Electrostatics : Electric Charges & Field

Electrostatics is the study of charge at rest

Charge (q):

- Charge is an intrinsic property of matter due to which it experiences electrostatic forces of attraction and repulsion.
- There are two types of charges ; positive (e.g. proton) and negative (eg. electron)
- Charge on a single electron is $\boxed{e = 1.6 \times 10^{-19} C}$ SI Unit - Coulomb(C)

Properties of Charge

eg → $\oplus \leftarrow \rightarrow \oplus$

$\oplus \rightarrow \leftarrow \ominus$

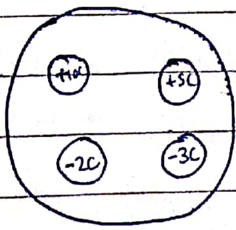
1- Attraction and Repulsion:

→ Like charges repel each other
→ Unlike charges attract each other

2- Additive nature of Charge:

→ Charges is additive in nature.
→ i.e. total charge on a body is the algebraic sum of all charges present on the body.

eg →



Total charge on body

$= +10C + 5C - 3C - 2C$

$= 10C$ Ans

3. Quantisation of charge:

→ Charge on a body is the integral multiple of charge on a single Electron.

i.e. $Q = ne$

(Where e is the charge on a single electron and $n = 1, 2, 3 \ldots$)

~~eg Q→ Cal Calculate the no. of e in~~

eg→

Q→ Calculate the no. of electrons in 1C charge.

Ans→ $q = ne$

$1 = n \times 1.6 \times 10^{-19}$

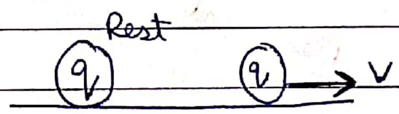
$n = \frac{1}{1.6 \times 10^{-19}} \Rightarrow \frac{10^{19}}{1.6}$

$n = 6.25 \times 10^{18}$ electrons.

4- Invariable of Charge:

→ Charge is invariable in nature.

→ i.e the charge of a body does not depend on its state of rest or motion.



Principal Of Conservation Of Charge

'In an isolated system, charge can neither be created nor destroyed.'

Note: → If a body has excess electrons, it has a negative charge.

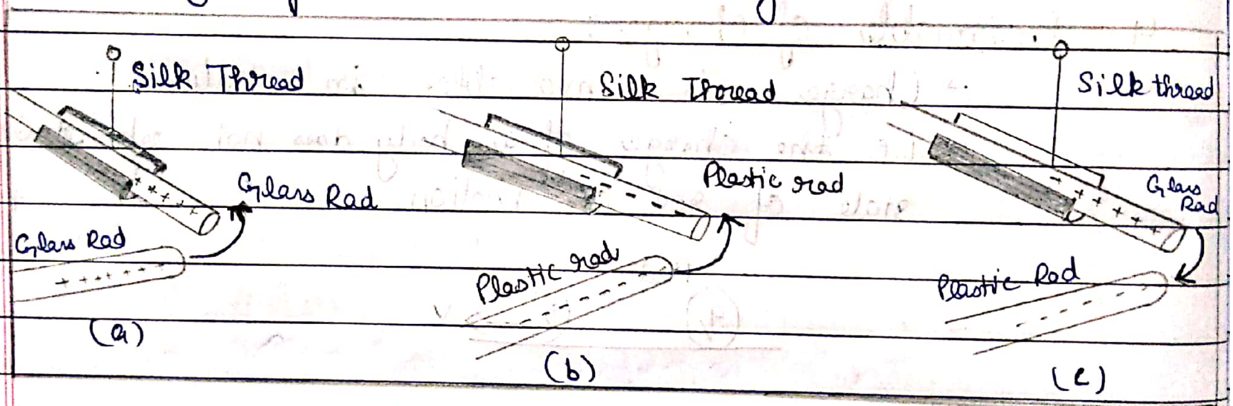
→ If a body has excess protons, it has a positive charge.

Method of Charging Bodies

1- Charging bodies by rubbing / friction :

When two bodies are rubbed together, the friction b/w the bodies causes transfer of electrons from one body to another and as a result both bodies become charged. The body which loses electrons becomes positively charged and the body which gains electrons becomes negative charged.

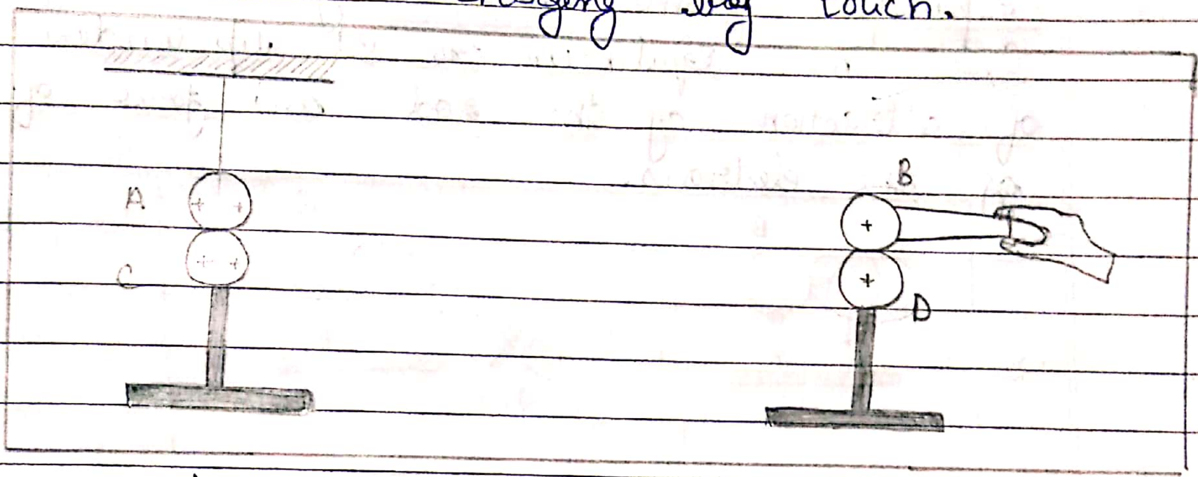
Eg- → rubbing glass rod with silk
→ rubbing plastic rod with fur



→ By convention, charge on glass rod and fur is positive and charge on silk cloth and plastic rod is negative.

2- Charging by touch :

When a charged body is made to touch an uncharged body, some of the charge from the charged body is transferred to the other body. This is called charging by touch.



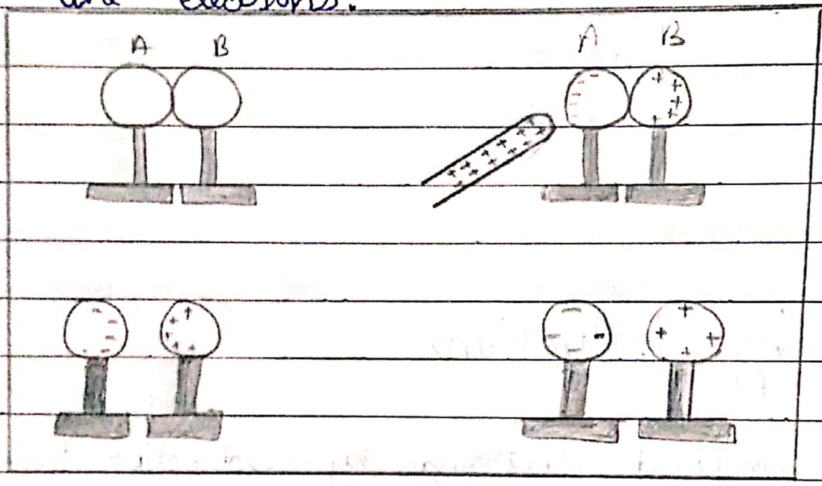
3- Charging by induction

Let us understand charging by induction through an example.

- Take two metallic spheres A and B (mounted on isulating stands) and bring them together.
- Now, bring a positively charged rod near the left end of sphere A (not touching).
- The +ve charge of the rod attracts the electrons of A as a result there is an excessive -ve charge on left side of A. At the same time, there is an accumulation of excessive positive charge on right side of sphere B due to repulsive forces.
- So, we see that at the end of this process, both spheres become charged. This process of charging is called charging by induction.

