$1+2+3+5 = 11$m

THREE DIMENSIONAL GEOMETRY

If a directed line L passing through origin makes angles α, β, γ with x, y, z axes respectively, then the angles are direction angles and cosine of these angles ~~ar~~ i.e $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called DC's of L.

If we reverse the direction of L, the direction angles are replaced by their suplements i.e $\pi-\alpha, \pi-\beta, \pi-\gamma$ and thus signs of DC's are reversed.

The 3 numbers proportional to DC's of a line are called DR's a, b, c and thus $\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda$

$$\begin{bmatrix} a, b, c \rightarrow \text{DR's} \\ l, m, n \rightarrow \text{dc's} \end{bmatrix}$$

Identities:

- $l^2 + m^2 + n^2 = 1$

  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

  $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

- If two points $A(x, y_1, z_1)$, $B(x_2\, y_2\, z_2)$ are given on a line L, then DR's of the line are $(x_2 - x_1)$ $(y_2 - y_1)$, $(z_2 - z_1)$
- Collinearity of 3 points:

  To prove –

$$\left.\begin{array}{l} \text{i] } \cancel{a}\ \vec{AB} \parallel \vec{BC} \\ \text{ii] } B \text{ is common point} \end{array}\right\} \Rightarrow A, B, C \text{ are collinear}$$

EXERCISE 11.1

Eg 1] If a line makes ∠'s 90°, 60°, 30° with +ve direction of x, y, z axis. Find its DC's

Sol] $l = \cos\alpha = \cos 90° = 0$

$m = \cos 60° = \frac{1}{2}$

$n = \cos 30° = \frac{\sqrt{3}}{2}$

1] If a line makes ∠'s 90°, 135°, 45° with x, y, z axis. Find DC

Sol] $l = \cos\alpha = \cos 90° = 0$

$m = \cos\beta = \cos 135° = \frac{-1}{\sqrt{2}}$ $\cos(180-45) = -\cos 45°$

$n = \cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

Eg 2] If a line has DR's $(2,-1,-2)$. Find its DC's

Sol] ~~DC's are~~ :

Consider : $\sqrt{4+1+4} = 3$

∴ DC's are : $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

** Eg 3] Find DC's of line passing through $(-2,4,-5),(1,2,3)$

Sol] $A \equiv (-2,4,-5)$, $B \equiv (1,2,3)$

Dr's $\equiv (x_2-x_1),(y_2-y_1),(z_2-z_1) = 3,-2,8$

consider $\sqrt{9+4+64} = \sqrt{77}$

∴ DC's are : $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

** (2M) 2] Find DC's of a line which makes equal angles with the coordinate axis.

Sol] Gn : equal angles $\Rightarrow \alpha = \beta = \gamma$

also $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$\cos^2\alpha = \frac{1}{3}$ $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

∴ DC's are : $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

3] If a line has DR's $-18, 12, -4$, what are its DC's ?

Sol] Consider $\sqrt{(18)^2+(12)^2+(4)^2} = 22$

∴ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Eg 5] ST the pt's $A(2,3,-4)$, $B(1,-2,3)$, $C(3,8,-11)$ are collinear.

Sol] DR's of $\vec{AB}$: $1-2, -2-3, 3+4$ i.e $-1, -5, 7$

DR's of $\vec{BC}$: $3-1, 8+2, -11-3$ i.e $2, 10, -14$

Clearly : $\frac{2}{-1} = \frac{10}{-5} = \frac{-14}{7} = -2$

∵ DR's of $\vec{AB}$ and $\vec{BC}$ are proportional.

$\vec{AB} \parallel \vec{BC}$ but pt. B is common

∴ A, B, C are collinear.

4] ST the pt's $A(2,3,4)$, $B(-1,-2,1)$ $C(5,8,7)$ are collinear.

Sol] DR's of $\vec{AB}$: $-1-2, -2-3, 1-4$ i.e $-3, -5, -3$

DR's of $\vec{BC}$: $5+1, 8+2, 7-1$ i.e $6, 10, 6$

Clearly $\frac{6}{-3} = \frac{10}{-5} = \frac{6}{-3} = -2$

∵ DR's of $\vec{AB}$ and $\vec{BC}$ are proportional

$\vec{AB} \parallel \vec{BC}$ but pt. B is common

∴ A, B, C are collinear.

5] Find the DC's of sides of the triangle whose vertices are $\overset{A}{(3,5,-4)}, \overset{B}{(-1,1,2)}$ and $\overset{C}{(-5,-5,-2)}$

Sol] $\bar{AB}$: DR's ≡ $-4, -4, 6$; $\sqrt{16+16+36} = \sqrt{68} = 2\sqrt{17}$

DC's of $\bar{AB}$ are : $\frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$ ≡ $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$

$\overline{BC}$: DR's $\equiv -4, -6, -4$; $\sqrt{16+36+16} = \sqrt{68} = 2\sqrt{17}$

DC's of $\overline{AB}$ are : $\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}} \equiv \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

$\overline{CA}$: DR's $\equiv +8, +10, -2$; $\sqrt{64+100+4} = \sqrt{168}$

DC's of $\overline{AC}$ are : $\frac{+8}{\sqrt{168}}, \frac{+10}{\sqrt{168}}, \frac{-2}{\sqrt{168}}$

$\equiv \frac{+8}{2\sqrt{42}}, \frac{+10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}} = \frac{+4}{\sqrt{42}}, \frac{+5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

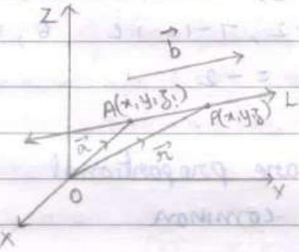
Equation of a line in space :

NOTE : A line is uniquely determined if

(a) it passes through a gn pt and has a gn direction $\left(\vec{r} = \underset{pt}{\vec{a}} + \lambda \underset{\parallel^{le}\text{ vector}}{\vec{b}}\right)$

(b) it passes through Two gn pt.s $[\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})]$

I Equation of a line passing through a point and $\parallel^{le}$ to a given vector $\vec{b}$.



Let the line L pass through the gn pt $A(x_1, y_1, z_1)$ with PV $\vec{a}$ and $\vec{B}$, be parallel to the gn vector $\vec{b}$.

Let $P(x, y, z)$ be any arbitrary point on the line L with PV $\vec{r}$.

Now L is parallel to $\vec{b}$

$\Rightarrow \vec{AP} \parallel \vec{b}$

$\Rightarrow \vec{AP} = \lambda\vec{b}$

$\vec{OP} - \vec{OA} = \lambda\vec{b}$

$$\vec{r} - \vec{a} = \lambda \vec{b}$$

Vector form : $\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$

Cartesian form : $\vec{r} = xi + yj + zk$; $\vec{a} = x_1 i + y_1 j + z_1 k$

Let $\vec{b} = ai + bj + ck$

Substituting these in the vector form,

$$xi + yj + zk = (x_1 i + y_1 j + z_1 k) + \lambda(ai + bj + ck)$$

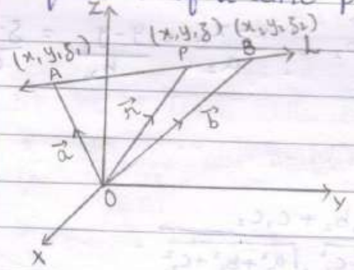
$$(x - x_1)i + (y - y_1)j + (z - z_1)k = \lambda(ai + bj + ck)$$

Equating corresponding components,

$x - x_1 = \lambda a$	$y - y_1 = \lambda b$	$z - z_1 = \lambda c$
$\frac{x - x_1}{a} = \lambda$	$\frac{y - y_1}{b} = \lambda$	$\frac{z - z_1}{c} = \lambda$

Cartesian form : $\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$

II Equation of a line passing through two points.



