunglasses.
7. To improve colour contrast.
8. In calculators, laptops, monitors, watchs which have LCD screens.
9. Chemical analysis -
   Dextro and laevo effect.

* Resolving and magnifying:



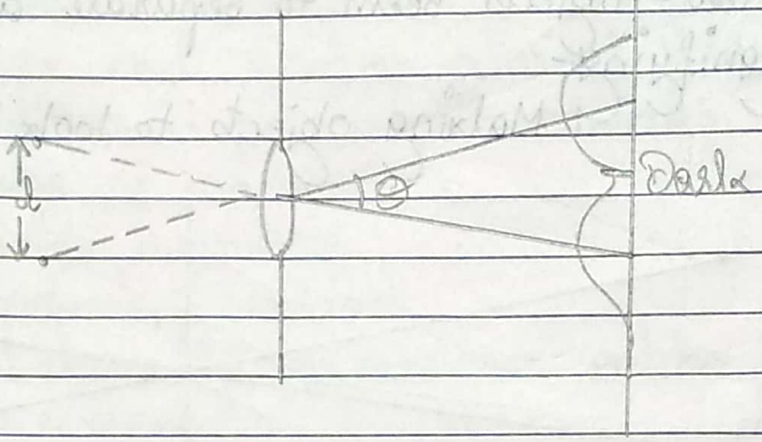
- **Resolving:**
  1. Making more details visible is resolving an image.
  2. "Two" objects seem to separate and resolve.
- **Magnifying:-**

  Making objects to look larger.



- **Condition for resolving:**
  1. Border case.
  2. Circular shape of opening.
  3. For both, $\theta = \frac{1.22\lambda}{a}$

* Resolving power of microscope:



Minimal angle for resolution $\Delta\theta$

$$\Delta\theta = \frac{1.22\lambda}{a}$$

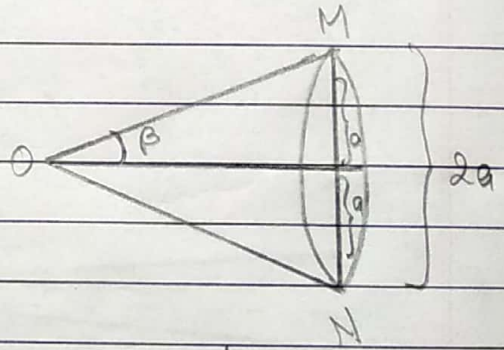
Where, λ - wavelength, a - aperture of opening.
d is the minimum distance of resolution.
Resolving power, $P = \frac{1}{d}$.

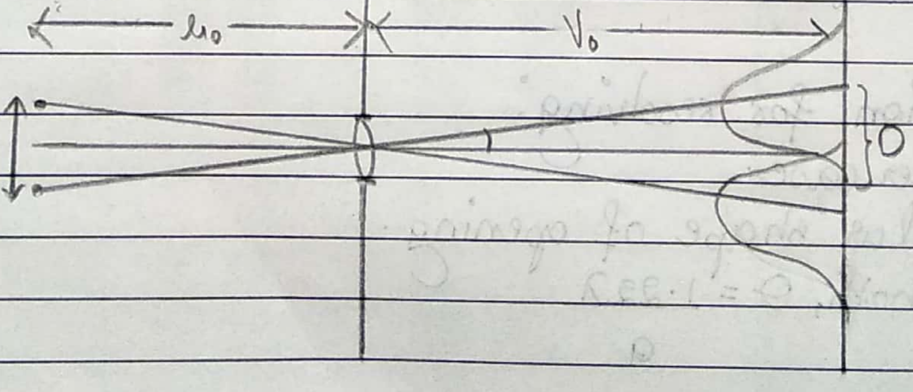
$$\frac{MN}{OM} = \sin\beta$$

$$a = OM\sin\beta$$

$$a = f_0\sin\beta$$

$$2a = 2f_0\sin\beta$$





$m = \frac{D}{d} = \frac{V_o}{u_o}$ $\therefore \frac{D}{d} = \frac{V_o}{f_o}$ or $d = \frac{Df_o}{V_o}$

We know that,

$\Delta\theta = \frac{1.22\lambda}{a}$

According to geometry,

$\Delta\theta = \frac{D}{V_o}$

$\therefore d = \Delta\theta f_o$

$d = \frac{1.22\lambda f_o}{a} = \frac{1.22\lambda \cancel{f_o}}{2\cancel{f_o}\sin\beta}$

$d = \frac{1.22\lambda}{2\sin\beta}$

We know,

$\lambda_{liquid} = \frac{\lambda_{air}}{\mu}$

With filled liquid between object and lens,

$d = \frac{1.22\lambda}{2\mu\sin\beta}$ or Power $= \frac{1}{d} = \frac{2\mu\sin\beta}{1.22\lambda}$

* Resolving power for telescope:

We know,

$\Delta\theta = \frac{1.22\lambda}{a}$

For $\Delta\theta$ should be smaller, 'a' should be large.

$\therefore$ R.P of telescope $= \frac{a}{1.22\lambda}$

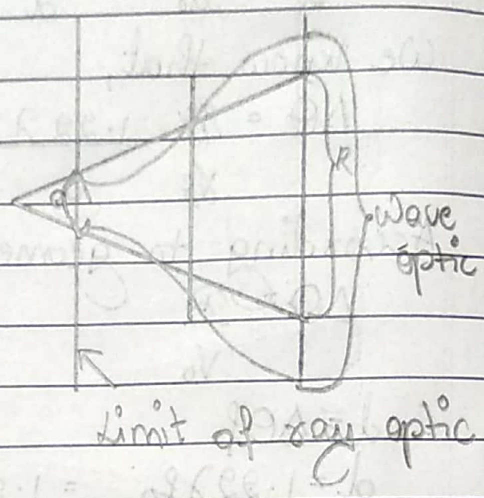
* Limit of ray optics:

$W_{DM} = \frac{2\lambda D}{a}$

Fresnel distance (z).

$\Delta\theta = \frac{a}{z} = \frac{\lambda}{a}$

$\therefore z_F = \frac{a^2}{\lambda}$



* Doppler effect:

- When there is relative motion between source and observer, the frequency as apparent to observer is different and there is change according to relation $\frac{\Delta\nu}{\nu} = \frac{-v}{c}$. Where, c is velocity of light, v is relative velocity. ν is original frequency and $\Delta\nu$ is change in frequency.
- Red shift -
  Red shift is shiffting of spectrum from start towards left end of spectrum due to increase in frequency.