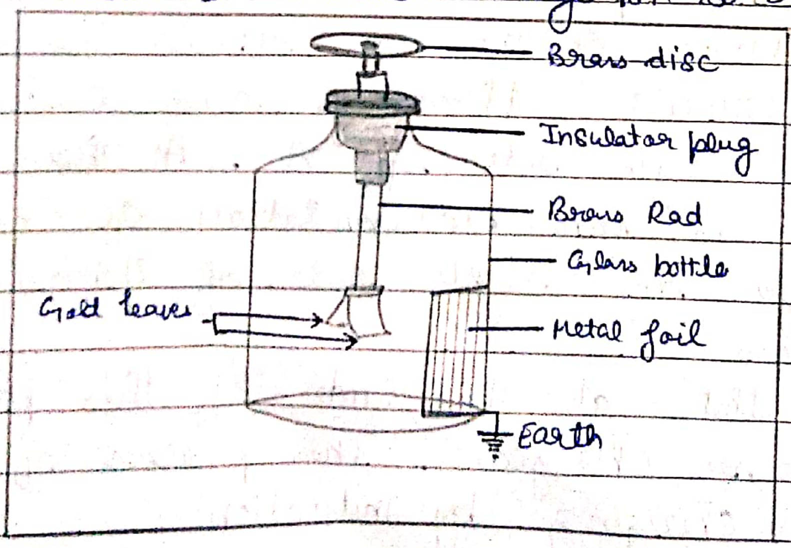
Note: 1. Charges on both spheres will be equal and opposite

2. Not all the e^- in the sphere accumulate on the side because as electrons keep getting accumulated the incoming electrons feel a strong force of repulsion from the already accumulated electrons. Over time equilibrium is set up under the force of attraction of the rod and force of repulsion of the electrons.



Gold Leaf Electroscope

→ Used to detect charge on a body



Working:

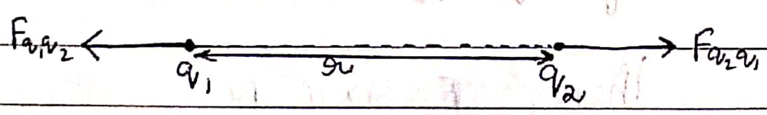
When a charged body is brought near or touched with the metal knob, charge travels to the leaves through the rod. Since both the leaves have the same charge they diverge (repel). The degree of divergence is an indicator of amount of charge.

Coulomb's Law

The force of attraction or repulsion between two stationary point charges is.

i-) directly proportional to the product of the magnitudes of the two charges, and

ii-) inversely proportional to the square of the distance b/w them. This force acts along the line joing the two charges.



i.e $F \propto |q_1q_2|$ & $F \propto \frac{1}{r^2}$

$\Rightarrow F \propto \frac{|q_1q_2|}{r^2} \Rightarrow F = \frac{K|q_1q_2|}{r^2}$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}}$$

K = electrostatic force constant

& $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, Nm^2/C^2$

Permittivity Of Free Space (ε_0)

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12} \, C^2 N^{-1} m^{-2}}$$

Unit Charges:

SI Unit ⇒ Coulomb (C)

$1 \mu C = 10^{-6} C$

$1 pC = 10^{-12} C$

pico

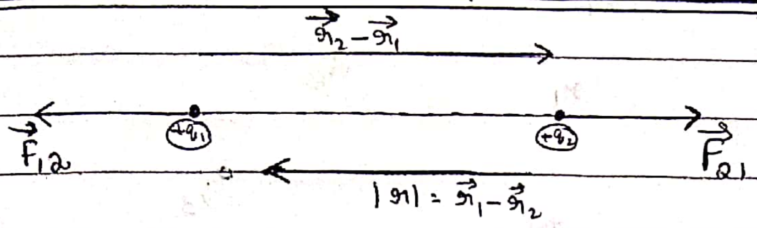
Defination of 1C ⇒

if $q_1 = q_2 = 1C$ & $r = 1m$

then, $F = 9 \times 10^9 \, N$

"1C is that amount of charge that repels an equal & similar charge with a force of $9 \times 10^9 N$ when placed in vacuum at a distance of 1m from it"

Coulomb's Law in Vector Form



unit vector

$$\vec{F}_{12} = \frac{Kq_1q_2}{|r|^2}\left[\widehat{(\vec{r}_1 - \vec{r}_2)}\right]$$

$$\left[\hat{A} = \frac{\vec{A}}{|A|}\right]$$

$$[|r| = \vec{r}_1 - \vec{r}_2]$$

$$\vec{F}_{12} = \frac{Kq_1q_2}{|r|^2}\left(\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}\right)$$

$$* \quad \boxed{\vec{F}_{12} = \frac{Kq_1q_2\,(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}}$$

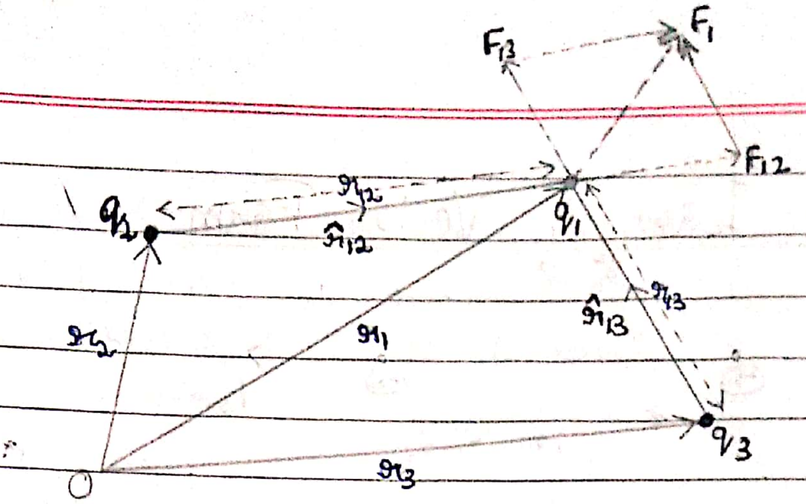
similarly

$$* \quad \boxed{\vec{F}_{21} = \frac{Kq_1q_2\,(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}}$$

Principal Of Superposition:

In a system of n charges the net force on any one charge is the vector sum of the forces due to all remaining charges on that charges.

* The force b/w any two charges is unaffected by the presence of any third charge.



(a)

Example 1.6 NCRT → Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, as shown in fig 1.9?

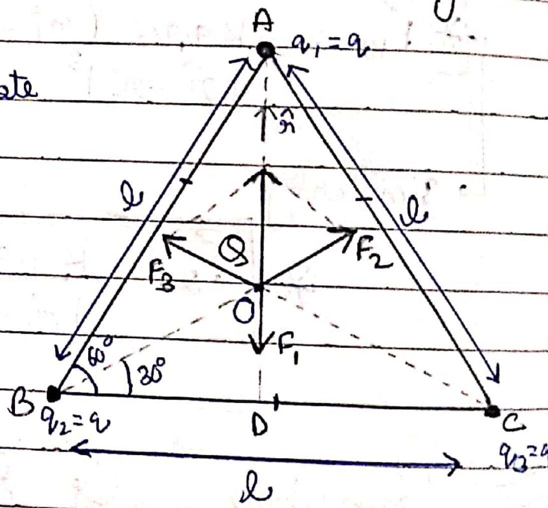


Fig 1.9

Sol^n → In a ΔABC, we can calculate

$AO = BO = CO = r$

$$\cos 30^\circ = \frac{BD}{BO}$$

$$\cos 30^\circ = \frac{l/2}{r}$$

$$r = \frac{l/2}{\cos 30} \Rightarrow \frac{l}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}} \Rightarrow \frac{l}{\sqrt{3}}$$

$$r = \frac{l}{\sqrt{3}}$$

$$AO = BO = CO = l/\sqrt{3}$$

Now, let us calculate the forces b/w the three charges at the vertices and charge Q at the centroid individually (principal of Superposition).