Let $\vec{a}$ and $\vec{b}$ be PV's of two given pt.s $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ lieing on a line L.

Let $\vec{r}$ be the PV of any arbitrary pt. $P(x, y, z)$.

P is a point on the line if and only if $\vec{AP}$ and $\vec{AB}$ are collinear vectors.

$\Rightarrow \vec{AP} \parallel \vec{AB}$

$\Rightarrow \vec{AP} = \lambda \vec{AB}$

$$\vec{OP} - \vec{OA} = \lambda(\vec{OB} - \vec{OA})$$

$$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$$

Vector form : $\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$

Let $\vec{r} = xi + yj + zk$, $\vec{a} = x_1i + y_1j + z_1k$, $\vec{b} = x_2i + y_2j + z_2k$

Substituting these in vector form,

$xi + yj + zk = (x_1i + y_1j + z_1k) + \lambda((x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k]$

$(x - x_1)i + (y - y_1)j + (z - z_1)k = \lambda[(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k]$

Equating the corresponding components

$x - x_1 = \lambda(x_2 - x_1)$	$y - y_1 = \lambda(y_2 - y_1)$	$z - z_1 = \lambda(z_2 - z_1)$
$\frac{x - x_1}{x_2 - x_1} = \lambda$	$\frac{y - y_1}{y_2 - y_1} = \lambda$	$\frac{z - z_1}{z_2 - z_1} = \lambda$

Eliminating the parameter λ,

Cartesian form : $\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$

Angle between two lines:

Formulae: $L_1 : \vec{r} = \underset{pt}{\vec{a}_1} + \lambda\vec{b}_1$; $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ → pts → $\vec{b}_1$

$l_2 : \vec{r} = \vec{a}_2 + \lambda\vec{b}_2$; $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ → pts → $\parallel^{le}$ vector DR's

Vector form: $\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

Cartesian form: $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

* If $\theta = 90°$, the two lines are $\perp^{le}$

Condition is : $\vec{b}_1 \cdot \vec{b}_2 = 0 \Rightarrow l_1 \perp^{le} l_2$

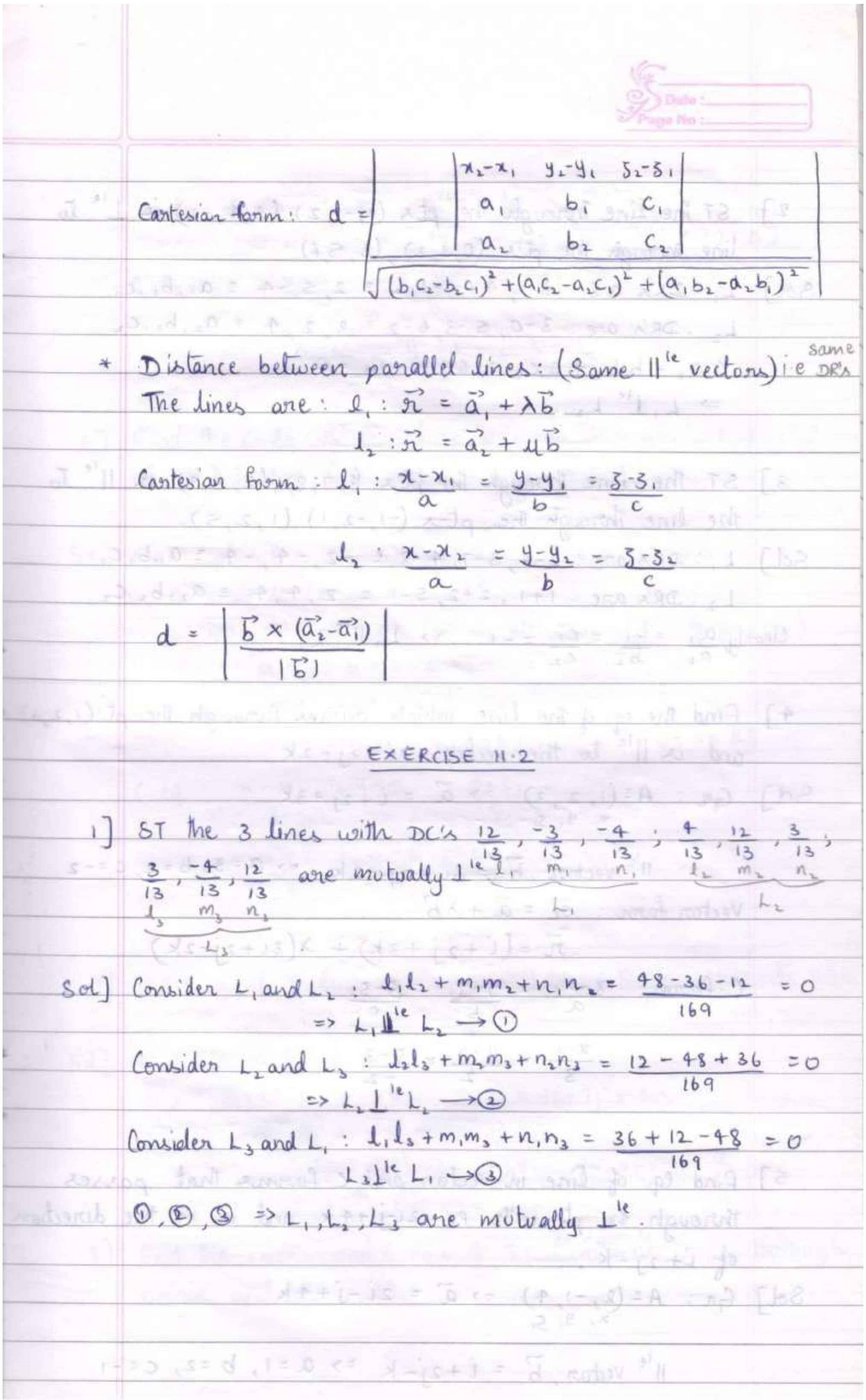
(or) $a_1a_2 + b_1b_2 + c_1c_2 = 0 \Rightarrow l_1 \perp^{le} l_2$

* If $\theta = 0°$, $l_1 \parallel^{le} l_2$

Conditions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda \Rightarrow l_1 \parallel^{le} l_2$

* Shortest distance between two skew lines:

vector form : $SD = d = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$



Cartesian form: $d = \left| \dfrac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(a_1c_2-a_2c_1)^2+(a_1b_2-a_2b_1)^2}} \right|$

* Distance between parallel lines: (Same $\parallel^{le}$ vectors) i.e same DR's

The lines are: $l_1 : \vec{r} = \vec{a}_1 + \lambda\vec{b}$

$l_2 : \vec{r} = \vec{a}_2 + \mu\vec{b}$

Cartesian form: $l_1 : \dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

$l_2 : \dfrac{x-x_2}{a} = \dfrac{y-y_2}{b} = \dfrac{z-z_2}{c}$

$d = \left| \dfrac{\vec{b} \times (\vec{a}_2-\vec{a}_1)}{|\vec{b}|} \right|$

EXERCISE 11.2

1] ST the 3 lines with DC's $\underbrace{\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}}_{\substack{l_1 \; m_1 \; n_1 \\ L_1}}$; $\underbrace{\frac{4}{13}, \frac{12}{13}, \frac{3}{13}}_{\substack{l_2 \; m_2 \; n_2 \\ L_2}}$; $\underbrace{\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}}_{\substack{l_3 \; m_3 \; n_3 \\ L_3}}$ are mutually $\perp^{le}$

Sol] Consider L_1 and L_2 : $l_1l_2 + m_1m_2 + n_1n_2 = \dfrac{48-36-12}{169} = 0$