$$F_1 = \frac{Kq_1Q(-\hat{j})}{\left(\frac{l}{\sqrt{3}}\right)^2} \Rightarrow \frac{3Kq_1Q}{l^2}(-\hat{j})$$

$$F_2 = \frac{3Kq_1Q}{l^2}(\sin 60\hat{i} + \cos 60\hat{j})$$

$$F_3 = \frac{3Kq_1Q}{l^2}(-\sin 60\hat{i} + \cos 60\hat{j})$$

Now to calculate the net force on Q:

$$F_{net} = F_1 + F_2 + F_3$$

$$= \frac{3Kq_1Q}{l^2}(-\hat{j}) + \frac{3Kq_1Q}{l^2}(\sin 60\hat{i} + \cos 60\hat{j}) + \frac{3Kq_1Q}{l^2}(-\sin 60\hat{i} + \cos 60\hat{j})$$

$$\Rightarrow \frac{3Kq_1Q}{l^2}\left[-\hat{j} + \cancel{\sin 60\hat{i}} + \cos 60\hat{j} - \cancel{\sin 60\hat{i}} + \cos 60\hat{j}\right]$$

$$= \frac{3Kq_1Q}{l^2}\left[-\hat{j} + 2\cos 60\hat{j}\right]$$

$$= \frac{3Kq_1Q}{l^2}\left[-\hat{j} + \cancel{2} \times \frac{1}{\cancel{2}}\hat{j}\right]$$

$$\Rightarrow \frac{3Kq_1Q}{l^2}\left[-\hat{j} + \hat{j}\right] \Rightarrow 0$$

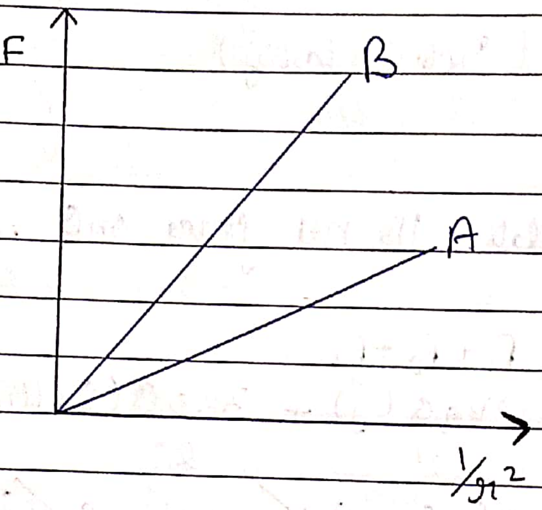
∴ We find the vector sum of the forces $F_1 + F_2 + F_3$ which amounts to zero.

⇒ We can also see by symmetry that net force on Q is zero.

Ques: Plot a graph showing variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance b/w the two charges of each pair of charges A(1μC, 2μC) and B(1μC, −3μC)
[PYQ 2011]

Solⁿ- We know,

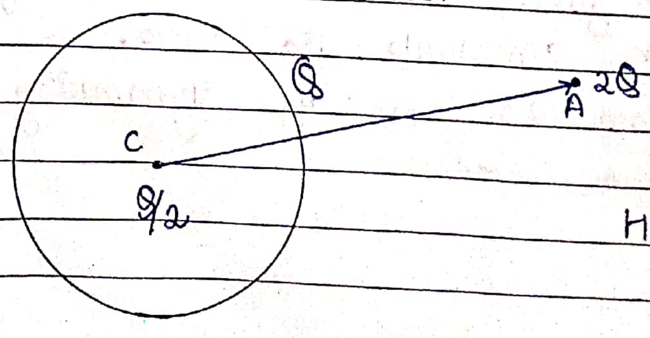
$$F \propto \frac{1}{r^2}, \text{ therefore}$$



Since, $F \propto q_1 q_2$, graph pair B will have greater slope, Than slope of pair A.

Ques- A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge Q/2 is placed at its centre C and another charge +2Q is placed at a distance r from the centre. Find the force on the charge Q/2 and 2Q. (PYQ 2015)

Do it Yourself question



Hint: field inside shell is zero

Dielectric Constant : Permittivity & Relative Permittivity

→ Permittivity :
* Permitivity is a property of the medium determines the electric force b/w two charges situated in that medium.
* It permits its own field to decrease the iteraction field b/w charges.

$$F_{vac} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \quad \text{——— (1)}$$

[ε_0 = permittivity of free space]

$$F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} \quad \text{——— (2)}$$

[ε = Absolute permittivity or permittivity of intervening medium]

Dividing eq^n (1) & (2).

$$\frac{F_{vac}}{F_{med}} = \frac{1/4\pi\varepsilon_0 \cdot \cancel{q_1q_2}/\cancel{r^2}}{1/4\pi\varepsilon \cdot \cancel{q_1q_2}/\cancel{r^2}}$$

$$\boxed{\frac{F_{vac}}{F_{med}} = \frac{\varepsilon}{\varepsilon_0}}$$

Note -

ε_0 → free space = $8.85 \times 10^{-12}\ C^2/Nm^2$ (minimum)

ε_w → water ⇒ $7.12 \times 10^{-10}\ C^2/Nm^2$

ε_{glass} (plexi.) → glass ⇒ $3.01 \times 10^{-11}\ C^2/Nm^2$

$$\boxed{\varepsilon_m > \varepsilon_0}$$

→ **Relative Permittivity (ε_r):**
(Dielectric Constant) It means that how many times the permittivity of medium is greater than permittivity of air.

Dielectric constant — $\boxed{\varepsilon_r = \dfrac{\varepsilon_m}{\varepsilon_0}}$ or $\boxed{\varepsilon_m = \varepsilon_r \times \varepsilon_0}$

or

$$\boxed{F_m = \frac{F_0}{\varepsilon_r}}$$

Proof → $F_0 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

$F_m = \dfrac{1}{4\pi\varepsilon_m}\dfrac{q_1 q_2}{r^2}$

$\dfrac{F_0}{F_m} = \dfrac{\varepsilon_m}{\varepsilon_0} = \varepsilon_r \Rightarrow$ Relative Permittivity $= \varepsilon_r$

$$\boxed{F_m = \frac{F_0}{\varepsilon_r}}$$

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eg → Q →

$q_1 \xleftrightarrow{\quad d \quad} q_2$ → vaccum ⇒ F

$q_1 \xleftrightarrow{\quad d \quad} q_2$ → liquid ⇒ F/4 Find ε_r?

$F = F_0 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{d^2}$

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$$\frac{F}{4} = F_m = \frac{1}{4\pi \varepsilon_m} \frac{q_1 q_2}{d^2}$$

Since, $\frac{F_0}{F_m} = \frac{\varepsilon_m}{\varepsilon_0} \Rightarrow \varepsilon_r$

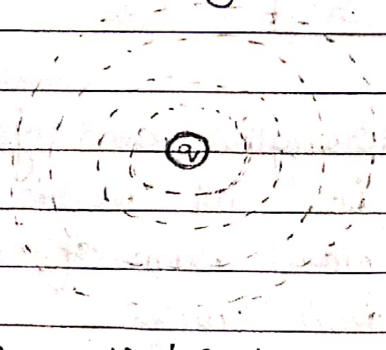
$$F_m = \frac{F_0}{\varepsilon_r}$$

$$\frac{F}{4} = \frac{F}{\varepsilon_r}$$

$$\boxed{\varepsilon_r = 4}$$

Electric Field

Electric field can be defined as the space around charge experience an electrostatic force of attraction and repulsion. An electric charge Q produces an electric field "everywhere" in its surrounding.

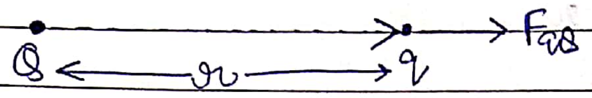


Electric field Strength / field intensity (E)

- Electric field strength at a point is defined as the force that a unit charge experiences when kept at that point.

- Mathematically, $\lim_{q \to 0} E = F/q$
- It is a vector quantity | SI unit - N/C or V/m

General expression for field strength.



Let us take a charge Q, at a distance r from it, there is another unit charge q, the force on q due to Q can be written as-

$$F = \frac{KQq}{r^2}$$

$$E = \frac{F}{q}$$

$$\boxed{E = \frac{KQ}{r^2}}$$

Note- Electric field strength also follows law of ~~sup~~ of superposition i.e. net electric field strength at a point is the vector sum of all field strengths due to individual charges.

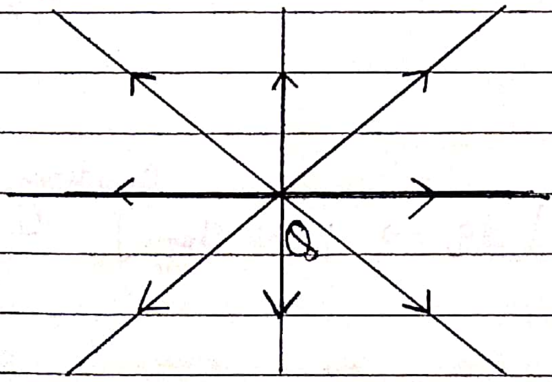
Characteristics of Electric field

1- Electric field at a point doesn't depend on charge q as the ratio F/q is independent of q.

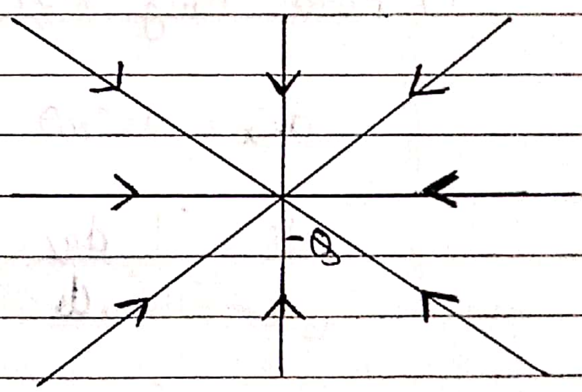
2- For a +ve point charge electric field is directed radially outwards.

3- For a negative point charge electric field directed radially inwards.

4- Magnitude of E due to a charge Q depends inversely on R^2, so at equal distances from the charge Q, E will have same magnitude i.e it shows radial symmetry.



radially outwards



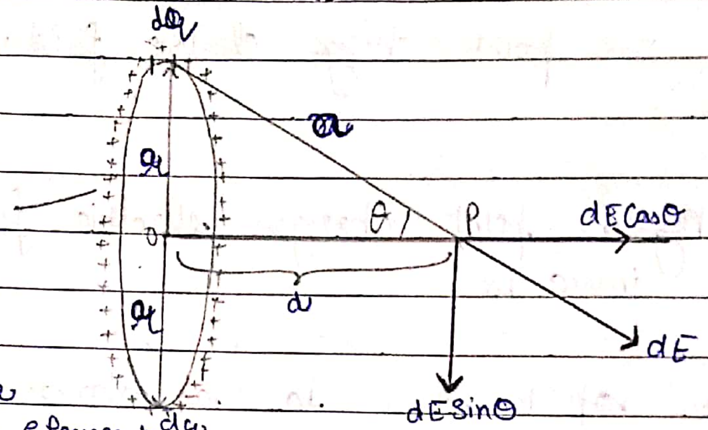
radially inwards

Ques. A charge is uniformly distributed over a ring of radius a. Obtain an expression for the electric field at its centre. Hence show that for large distances it behaves like a point charge. (PYQ 2016)

Soln

$\cos\theta = \frac{d}{a}$

Charge distributed



Consider a differential element dq of ~~length dl~~ And charge dq

$\cancel{\lambda = \frac{q}{2\pi a}}$

i.) Field along x-axis

$dq = \lambda dl$

$$dE_x = dE\cos\theta$$

$$dE = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{K}\frac{dq}{a^2} \qquad [dq \rightarrow \text{point charge}] \text{ Considering}$$

$$E_x = \int dE\cos\theta$$

$$= \int \frac{K\,dq\cos\theta}{a^2} \qquad \left[\cos\theta = \frac{d}{a}\right]$$

$$\Rightarrow \int \frac{K\,dq}{a^2} \times \frac{d}{a}$$

$$\Rightarrow \frac{Kd}{a^3}\int dq$$

$$E_x = \frac{KQd}{a^3}$$

$a^2 = r^2 + d^2$

$a = \sqrt{r^2+d^2}$

$$\boxed{E_x \Rightarrow \frac{KQd}{(r^2+d^2)^{3/2}}}$$

$$\boxed{E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qd}{(r^2+d^2)^{3/2}}}$$

2) Field along y-axis

$E_y = 0$ [Due to Symmetry)

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{Qd}{(r^2+d^2)^{3/2}}$$

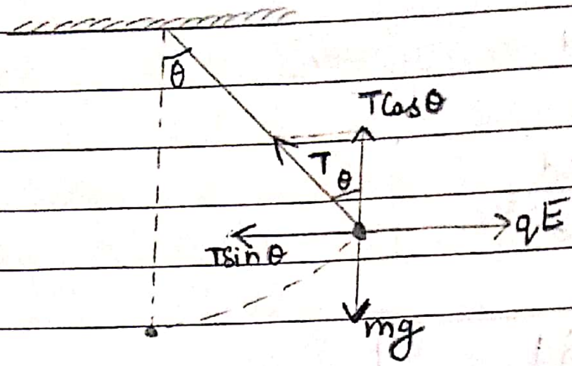
So, field at centre

$$\boxed{E_c = 0}$$

Ques. A pendulum of mass 80 milligram carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 2×10^{4} Vm^{-1}. Find the tension in the thread of the pendulum and the angle it makes with the vertical.

Sol^n- Given: $m = 80\,mg = 80\times10^{-6}$ Kg

$q = 2\times10^{-8}C$; $E = 2\times10^{4}\,Vm^{-1}$



Let T be the tension in the thread and θ be the angle it makes with vertical;
when bob is in equillibrium

$$T\sin\theta = qE \quad (i) \quad ; \quad T\cos\theta = mg \quad (ii)$$

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{qE}{mg}$$

$$\tan\theta = \frac{qE}{mg}$$

$$= \frac{2\times10^{-8}\times2\times10^{4}}{80\times10^{-6}\times9.8} = 0.51$$

$$\tan\theta = 0.51$$

$$\theta = 27^\circ$$

Also, $T = \frac{qE}{\sin\theta}$ from eq^n (i)

$$T = \frac{2\times10^{-8}\times2\times10^{4}}{\sin 27^\circ}$$

$$= 8.81\times10^{-4}\ N.$$

Ques- An electric falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0 \times 10^4\ NC^{-1}$ (Fig 1) The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig 2]. Compute the time of fall in each case. ~~Contrast the situation~~.

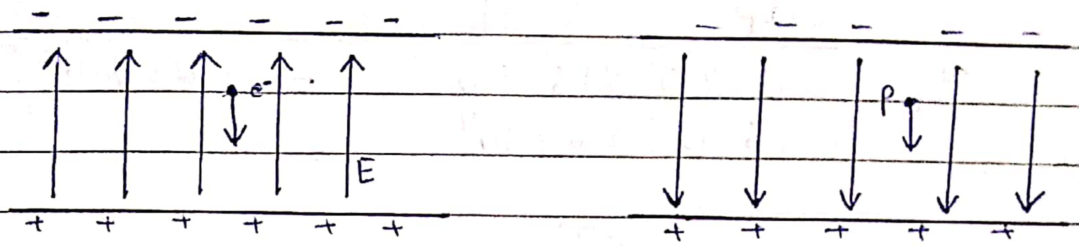


Fig 1

a-) Fig 1 The upward field exerts a downward force eE on the electron

$\therefore$ Acceleration of the electron, $a_e = \frac{eE}{m_e}$

As, $u=0$, $S = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$

$\therefore$ Time of fall on the electrons is

$$t_e = \sqrt{\frac{2S}{a_e}} = \sqrt{\frac{2Sm_e}{eE}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^{4}}}$$

$\boxed{t_e = 2.9 \times 10^{-9}\ s}$ Ans

b-) Fig 2- The downward field exerts a downward force eE on the proton

$$a_p = \sqrt{\frac{eE}{m_p}}$$

Tim of fall of the proton is

$$t_p = \sqrt{\frac{2s}{a_p}} = \sqrt{\frac{2sm}{eE}}$$

$$t_p = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 2 \times 10^{4}}} = 1.25 \times 10^{-7}$$

$$\boxed{t_p = 1.25 \times 10^{-7}}$$ Ans.

Electric Field Lines

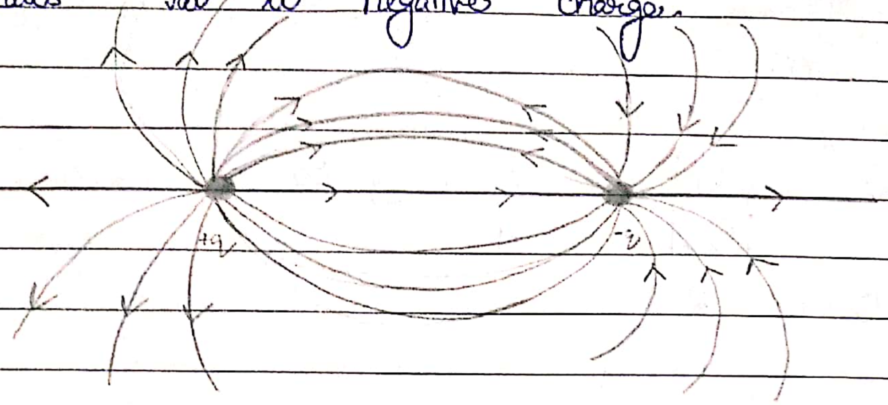
- The concept of electric field lines was given by Faraday to visualise the strength of electric field

- Electric field lines are imaginary, straight or curved lines around charged bodies such that tangent to it at a point gives direction of electric field at that point.

- It is the pictorial way of representing electric field around the charge.

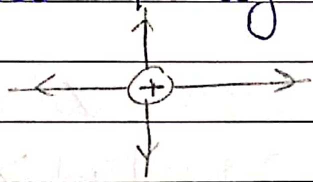
Properties Of Electric Field

1) Field lines orignate from a positive charge and terminates at a negative charge.