$\Rightarrow L_1 \perp^{le} L_2 \rightarrow$ ①

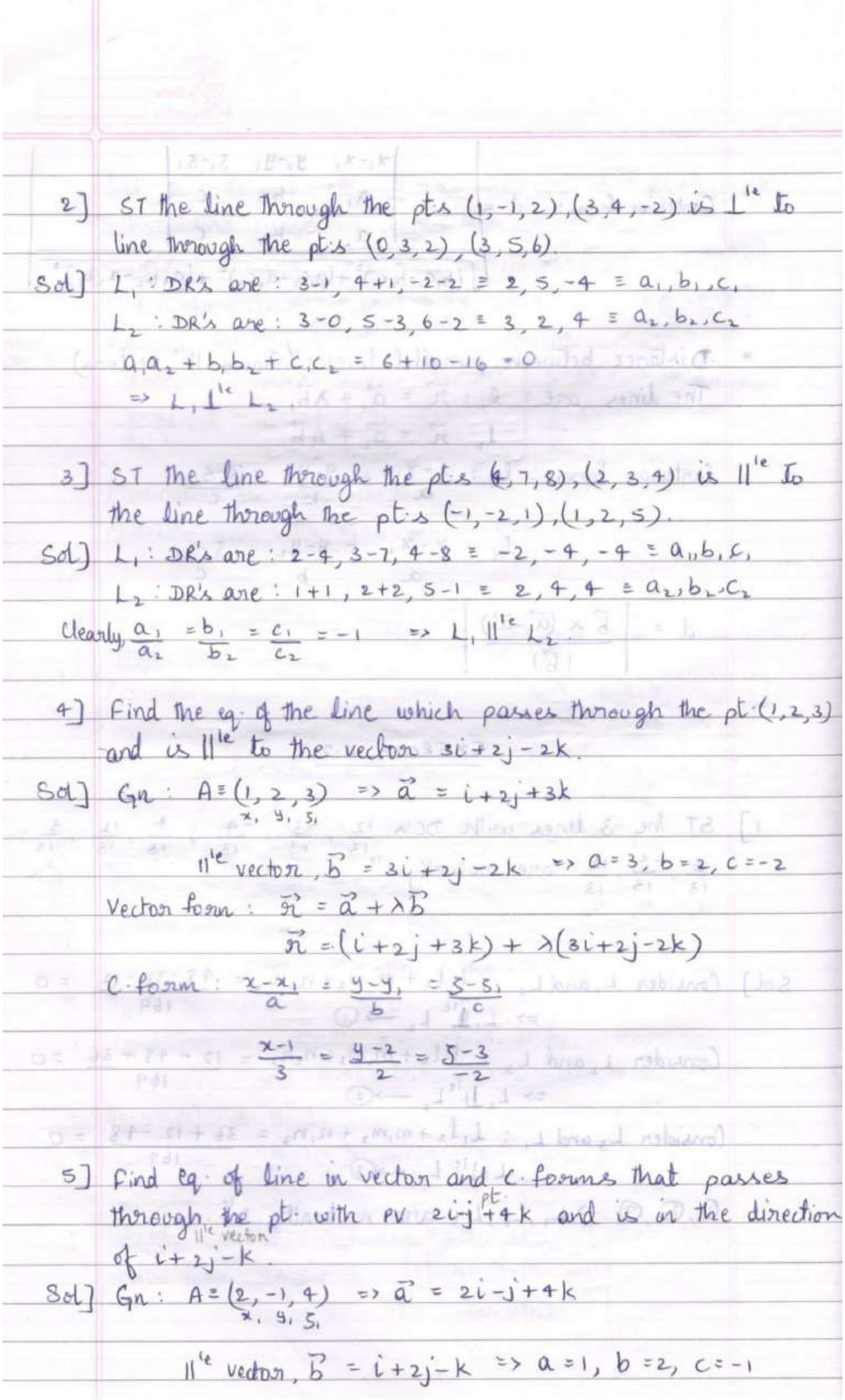
Consider L_2 and L_3 : $l_2l_3 + m_2m_3 + n_2n_3 = \dfrac{12-48+36}{169} = 0$

$\Rightarrow L_2 \perp^{le} L_2 \rightarrow$ ②

Consider L_3 and L_1 : $l_1l_3 + m_1m_3 + n_1n_3 = \dfrac{36+12-48}{169} = 0$

$L_3 \perp^{le} L_1 \rightarrow$ ③

①, ②, ③ $\Rightarrow L_1, L_2, L_3$ are mutually $\perp^{le}$.



2] ST the line through the pts $(1,-1,2),(3,4,-2)$ is $\perp^{le}$ to line through the pts $(0,3,2),(3,5,6)$.

Sol] L_1: DR's are: $3-1,\ 4+1,\ -2-2 \equiv 2, 5, -4 \equiv a_1, b_1, c_1$

L_2: DR's are: $3-0,\ 5-3,\ 6-2 \equiv 3, 2, 4 \equiv a_2, b_2, c_2$

$a_1a_2 + b_1b_2 + c_1c_2 = 6 + 10 - 16 = 0$

$\Rightarrow L_1 \perp^{le} L_2$.

3] ST the line through the pts $(4,7,8),(2,3,4)$ is $\|^{le}$ to the line through the pts $(-1,-2,1),(1,2,5)$.

Sol] L_1: DR's are: $2-4,\ 3-7,\ 4-8 \equiv -2, -4, -4 \equiv a_1, b_1, c_1$

L_2: DR's are: $1+1,\ 2+2,\ 5-1 \equiv 2, 4, 4 \equiv a_2, b_2, c_2$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -1 \Rightarrow L_1 \|^{le} L_2$

4] Find the eq. of the line which passes through the pt $(1,2,3)$ and is $\|^{le}$ to the vector $3i+2j-2k$.

Sol] Gn: $A \equiv (\underset{x_1}{1}, \underset{y_1}{2}, \underset{z_1}{3}) \Rightarrow \vec{a} = i+2j+3k$

$\|^{le}$ vector, $\vec{b} = 3i+2j-2k \Rightarrow a=3,\ b=2,\ c=-2$

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

$$\vec{r} = (i+2j+3k) + \lambda(3i+2j-2k)$$

C·form: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

$$\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$$

5] Find eq. of line in vector and C·forms that passes through the $\underset{\|^{le}\text{ vector}}{\text{pt·}}$ with PV $2i-j\overset{pt.}{+}4k$ and is in the direction of $i+2j-k$.

Sol] Gn: $A \equiv (\underset{x_1}{2}, \underset{y_1}{-1}, \underset{z_1}{4}) \Rightarrow \vec{a} = 2i-j+4k$

$\|^{le}$ vector, $\vec{b} = i+2j-k \Rightarrow a=1,\ b=2,\ c=-1$

Vector form : $\vec{r} = \vec{a} + \lambda \vec{b}$

$= (2i - j + 4k) + \lambda(i + 2j - k)$

C. form : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

6] Find the C. eq. of the line passing through (-2, 4, -5) and ||'e to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Sol] Gn : pt ≡ (-2, 4, -5) => $\vec{a} = -2i + 4j - 5k$

||'e ~~vecto~~ to => same DR's

∴ $\vec{b} = 3i + 5j + 6k$ (using denominator of gn eq)

=> a = 3, b = 5, c = 6

V. form : $\vec{r} = \vec{a} + \lambda \vec{b}$

$= (-2i + 4j - 5k) + \lambda(3i + 5j + 6k)$

C eq : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

=> $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

7] The C eq of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ write its V form

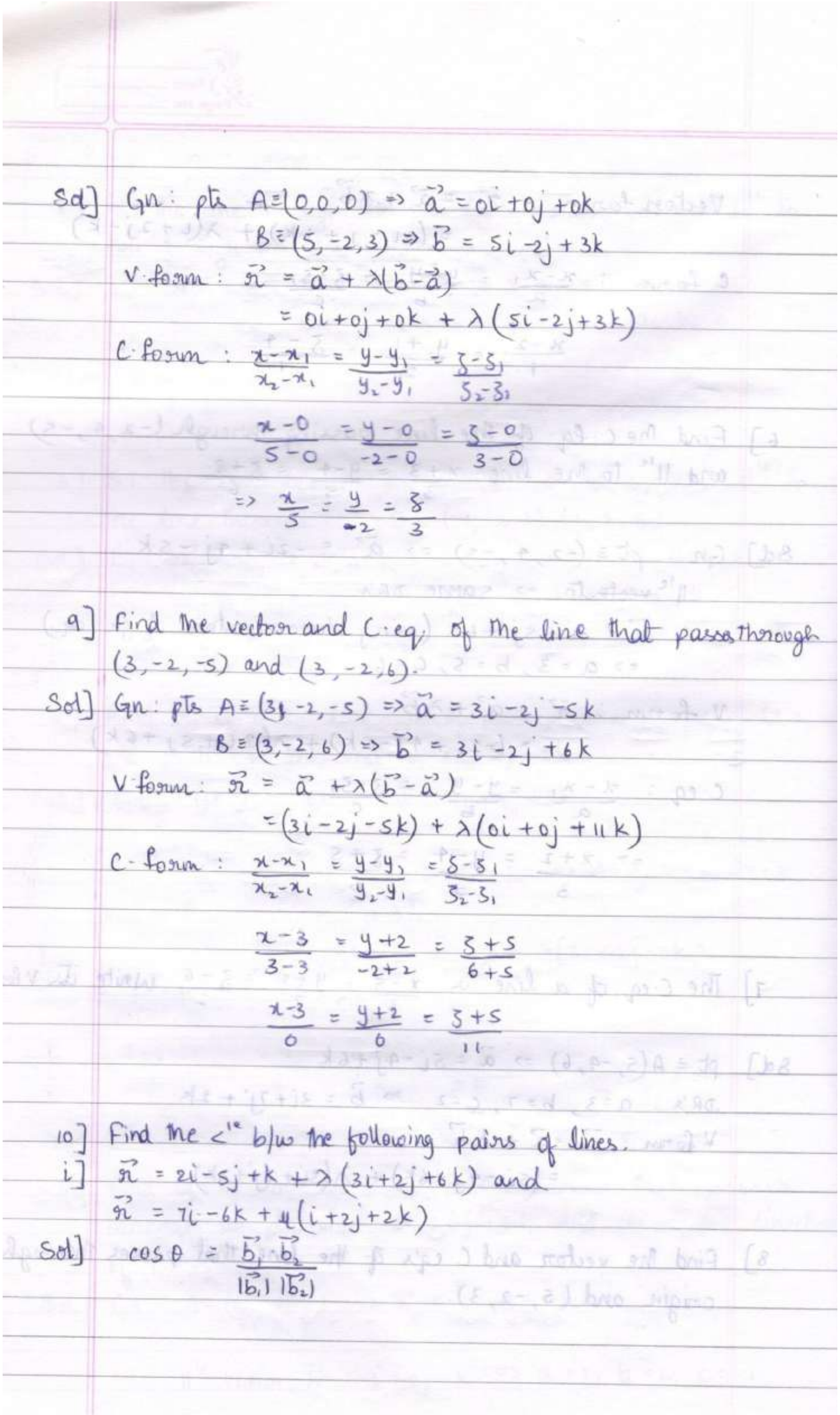
Sol] pt ≡ A(5, -4, 6) => $\vec{a} = 5i - 4j + 6k$

DR's : a = 3, b = 7, c = 2 => $\vec{b} = 3i + 7j + 2k$

V form : $\vec{r} = \vec{a} + \lambda \vec{b}$

$= (5i - 4j + 6k) + \lambda(3i + 7j + 2k)$

8] Find the vector and C eq's of the line that passes through origin and (5, -2, 3).



Sol] Gn: pts $A \equiv (0,0,0) \Rightarrow \vec{a} = 0i + 0j + 0k$

$B \equiv (5,-2,3) \Rightarrow \vec{b} = 5i - 2j + 3k$

V. form: $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$= 0i + 0j + 0k + \lambda(5i - 2j + 3k)$

C. form: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$$\frac{x-0}{5-0} = \frac{y-0}{-2-0} = \frac{z-0}{3-0}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

9] Find the vector and C. eq. of the line that passa through (3, -2, -5) and (3, -2, 6).

Sol] Gn: pts $A \equiv (3, -2, -5) \Rightarrow \vec{a} = 3i - 2j - 5k$

$B \equiv (3, -2, 6) \Rightarrow \vec{b} = 3i - 2j + 6k$

V form: $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$= (3i - 2j - 5k) + \lambda(0i + 0j + 11k)$

C. form: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$$\frac{x-3}{3-3} = \frac{y+2}{-2+2} = \frac{z+5}{6+5}$$

$$\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$$

10] Find the <^{le} b/w the following pairs of lines.

i] $\vec{r} = 2i - 5j + k + \lambda(3i + 2j + 6k)$ and

$\vec{r} = 7i - 6k + \mu(i + 2j + 2k)$

Sol] $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}| \, |\vec{b_2}|}$

$$\cos\theta = \frac{(3i + 2j + 6k)\cdot(i + 2j + 2k)}{\sqrt{9 + 4 + 36}\,\sqrt{1 + 4 + 4}}$$

$$= \frac{3 + 4 + 12}{\sqrt{49}\,\sqrt{9}}$$

$$= \frac{19}{7\cdot 3} = \frac{19}{21}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

** ii] $\vec{r} = (\overset{\vec{a}_1}{3i + j \mp 2k}) + \lambda(\overset{\vec{b}_1}{i - j - 2k})$ and

$\vec{r} = (\underset{\vec{a}_2}{2i - j - 56k}) + \mu(\underset{\vec{b}_2}{3i - 5j - 4k})$

Sol] $\cos\theta = \dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1|\,|\vec{b}_2|} = \dfrac{3 + 5 + 8}{\sqrt{1+1+4}\,\sqrt{9+25+16}} = \dfrac{16}{\sqrt{6}\,\sqrt{50}} = \dfrac{16}{5\sqrt{2}\sqrt{3}\sqrt{6}}$

$$\cos\theta = \frac{16}{5\cdot 2\cdot\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

11] Find the $<^{le}$

i] $\dfrac{x-2}{\underset{a_1}{2}} = \dfrac{y-1}{\underset{b_1}{5}} = \dfrac{z+3}{\underset{c_1}{-3}}$ & $\dfrac{x+2}{\underset{a_2}{-1}} = \dfrac{y-4}{\underset{b_2}{8}} = \dfrac{z-5}{\underset{c_2}{4}}$

Sol] $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$

$$= \frac{-2 + 40 - 12}{\sqrt{4+25+9}\,\sqrt{1+64+16}} = \frac{26}{\sqrt{38}\,\sqrt{81}} = \frac{26}{9\sqrt{38}}$$

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

ii] $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ & $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

Sol] $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$= \frac{8 + 2 + 8}{\sqrt{4+4+1}\sqrt{16+1+64}}$

$= \frac{18}{\sqrt{9}\sqrt{81}} = \frac{18}{3 \cdot 9} = \frac{18}{27}$

$\therefore \theta = \cos^{-1}\left(\frac{18}{27}\right) = \cos^{-1}\left(\frac{2}{3}\right)$