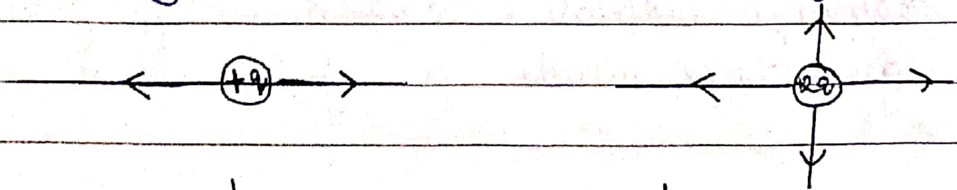


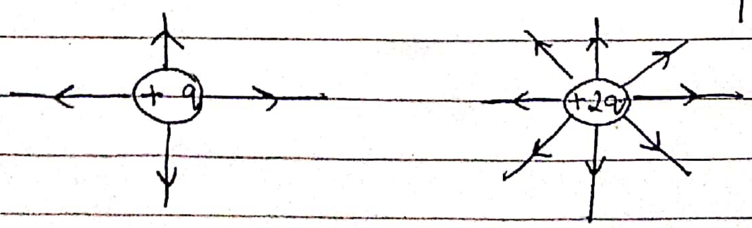
2) Electrostatic field lines do not form closed loops because of the conservative nature of electric field. (i.e work done by ~~electro~~· electric field depends on final and initial position and not path followed)

3) They do not have any breaks ; they are continuous in nature.

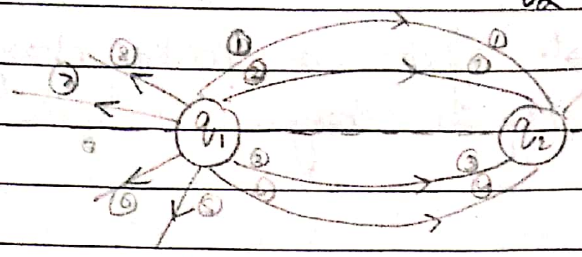


4) The no. of field lines from/ to a charge is directly proportional to magnitude of charge.

(1)


(2)


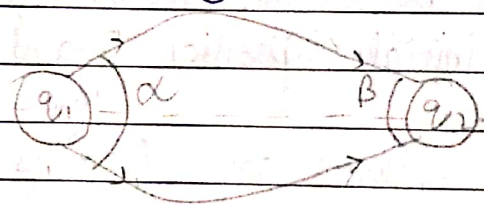
Q- Find the ratio of $\frac{q_1}{q_2}$



$$\frac{|q_1|}{|q_2|} = \frac{8}{4} \Rightarrow 2$$

$q_1 : q_2 = 2:1$

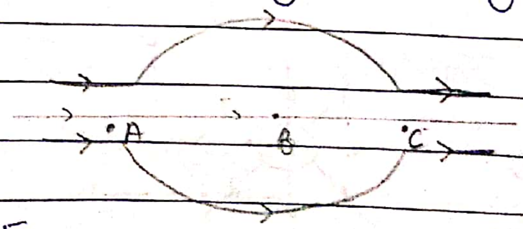
(6) The field lines are symmetric about the line joining two charges.



$\alpha > \beta$ — $|q_1|$ is less

so, $|q_2| > |q_1|$

(7) The density (relative closeness) of field line gives Magnitude of field.

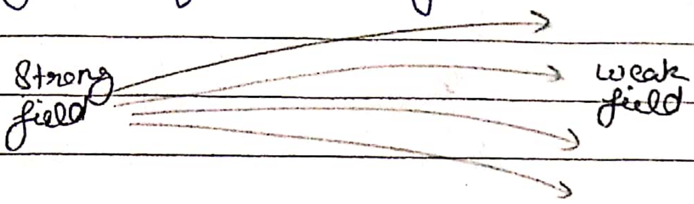


* $E_A > E_B$

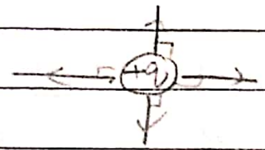
$E_C > E_B$

$E_A = E_C$

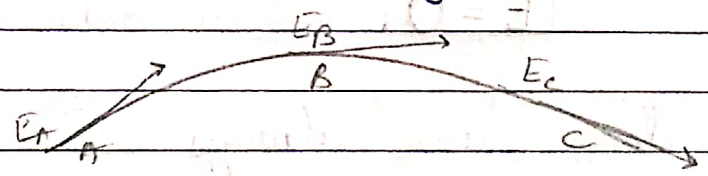
* greater is the density of field lines, greater is the strength of electric field in the region.



(8) The electric field lines are $\perp^{ar}$ to surface of charge

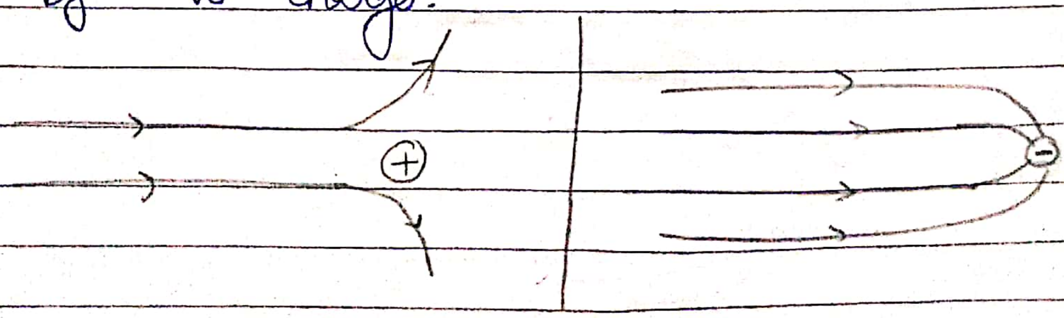


(9) The tangent drawn on a field lines gives the direction of $\vec{E}$ field at that point.

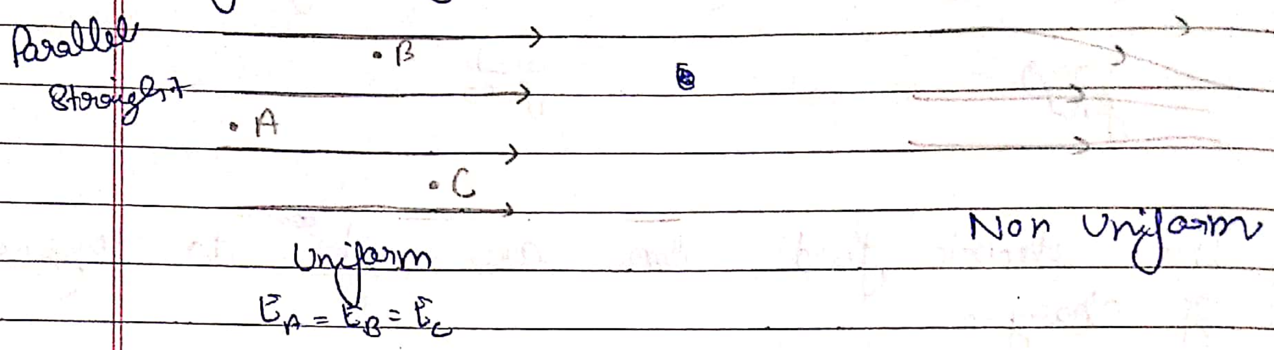


(10) Two field lines can never intersect each other (This is because at the point of interaction we can draw two tangents which means electric field at the point of intersection will have two direction, which is not possible.)

(11) Field lines repel by +ve charge / attracted by -ve charge.



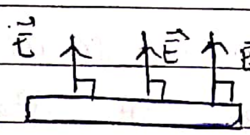
(12) Parallel, Straight, Equispace field lines represent uniform $\vec{E}$ field.



(13) Under electrostatic conditions $\vec{E}$ field inside a conductor is Absent.

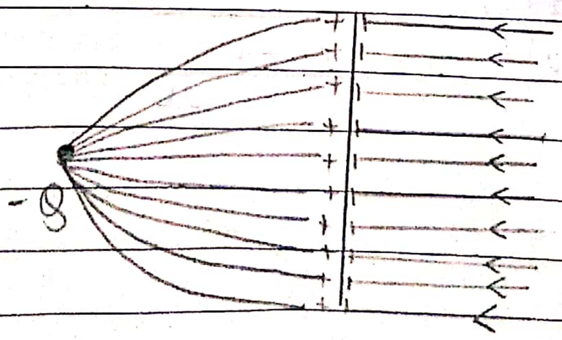
$\boxed{E = 0}$ electrostatic condition

(14) $\vec{E}$ field lines are always $\perp^{ar}$ to surface of a conductor



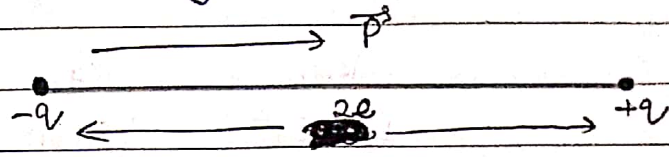
Ques- Draw a pattern of electric field lines, which a point charge $-Q$ is kept near an uncharged conducting plate.