** 12] Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at $\perp^{r}$

Sol] $L_1 : \frac{x-1}{\underset{a_1}{-3}} = \frac{y-2}{\underset{b_1}{\frac{2p}{7}}} = \frac{z-3}{\underset{c_1}{2}}$, $L_2 : \frac{x-1}{\underset{a_2}{\frac{-3p}{7}}} = \frac{y-5}{\underset{b_2}{1}} = \frac{z-6}{\underset{c_2}{-5}}$

Since $L_1 \perp^{r} L_2$

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow \frac{9p}{7} + \frac{2p}{7} - 10p = 0$

$\frac{11p}{7} - 10 = 0$

$11p = 70$

$\boxed{p = \frac{70}{11}}$

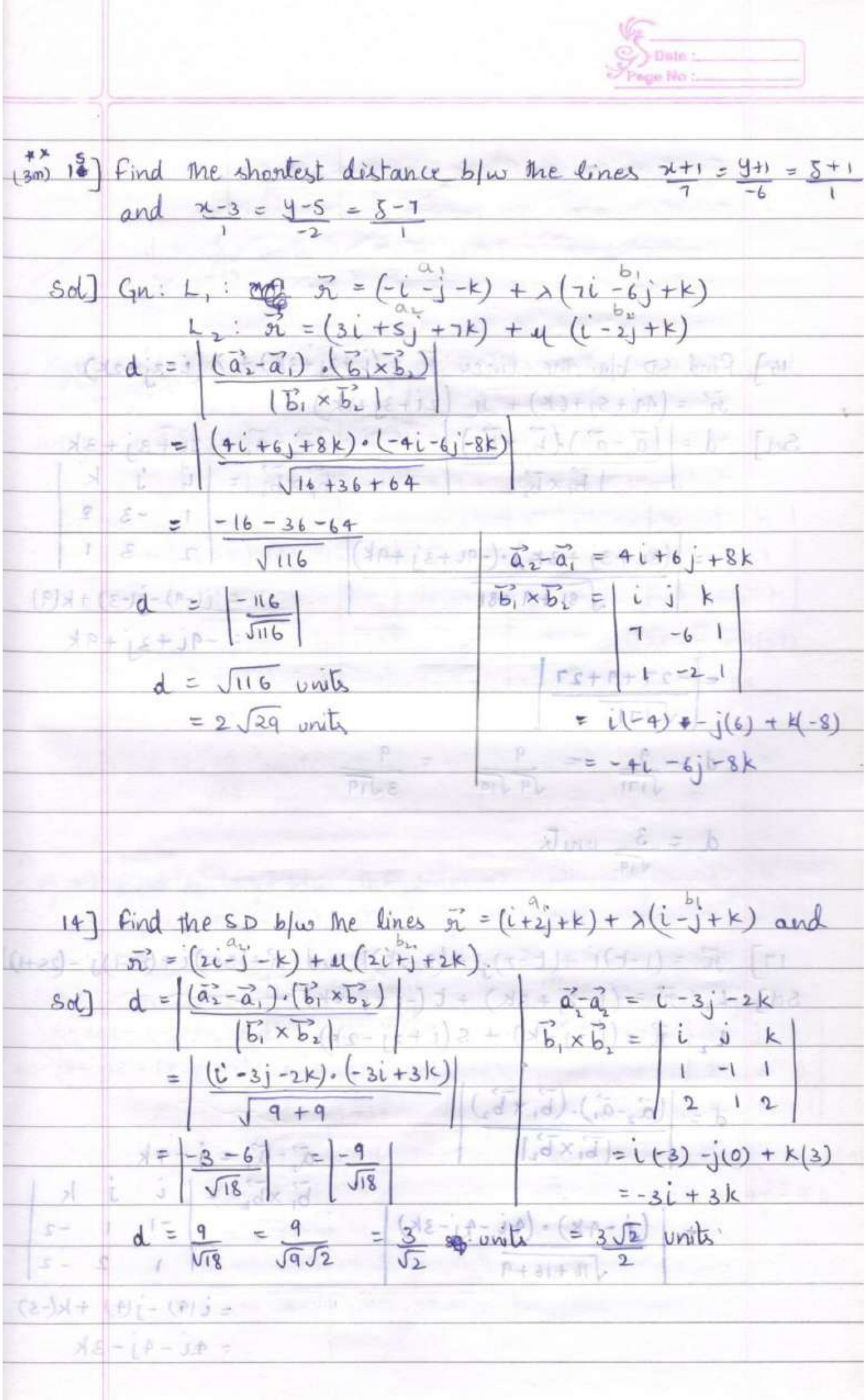
13] ST the lines $\frac{x-5}{\underset{a_1}{7}} = \frac{y+2}{\underset{b_1}{-5}} = \frac{z}{\underset{c_1}{1}}$ and $\frac{x}{\underset{a_2}{1}} = \frac{y}{\underset{b_2}{2}} = \frac{z}{\underset{c_2}{3}}$ are $\perp^{r}$.

Sol] Consider , $a_1a_2 + b_1b_2 + c_1c_2$

$= 7 - 10 + 3 = 0$

$\because \vec{a} \cdot \vec{b} = 0$

$\therefore$ The lines are $\perp^{r}$ to each other.



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** (3m) 15] Find the shortest distance b/w the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Sol] Gn: $L_1 : \vec{r} = (\overset{a_1}{-i-j-k}) + \lambda(\overset{b_1}{7i-6j+k})$

$L_2 : \vec{r} = (\overset{a_2}{3i+5j+7k}) + \mu(\overset{b_2}{i-2j+k})$

$d = \left| \frac{(\vec{a_2}-\vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$= \left| \frac{(4i+6j+8k) \cdot (-4i-6j-8k)}{\sqrt{16+36+64}} \right|$

$= \frac{-16-36-64}{\sqrt{116}}$

$d = \left| \frac{-116}{\sqrt{116}} \right|$

$d = \sqrt{116}$ units

$= 2\sqrt{29}$ units

$\vec{a_2}-\vec{a_1} = 4i+6j+8k$

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= i(-4) - j(6) + k(-8)$

$= -4i - 6j - 8k$

14] Find the SD b/w the lines $\vec{r} = (\overset{a_1}{i+2j+k}) + \lambda(\overset{b_1}{i-j+k})$ and $\vec{r} = (\overset{a_2}{2i-j-k}) + \mu(\overset{b_2}{2i+j+2k})$.

Sol] $d = \left| \frac{(\vec{a_2}-\vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$= \left| \frac{(i-3j-2k) \cdot (-3i+3k)}{\sqrt{9+9}} \right|$

$= \left| \frac{-3-6}{\sqrt{18}} \right| = \left| \frac{-9}{\sqrt{18}} \right|$

$d = \frac{9}{\sqrt{18}} = \frac{9}{\sqrt{9}\sqrt{2}} = \frac{3}{\sqrt{2}}$ units $= \frac{3\sqrt{2}}{2}$ units

$\vec{a_2}-\vec{a_2} = i-3j-2k$

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= i(-3) - j(0) + k(3)$

$= -3i + 3k$

16] Find SD b/w the lines $\vec{r} = (\overset{a_1}{i+2j+3k}) + \lambda(\overset{b_1}{i-3j+2k})$

$\vec{r} = (\overset{a_2}{4i+5j+6k}) + \mu\,(\overset{b_2}{2i+3j+1k})$

Sol] $d = \left|\dfrac{(\vec{a}_2-\vec{a}_1)(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$\vec{a}_2-\vec{a}_1 = 3i+3j+3k$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= i(-9)-j(-3)+k(9)$

$= -9i+3j+9k$

$= \left|\dfrac{(3i+3j+3k)\cdot(-9i+3j+9k)}{\sqrt{81+9+81}}\right|$

$= \left|\dfrac{-27+9+27}{\sqrt{171}}\right|$

$d = \dfrac{9}{\sqrt{171}} = \dfrac{9}{\sqrt{9}\sqrt{19}} = \dfrac{9}{3\sqrt{19}}$

$d = \dfrac{3}{\sqrt{19}}$ units

17] $\vec{r} = (1-t)i + (t-2)j + (3-2t)k$ and $\vec{r} = (s+1)i + (2s-1)j - (2s+1)k$

Sol] $L_1 : \vec{r} = (\overset{a_1}{i-2j+3k}) + t(\overset{b_1}{-i+j-2k})$

$L_2 : \vec{r} = (\overset{a_2}{i-j-k}) + s(\overset{b_2}{i+2j-2k})$

$si+i+2sj-j-2sk-k$

$(i-j-k)+s(i+2j-2k)$

$d = \left|\dfrac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$\vec{a}_2-\vec{a}_1 = j-4k$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$= i(2)-j(4)+k(-3)$

$= 2i-4j-3k$

$= \left|\dfrac{(j-4k)\cdot(4i-4j-3k)}{\sqrt{4+16+9}}\right|$

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$$d = \left|\frac{-4+12}{\sqrt{29}}\right|$$

$$d = \frac{8}{\sqrt{29}} \text{ units}$$

Eg11) Find SD b/w $\vec{r} = (\overset{a_1}{1i+1j+0k}) + \lambda(\overset{b_1}{2i-j+k})$ and
$\vec{r} = (\overset{a_2}{2i+j+(-1k)}) + \mu(\overset{b_2}{3i-5j+2k})$

sol] $$d = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$= \left|\frac{(i-k)\cdot(3i-j-7k)}{\sqrt{9+1+49}}\right|$$

$$= \frac{3+7}{\sqrt{59}}$$

$$d = \frac{10}{\sqrt{59}} \text{ units}$$

$\vec{a}_2-\vec{a}_1 = i-k$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= i(3) - j + k(-7)$

$= 3i - j - 7k$

** Eg12] Find distance b/w the lines L_1 and L_2 given by
$\vec{r} = (\overset{a_1}{i+2j-4k}) + \lambda(\overset{b}{2i+3j+6k})$ and
$\vec{r} = (\overset{a_2}{3i+3j-5k}) + \mu(2i+3j+6k)$

Sol] $$SD = d = \left|\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}\right|$$

$$d = \left|\frac{|-9i+14j-4k|}{\sqrt{4+9+36}}\right|$$

$$= \frac{\sqrt{81+196+16}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7}$$

$\vec{a}_2-\vec{a}_1 = 2i+j-k$

$$\vec{b}\times(\vec{a}_2-\vec{a}_1) = \begin{vmatrix} i & j & k \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$= i(-9) - j(-14) + k(-9)$

$= -9i + 14j - 4k$

Equation of a Plane:

* Eq. of a plane in normal form;

V. form : $\vec{r}\cdot\hat{n} = d$ / $\vec{r}\cdot\vec{N} = d$

C. form : $lx + my + nz = d$ / $ax + by + cz = d$

d = distance of the plane from origin.

* Eq. of a plane ⊥^{r} to a given vector and passing through a gn pt.

V. form : $(\vec{r}-\vec{a})\cdot\vec{N} = 0$ $[(\vec{r} - pt)\cdot \perp^{r} \text{vector} = 0]$

C. form : $A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$

* Eq. of a plane passing through 3 non-collinear pts: (STP)

V. form: $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})] = 0$

C. form :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

* Intercept form of the plane :

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

(3m) * Eq. of the plane passing through intersection of two planes π_1 and π_2 is:

V. form: $\pi = \pi_1 + \lambda\pi_2 = 0$

Gn 2 planes : C. form : $\pi_1 : A_1x + B_1y + C_1z + D_1 = 0$

$\pi_2 = A_2x + B_2y + C_2z + D_2 = 0$

New plane $\pi : (A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2)$

* Angle b/w two planes :

$\pi_1 = \vec{r}\cdot\vec{N_1} = d_1$

$\pi_2 = \vec{r}\cdot\vec{N_2} = d_2$

V. form : $\cos\theta = \dfrac{\vec{N_1}\cdot\vec{N_2}}{|\vec{N_1}||\vec{N_2}|}$

C. form : $\cos\theta = \left|\dfrac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

** * Angle b/w a line and a plane :

$L : \vec{r} = \vec{a} + \lambda \vec{b}$

$\pi : \vec{r} \cdot \vec{n} = d$

$$\sin \phi = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|}$$

(1m) * Distance of a pt. from a plane :

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

EXERCISE 11.3

14] In the following cases, find the distance of gn. pts from the plane.

(a) pt $\equiv (0,0,0)$; $\pi : 3x - 4y + 12z = 3$

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right| = \left| \frac{0 + 0 + 0 - 3}{\sqrt{9 + 16 + 144}} \right|$$

$$d = \left| \frac{-3}{\sqrt{169}} \right| = \frac{3}{13} \text{ units}$$

(b) pt $\equiv (3, -2, 1)$; $\pi : 2x - y + 2z + 3 = 0$

$$d = \left| \frac{6 + 2 + 2 + 3}{\sqrt{4 + 1 + 4}} \right| = \left| \frac{13}{\sqrt{9}} \right| = \frac{13}{3} \text{ units}$$

(c) pt $\equiv (2, 3, -5)$; $\pi = x + 2y - 2z = 9$

$$d = \left| \frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}} \right| = \left| \frac{9}{\sqrt{9}} \right| = \frac{9}{3} = 3 \text{ units}$$

(d) pt $\equiv (-6, 0, 0)$; $\pi = 2x - 3y + 6z - 2 = 0$

$$d = \left| \frac{-12 - 2}{\sqrt{4 + 9 + 36}} \right| = \left| \frac{-14}{\sqrt{49}} \right| = \frac{14}{7} = 2 \text{ units}$$

13] In the following cases determine whether the given planes are $\parallel^{le}$ or $\perp^{le}$ and in case they are neither, find $\angle^{le}$ b/w them

NOTE: (a) $A_1A_2 + B_1B_2 + C_1C_2 = 0 \Rightarrow \Pi_1 \perp^{le} \Pi_2$

(b) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \lambda \Rightarrow \Pi_1 \parallel^{le} \Pi_2$

(c) $\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|}\right|$

(a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

Sol] Clearly Π_1 and Π_2 are neither $\parallel^{le}$ nor $\perp^{le}$.

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\,\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

$$= \left|\frac{21 - 5 - 60}{\sqrt{49+25+36}\,\sqrt{9+1+100}}\right| = \left|\frac{-44}{\sqrt{110}\,\sqrt{110}}\right| = \left|\frac{-44}{110}\right| = \frac{+4}{10}$$

$$\cos\theta = \frac{+2}{5}$$

$$\theta = \cos^{-1}\left(+\frac{2}{5}\right)$$

(b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$

Sol] ~~Clearly Π_1 and Π_2 are neither $\parallel^{le}$ nor $\perp^{le}$~~

consider: $A_1A_2 + B_1B_2 + C_1C_2$

$= 2 - 2 + 0 = 0$

$\therefore \Pi_1 \perp^{le} \Pi_2$

(c) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

Sol] Consider $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \Rightarrow \frac{2}{3} = \frac{-2}{-3} = \frac{4}{6} = \frac{2}{3}$

$\therefore \Pi_1 \parallel^{le} \Pi_2$

(d) $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

Sol] $\frac{2}{2} = \frac{-1}{-1} = \frac{3}{3} = 1$

$\therefore \Pi_1 \parallel^{le} \Pi_2$

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(e) $4x+8y+z-8=0$ and $y+z-4=0$