Soln.



Electric Dipole

An electric dipole is an arrangement of 2 equal and opposite charges kept at some finite distance.



Dipole Moment

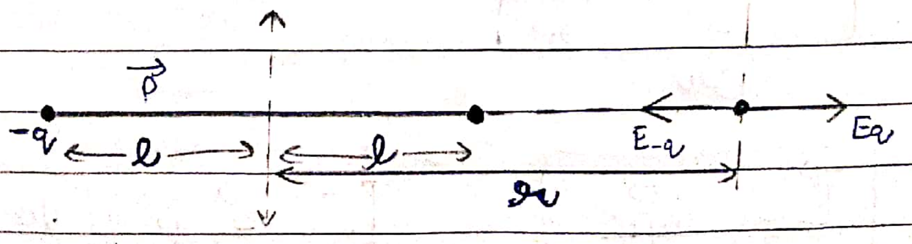
- The dipole moment (vector quantity) of an electric dipole measures the strength of the dipole.
- Its magnitude is equal to the product of the charge and the separation b/w the two charges.

$$\boxed{P = q.2l}$$

- Its direction is along the dipole axis, from the negative charge to the positive charge.

Electric Field due to dipole

a. Field on axial position / End on position (PYQ 2015)



$\vec{E}_1 = \frac{kq}{(r+l)^2}$, $\vec{E}_2 = \frac{kq}{(r-l)^2}$

$E_{net} = \vec{E}_2 - \vec{E}_1$

$\left[K = \frac{1}{4\pi\varepsilon_0} \right]$

$\Rightarrow \frac{kq}{(r-l)^2} - \frac{kq}{(r+l)^2}$

$\Rightarrow kq\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$

$\Rightarrow kq\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2(r+l)^2}\right]$

$\Rightarrow kq\left[\frac{\cancel{r^2} + \cancel{l^2} + 2rl - \cancel{r^2} - \cancel{l^2} + 2rl}{(r^2-l^2)^2}\right]$

$\Rightarrow kq\left[\frac{4rl}{(r^2-l^2)^2}\right]$

$\Rightarrow \frac{2kq(2l)r}{(r^2-l^2)^2}$

since $[r >>> l]$

$\Rightarrow \frac{2kpr}{r^4}$

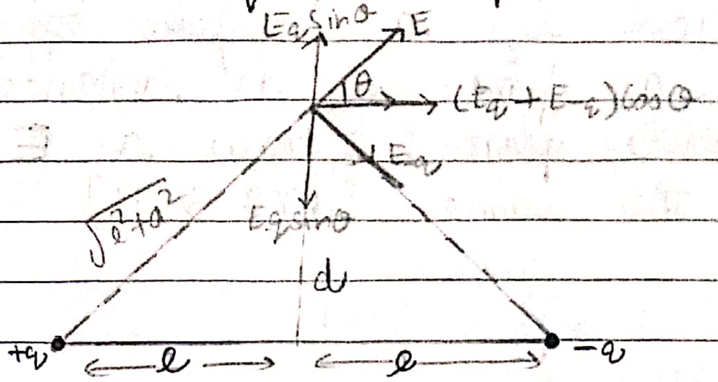
so, by neglecting l

$\boxed{E \Rightarrow \frac{2kp}{r^3}}$

or

$\boxed{\vec{E}_{axis} = \frac{2\vec{p}}{4\pi\varepsilon_0 \times r^3}}$ $[\because \vec{E} \text{ is } \parallel \text{ to } \vec{p}]$

b- Electric field on equatorial position / Broad on position (PYQ 2017)



$$E = E_1 = E_2 = \frac{Kq}{(\sqrt{l^2+d^2})^2} \Rightarrow \frac{Kq}{l^2+d^2}$$

$$E_{net} = E\cos\theta + E\cos\theta$$

$$\Rightarrow 2E\cos\theta$$

$$\Rightarrow 2 \times \frac{Kq}{l^2+d^2}\cos\theta$$

$$\left[\cos\theta = \frac{l}{\sqrt{l^2+d^2}}\right]$$

$$\Rightarrow \frac{2Kq}{l^2+d^2} \times \frac{l}{\sqrt{l^2+a^2}}$$

$$\Rightarrow \frac{2Kql}{(l^2+d^2)^{3/2}}$$

$$[d >>> l]$$

$$\boxed{E \Rightarrow \frac{Kp}{d^3}} \quad [p = 2ql]$$

Or

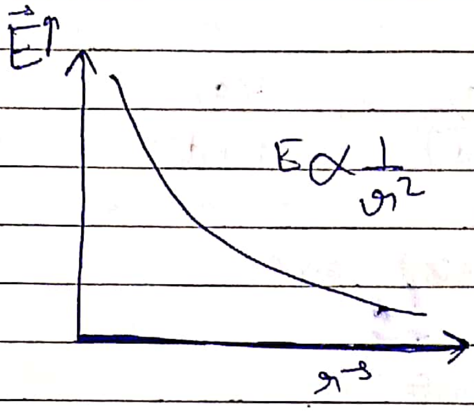
$$\boxed{\vec{E}_{Equa} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{d^3}}$$

($\vec{E}$ is antill^{ar} to $\vec{p}$)

Note ; $\boxed{E_{axis} = 2\,E_{quitarial}}$

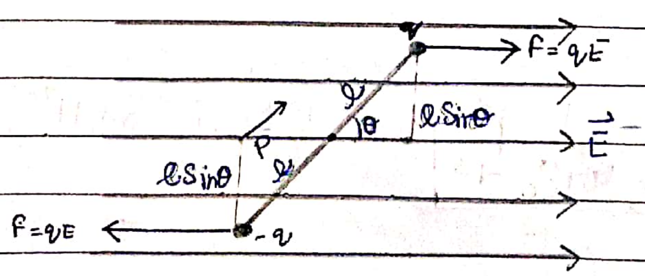
Ques. What is the expression for the electric field produced by a dipole of moment p at a point at a distance r in the equitorial plane? Draw a E v/s r graph for the same [PYQ 2015]

Solⁿ:
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{\vec{p}}{r^3}$$



Behaviour Of Dipole in external electric field.

Let us assume that a dipole of dipole moment P is kept in a uniform external field



As we can see from the figure, the net force on the dipole will be zero. But since line of action of the two forces is not the same, the dipole will experience a torque τ which can be written as,

$\boxed{F_{net} = 0}$ Always → Translation equilibrium

" Two equal & opposite forces which do not have same line of Action "
↳ It will rotate about Centre of Mass (COM) (Spin)

$\boxed{\tau = F \times \perp^{ar} \text{ distance}}$ Between F and axis of rotation

$$\vec{\tau} = \vec{\tau_1} + \vec{\tau_2}$$
$$= F \perp d + F \perp d$$
$$= qEl\sin\theta + qEl\sin\theta$$
(clockwise) (clockwise)

$$\Rightarrow 2qEl\sin\theta$$
$$= q(2l)E\sin\theta$$

$\boxed{p = q(2l)}$

$\boxed{\vec{\tau}_{n} = p \times E\sin\theta}$

or

$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

Points

- From this we can see that net ~~f~~ torque on the dipole is zero when it makes an ~~angle~~ angle of 0° (parallel) or 180° (antiparallel) with the field.

- When the dipole is parallel to the field, it is known as the position of stable equilibrium and when it anti-parallel, it is known as the position of unsable equilibrium.

- The torque on a dipole is maximum when it is perpendicular to the field. $\boxed{\tau = pE}$

Dipole in Non-Uniform field.

In a non-uniform field, the two charges $+q, -q$ ~~expier~~ experience different forces therefore net force is not zero

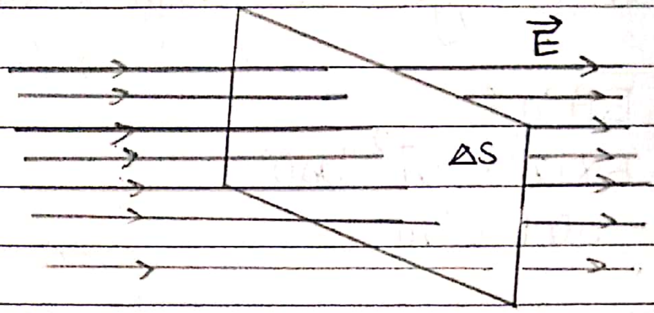
[illegible] Area vector

→ Direction of Area vector is ⊥ to plane surface

→ The direction of a planar area vector is specified by the normal to the plane.