Sol] $\cos\theta = \left|\dfrac{8+1}{\sqrt{16+64+1}\,\sqrt{1+1}}\right| = \left|\dfrac{9}{\sqrt{81}\sqrt{2}}\right| = \dfrac{1}{\sqrt{2}}$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

12] Find the angle b/w the planes whose vector eq.s are $\vec{r}\cdot(\underbrace{2i+2j-3k}_{\vec{n}_1})=5$ and $\vec{r}\cdot(\underbrace{3i-3j+5k}_{\vec{n}_2})=3$

Sol] $\cos\theta = \left|\dfrac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right| = \left|\dfrac{6-6-15}{\sqrt{4+4+9}\,\sqrt{9+9+25}}\right| = \left|\dfrac{-15}{\sqrt{17}\sqrt{43}}\right|$

$$\theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$$

** 9] Find eq. of plane through the intersection of the planes $\overset{\pi_1}{3x-y+2z-4=0}$ and $\overset{\pi_2}{x+y+z-2=0}$ and the pt $(2,2,1)$

Sol] Eq. of plane $= \pi_1 + \lambda\pi_2 = 0$

$(3x-y+2z-4)+\lambda(x+y+z-2)=0 \longrightarrow ①$

① passes through $(2,2,1)$

$\Rightarrow (6-2+2-4)+\lambda(2+2+1-2)=0$

$\Rightarrow 2+\lambda 3=0$

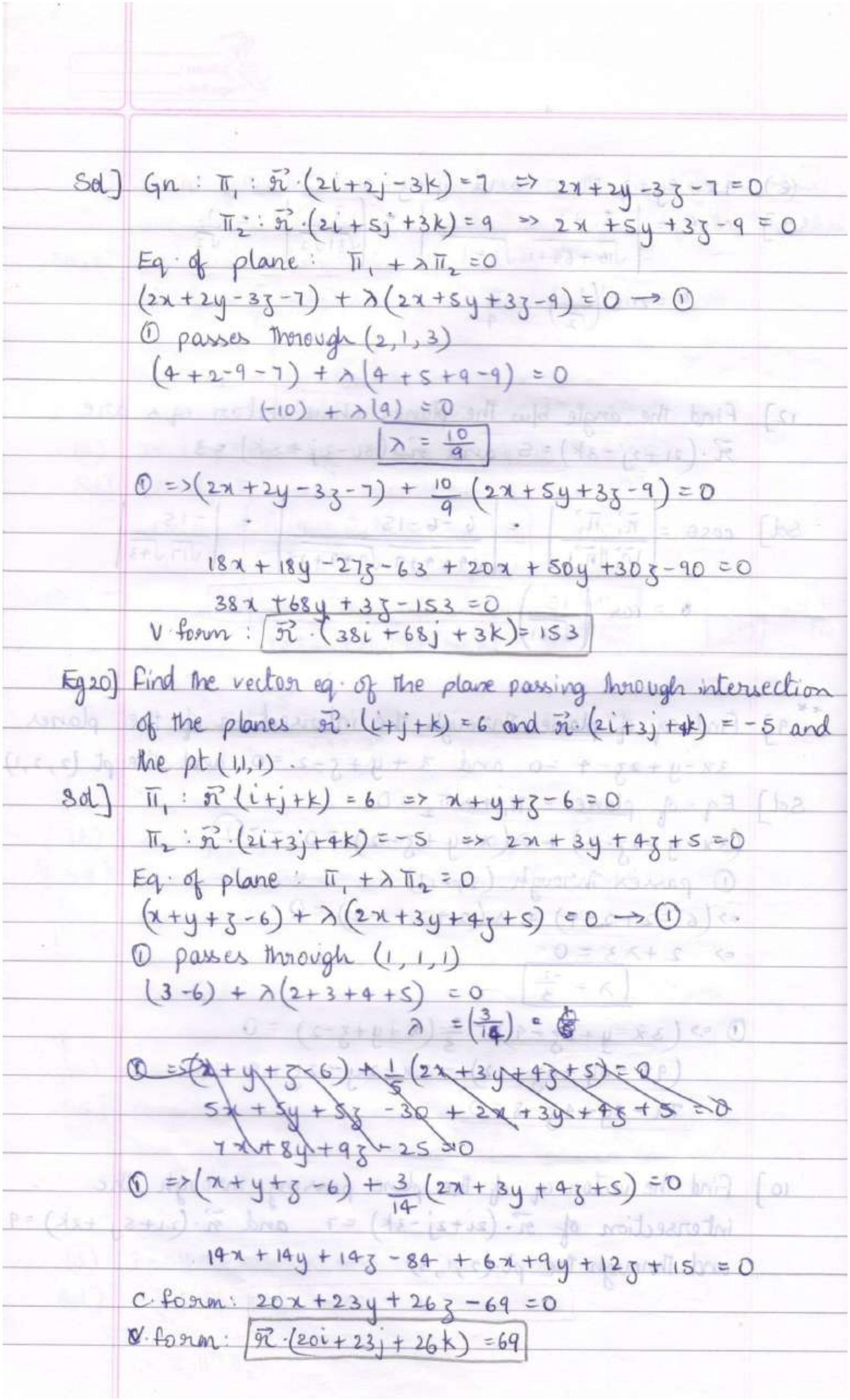
$\boxed{\lambda = \frac{-2}{3}}$

① $\Rightarrow (3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$

$(9x-3y+6z-12)-2x-2y-2z+4=0$

$\boxed{7x-5y+4z-8=0}$

10] Find the vector eq. of the plane passing through the intersection of $\vec{r}\cdot(2i+2j-3k)=7$ and $\vec{r}\cdot(2i+5j+3k)=9$ and through the pt. $(2,1,3)$.



Sol] Gn : $\pi_1 : \vec{r}\cdot(2i+2j-3k)=7 \Rightarrow 2x+2y-3z-7=0$

$\pi_2 : \vec{r}\cdot(2i+5j+3k)=9 \Rightarrow 2x+5y+3z-9=0$

Eq. of plane : $\pi_1+\lambda\pi_2=0$

$(2x+2y-3z-7)+\lambda(2x+5y+3z-9)=0 \rightarrow$ ①

① passes through $(2,1,3)$

$(4+2-9-7)+\lambda(4+5+9-9)=0$

$(-10)+\lambda(9)=0$

$\boxed{\lambda=\frac{10}{9}}$

① $\Rightarrow (2x+2y-3z-7)+\frac{10}{9}(2x+5y+3z-9)=0$

$18x+18y-27z-63+20x+50y+30z-90=0$

$38x+68y+3z-153=0$

V. form : $\boxed{\vec{r}\cdot(38i+68j+3k)=153}$

Eg20] Find the vector eq. of the plane passing through intersection of the planes $\vec{r}\cdot(i+j+k)=6$ and $\vec{r}\cdot(2i+3j+4k)=-5$ and the pt $(1,1,1)$.