Electric flux

Electric flux can be defined as the number of field lines crossing per unit ~~p~~ area, perpendicular to it.

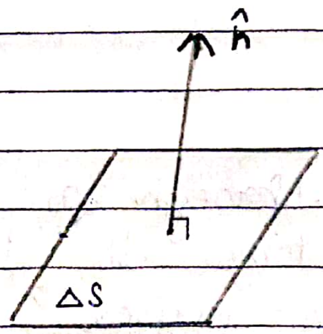


Mathematically

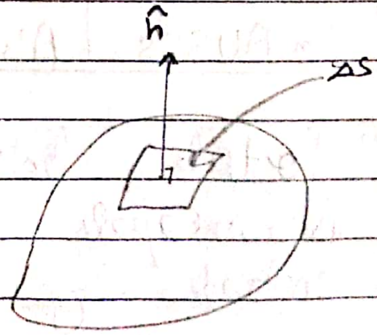
$$\boxed{\phi = E.A}$$

$= EA\cos\theta$ [when E is the field vector and A is the area vector)

Note: the direction of the area vector is along the normal to the area.



$\Delta S = \Delta S\hat{n}$



·Points

→ Electric field ↑ → Field Lines density ↑ $(\phi_E \propto A)$

→ Unit → $\frac{Nm^2}{C}$

Ques- Given a uniform electric field $E = 5 \times 10^3 \hat{i}$ N/C. Find flux of this field through a square of side 10cm whose plane is parallel to the y-z plane. What would be the flux through the same square if its plane makes an angle 30° with x-axis? [PYQ 2014]

Sol^n → i-)

$$\phi = EA\cos\theta$$
$$= 5 \times 10^3 \times 10^{-2} \cos 0°$$
$$= 50 \, NC^{-1} m^2$$

ii-) Since plane makes angle of 30°, normal will make angle of $90° - 30° = \underline{60°}$

$$\phi = EA\cos\theta$$
$$= 5 \times 10^3 \times 10^{-2} \cos 60°$$
$$= 25 \, NC^{-1} m^2$$

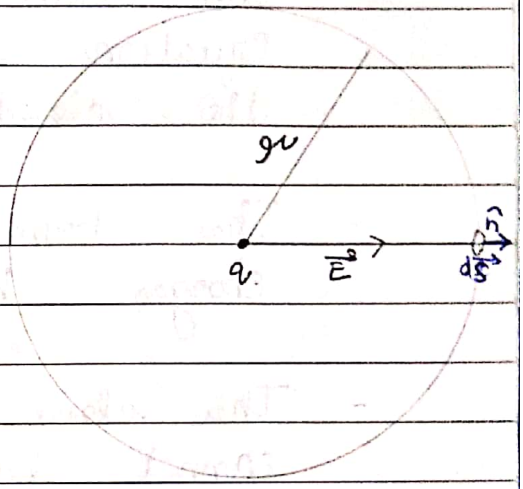
GAUSS's LAW

* "Total electric flux through a closed surface is equal to $1/\varepsilon_0$ times the total charge enclosed by the closed surface."

$$\boxed{\phi = \oint E.ds = \frac{q}{\varepsilon_0}}$$

Derivation:

Let us assume a sphere of radius r which encloses charges q. Consider differential area ds. The flux through ds can be written as:



$$\phi = \oint \vec{E}.d\vec{s}$$

$$= \oint E\,ds\cos 0^\circ$$

$$\cancel{= \frac{1}{4\pi\varepsilon_0}} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} \oint ds$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0}\frac{q}{\cancel{r^2}} \times \cancel{4\pi r^2}$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

$$\Rightarrow \boxed{\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}}$$

Note: if net charge enclosed by a ~~surf~~ surface is zero, then the flux through that surface is also zero $\left[\text{since } \phi = \frac{q}{\varepsilon_0}\right]$.

Important Points regarding Gauss's Law

- Gauss' Law is true for any closed surface irrespective of its shape.

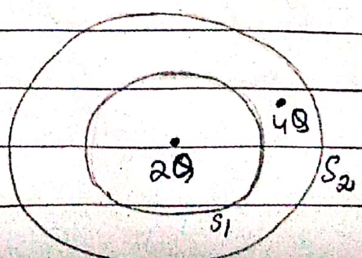
- The term q on the right side of the equation refers to all the charges inside the closed surface.

- The term E on the left side is due to all charges both inside and outside the surface.

- The close surface called Gausian surface and it cannot pass through a point charge. (can pass through continuous charge)

- Gauss law is based on inverse square dependence on distance as seen in Coulomb's law. Any violation of gauss law will result in departure from the inverse square law.

Ques. Consider two hollow concentric sphere S_1 and S_2 (l_2 sms) enclosing charge 2Q and 4Q resp. Find the ratio of flux through them. How will the flux through S_1 change if a medium of dielectric constant K is introduced in S_1? [PYQ 2014]



Ans- According to gauss law, $\phi = \frac{q}{\varepsilon_0}$

i-) $\phi_1 = \frac{2Q}{\varepsilon_0}$, $\phi_2 = \frac{6Q}{\varepsilon_0}$

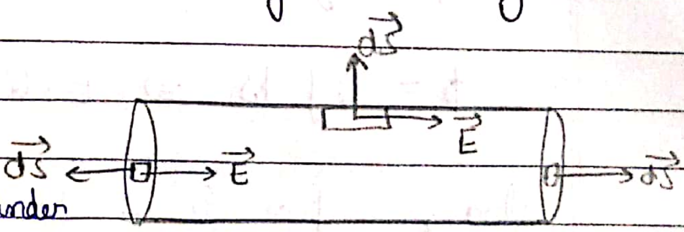
$\frac{\phi_1}{\phi_2} = \frac{1}{3}$

ii-) If a medium of dielectric constant K is introduced in S_1

$\varepsilon = K\varepsilon_0$

$\phi = \frac{2Q}{\varepsilon_0}$

Ques- A cylinder is placed in a uniform electric field $\vec{E}$ with its axis parallel to the field. Show that the total electric flux through the cylinder is zero.



Flux through the entire cylinder

$$\phi_E = \underset{\text{[left plane face]}}{\int \vec{E}.\vec{ds}} + \underset{\text{[right plane face]}}{\int \vec{E}\vec{ds}} + \underset{\text{[Curved surface]}}{\int \vec{E}\vec{ds}}$$

$$= \int E dS \cos 180^\circ + \int E ds \cos 0^\circ + \int E dS \cos 90^\circ$$

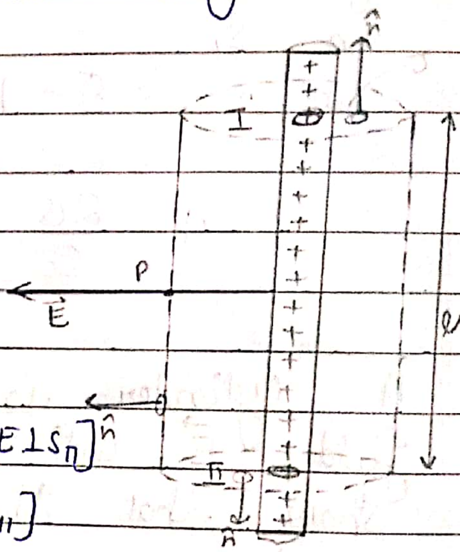
$$= -E\int dS + E\int dS + 0$$

$$= -E \times \pi r^2 + E \times \pi r^2 = 0$$

Application Of Gauss's Law

I-] Field due to an infinitely long straight uniformly charged wire [PYQ 2017]

Consider an infinite straight wire with uniform charge density, draw a cylinder Gaussian surface of radius r and length l around it



$$\phi = \oint \vec{E}.\vec{ds}$$

$$= \oint \vec{E}.\vec{ds}$$

$$= \int_I \cancel{\vec{E}\,\vec{ds}}^{\,0} + \int_{II} \cancel{\vec{E}\,\vec{ds}}^{\,0} + \int_{III} \vec{E}\,\vec{ds}$$

$[\because E \perp S_I, E \perp S_{II}]$

$$\Rightarrow \phi = \int E\, ds \cos 0^\circ \qquad [\because E \parallel S_{III}]$$

$$\Rightarrow \phi = E \int ds \Rightarrow E \times 2\pi r l \quad \text{——} (1)$$

also, $$\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \quad \text{——} (2)$$

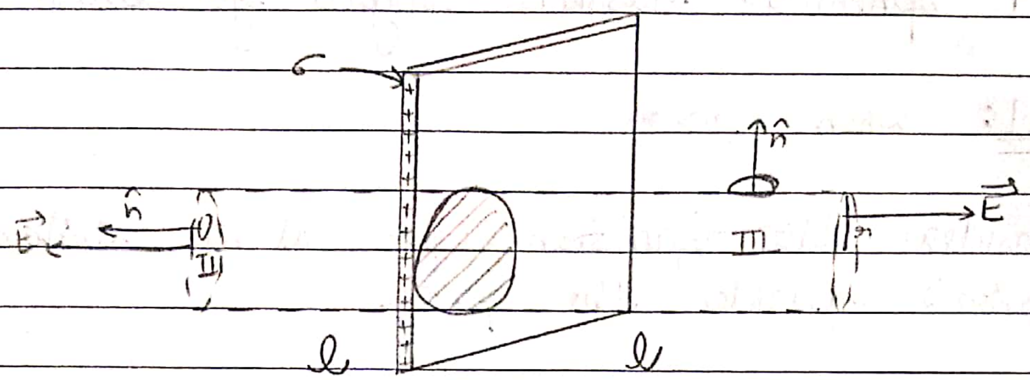
Equating 1,2

$$\Rightarrow E \times 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

II-] Field Due To Uniformly Charged Plate [PTU 2017]

Consider an infinite plane sheet with uniform charge density σ, draw a cylinder Gaussian surface of radius r and length $2l$ as shown



$$\phi = \oint \vec{E}\,d\vec{s} \qquad [\because E \perp S_{III}]$$

$$\phi = \int_{I} \vec{E}\,d\vec{s} + \int_{II} \vec{E}\cdot d\vec{s} + \cancel{\int_{III} \vec{E}\,d\vec{s}}^{\,0} \qquad [\because E \parallel S_I,\ E \parallel S_{II}]$$

$$= \int_{I} E\,ds\cos 0^\circ + \int_{II} E\,ds\cos 0^\circ$$

$$= E\int ds + E\int ds$$

$$\phi = 2E\pi r^2 \text{ ——— } (1)$$

Also, $$\phi = \frac{q}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} \text{ ——— } (2)$$

Equating 1, 2

$$\Rightarrow 2E\pi r^2 = \frac{\sigma\pi r^2}{\varepsilon_0}$$

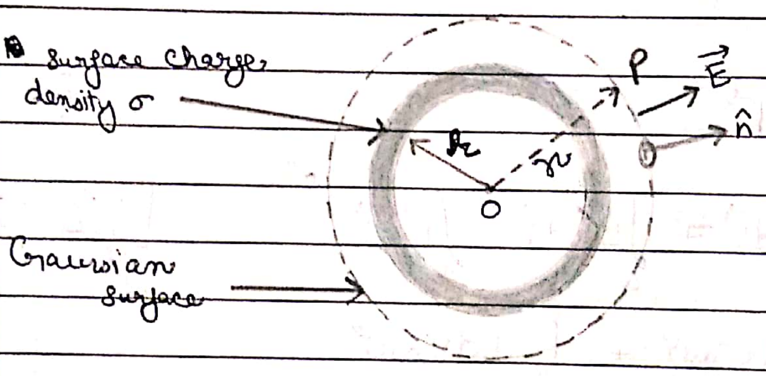
$$\Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

III Field Due To a Uniformly Charged Thin Spherical shell [PYQ 2020]

Consider a uniformly charge spherical shell of radius r with uniform charge density, draw a spherical Gaussian surface of radius x

Case 1: when $x > r$

Consider the Gaussian surface at a distance x (from centre) outside the sphere.



According to Gauss's law

$$\phi = \oint \vec{E}.\vec{ds}$$

$$\phi = \oint E ds \cos 0^\circ = E \oint ds \qquad [\because \vec{E} \parallel \hat{n}]$$

$$\phi = E.4\pi x^2 \quad \text{———} (1)$$

also,

$$\phi = \frac{q}{\varepsilon_0} = \frac{\sigma 4\pi r^2}{\varepsilon_0} \quad \text{———} (2)$$

Equating ① & ②

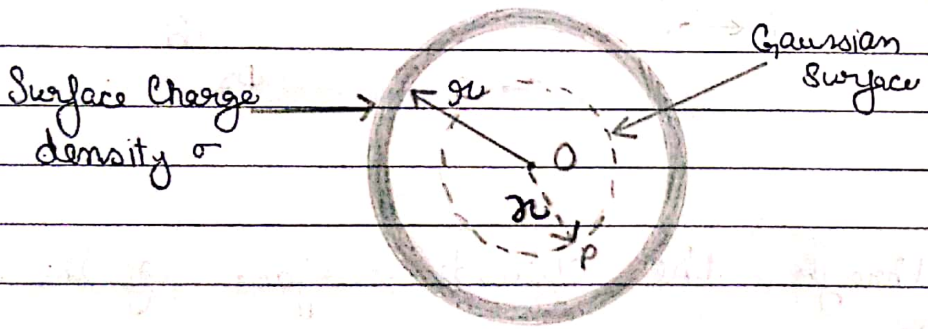
$\Rightarrow E \cdot 4\pi r^2 = \frac{\sigma 4\pi R^2}{\varepsilon_0}$

$\Rightarrow \boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$

For $r = R$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Case 2: When $r < R$



Consider the Gaussian surface inside the sphere. As show in the previous part

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Charge enclosed in this Gaussian surface is 0, therefore

$$\underline{E = 0}$$

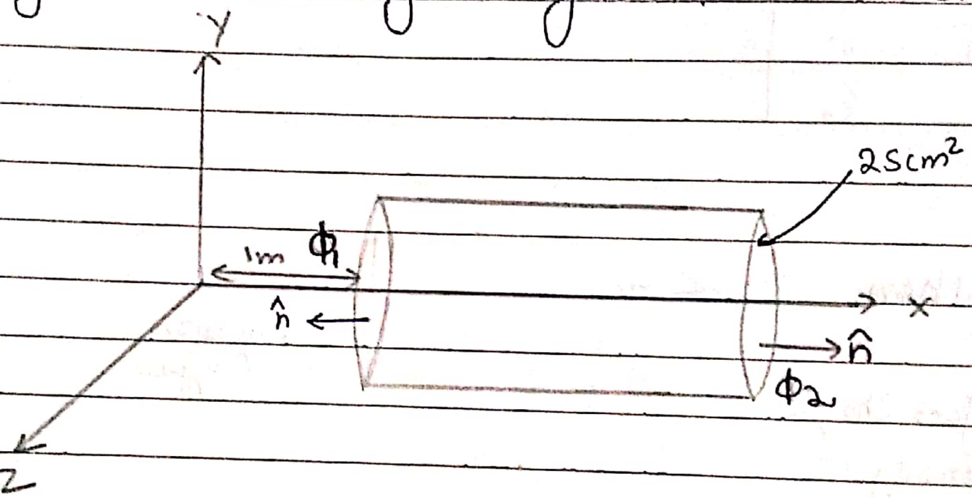
Ques- A hollow cylindrical box of length 1m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the fig. The electric field in the region is given by $E = 50x\hat{i}$, where E is in NC^{-1} and x is in meters.

Find.

i) Net flux through the cylinder

ii) Charge enclosed by the cylinder. [PYQ 2013]

Ans



i) Flux through the curved surface of the cylinder is zero.

→ Magnitude of the electric field at the left surface.

$$E = 50 \times 1 = 50 NC^{-1}$$

Flux through the left face,

$$\phi_L = ES\cos\theta = 50 \times 25 \times 10^{-4} \cos 180^\circ$$
$$= -1250 \times 10^{-4}\ Nm^2C^{-1}$$

→ Magnitude of the electric field at the right face,

$$E = 50 \times 2 = 100\, NC^{-1}$$

∴ Flux through the right face,

$$\phi_R = 100 \times 25 \times 10^{-4} \cos 0^\circ$$
$$= 2500 \times 10^{-4}\, Nm^2 C^{-1}$$

→ Net flux through the cylinder,

$$\phi_E = \phi_L + \phi_R = (2500 - 1250) \times 10^{-4}\, Nm^2 C^{-1}$$
$$= 1250 \times 10^{-4}\, Nm^2 C^{-1}$$
$$= 1.250 \times 10^{-1}\, Nm^2 C^{-1}$$

ii) Total charge enclosed by the cylinder

$$q = \varepsilon_0 \phi_E$$
$$= 8.854 \times 10^{-12} \times 1250 \times 10^{-4} C$$
$$= 11067.5 \times 10^{-16} C = 1.107\, pC.$$