Sol] $\pi_1 : \vec{r}(i+j+k)=6 \Rightarrow x+y+z-6=0$

$\pi_2 : \vec{r}\cdot(2i+3j+4k)=-5 \Rightarrow 2x+3y+4z+5=0$

Eq. of plane : $\pi_1+\lambda\pi_2=0$

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0 \rightarrow$ ①

① passes through $(1,1,1)$

$(3-6)+\lambda(2+3+4+5)=0$

$\lambda=\left(\frac{3}{14}\right)=\cancel{\frac{1}{5}}$

~~① $\Rightarrow (x+y+z-6)+\frac{1}{5}(2x+3y+4z+5)=0$~~

~~$5x+5y+5z-30+2x+3y+4z+5=0$~~

~~$7x+8y+9z-25=0$~~

① $\Rightarrow (x+y+z-6)+\frac{3}{14}(2x+3y+4z+5)=0$

$14x+14y+14z-84+6x+9y+12z+15=0$

C. form: $20x+23y+26z-69=0$

V. form: $\boxed{\vec{r}\cdot(20i+23j+26k)=69}$

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11] Find the eq. of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is $\perp^{le}$ to the plane $x-y+z=0$

Sol] $\pi_1 : x+z+y=1$

$\pi_2 : 2x+3y+4z=5$

Eq. of plane through intersection of π_1 and π_2 is

$\pi_1 + \lambda\pi_2 = 0$

$(x+y+z-1) + \lambda(2x+3y+4z-5) = 0 \longrightarrow ①$

① is $\perp^{le}$ to $x-y+z=0$

$(\vec{a}\cdot\vec{b}=0)$ $a_1a_2+b_1b_2+c_1c_2=0$

$1(1+2\lambda) - 1(1+3\lambda) + 1(1+4\lambda) = 0$

$1+2\lambda-1-3\lambda+1+4\lambda=0$

$1+3\lambda=0$

$\lambda = \frac{-1}{3}$

① $\Rightarrow (x+y+z-1) + \left(\frac{-1}{3}\right)(2x+3y+4z-5) = 0$

$3x+3y+3z-3 -2x-3y-4z+5=0$

$\boxed{x-z+2=0}$

** (im) 8] (a) Find the eq. of the plane with intercept 3 on y-axis and parallel to zox plane.

Sol] $y=3$.

(b) Find eq. of plane making intercept of -2 on z-axis and parallel to xoy plane.

$z=-2 \Rightarrow z+2=0$

(c) Find eq. of plane making intercel of 5 on x-axis and $\parallel^{le}$ to yoz plane

$x=5$.

7] (a) Find the intercepts cutt off by the plane $2x+y-z=5$

Sol] $\div$ by 5 : $\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

$\Rightarrow$ x intercept $= a = \frac{5}{2}$ z intercept $= c = -5$

y " $= b = 5$

(b) Find the eq. of plane making intercepts 1, -1, 2 on coordinate axis.

$$\frac{x}{1} + \frac{y}{-1} + \frac{z}{2} = 1$$

(c) Find the intercepts made by the plane $x - 3y + 6z = 6$

$$\frac{x}{6} + \frac{y}{-6/3} + \frac{z}{6/6} = 1 \Rightarrow \frac{x}{6} + \frac{y}{-2} + \frac{z}{1} = 1$$

$\therefore$ x intercept $= a = 6$

y " $= b = -2$

z " $= c = 1$

** 6] Find eq.s of the planes passing through the 3 pt.s

(a) $(1,1,-1), (6,4,-5), (-4,-2,3)$

Eq. of plane through 3 ~~pts~~ non collinear pts.

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y - 1 & z + 1 \\ 6 - 1 & 4 - 1 & -5 + 1 \\ -4 - 1 & -2 - 1 & 3 + 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 1 & y - 1 & z + 1 \\ 5 & 3 & -4 \\ -5 & -3 & 4 \end{vmatrix} = 0$$

$R_2 = -R_3 \Rightarrow \Delta = 0$

$\Rightarrow$ Gn 3 pt.s are colliner.

$\Rightarrow$ Infinite no. of planes pass through the gn. 3 pt.s.

(b) $(1,1,0), (1,2,1), (-2,2,-1)$

$$\Rightarrow \begin{vmatrix} x - 1 & y - 1 & z - 0 \\ 1 - 1 & 2 - 1 & 1 - 0 \\ -2 - 1 & 2 - 1 & -1 - 0 \end{vmatrix} = 0$$

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$$\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$x-1(-1-1) - (y-1)(3) + z(3) = 0$$

$$-2x + 2 - 3y + 3 + 3z = 0$$

$$2x + 3y - 3z - 5 = 0$$

NOTE: If $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$ $\begin{matrix} \rightarrow \text{dr's of line } L_1 \\ \\ \rightarrow \text{dr's of line } L_2 \end{matrix}$

$$\Rightarrow \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$\Rightarrow$ Gn lines are coplanar.

Eg 18] Find eq. of plane passing through $R(2,5,-3)$, $S(-2,-3,5)$ and $T(5,3,-3)$ in vector and c. form.

Sol] V form : $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})] = 0$

$\vec{a} = 2i + 5j + 3k$, $\vec{b} = -2i - 3j + 5k$, $\vec{c} = 5i + 3j - 3k$

$\therefore [\vec{r} - (2i + 5j + 3k)] \cdot \{(-4i - 8j + 8k) \times (3i - 2j + 0k)\} = 0$

c. form : $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$(x-2)(16) - (y-5)(-24) + (z+3)(32) = 0$$

$$16x - 32 + 24y - 120 + 32z + 96 = 0$$

$$\boxed{16x + 24y + 32z - 56 = 0}$$

Eg 21] ST the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

Sol] Here $(x_1, y_1, z_1) \equiv (-3, 1, 5)$

$(x_2\ y_2\ z_2) \equiv (-1, 2, 5)$

~~$a, b, c \equiv$~~

$a_1, b_1, c_1 \equiv -3, 1, 5$ and $a_2, b_2, c_2 \equiv -1, 2, 5$

Consider $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$$\begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 2(5-10) - 1(-15+5)$$

$$= -10 + 10$$

$$= 0$$

$\because \Delta = 0 \Rightarrow$ Gn lines are coplanar.

Eg 22] Find the <le b/w the 2 planes $2x+y-2z=5$ and $3x-6y-2z=7$ using vector method.

Sol] Gn $\vec{n_1} = 2i+j-2k$; $\vec{n_2} = 3i-6j-2k$

$$\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right| = \left|\frac{6-6+4}{\sqrt{4+1+4}\sqrt{9+36+4}}\right|$$

$$\cos\theta = \left|\frac{4}{\sqrt{9}\sqrt{49}}\right| = \frac{4}{21}$$

$$\theta = \cos^{-1}\left(\frac{4}{21}\right)$$

Eg 23] Find the <le b/w 2 planes $3x-6y+2z=7$ and $2x+2y-2z=5$

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$= \left|\frac{6-12-4}{\sqrt{9+36+4}\sqrt{4+4+4}}\right| = \left|\frac{-10}{\sqrt{49}\sqrt{12}}\right|$$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\cos\theta = \frac{10}{7\sqrt{12}} = \frac{5}{7\sqrt{3}}$$

$\sqrt{12} = 2\sqrt{2}$

$$\theta = \cos^{-1}\left(\frac{5}{7\sqrt{3}}\right)$$

**Eg 24] Find the distance of a pt $(2, 5, -3)$ from the plane $\vec{r}\cdot(6i-3j+2k)=4$.

Sol] Gn : $\pi : 6x-3y+2z-4=0$ pt $(2,5,-3)$

$$d = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{12-15-6-4}{\sqrt{36+9+4}}\right| = \frac{13}{7} \text{ units}$$

**Eg 25] Find the $\angle^{le}$ b/w the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$

Sol] $\vec{b} = 2i+3j+6k \;;\; \vec{n} = 10i+2j-11k$

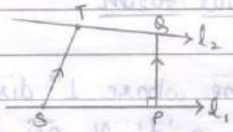
$$\sin\phi = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\,|\vec{n}|}\right| = \left|\frac{20+6-66}{\sqrt{4+9+36}\,\sqrt{100+4+121}}\right|$$

$$\sin\phi = \left|\frac{-40}{7(15)}\right| = \left(\frac{\overset{8}{\cancel{40}}}{\underset{21}{\cancel{105}}}\right)$$

$$\phi = \sin^{-1}\left(\frac{8}{21}\right)$$

III Shortest distance between skew lines:

Consider 2 skew lines $l_1 : \vec{r} = \vec{a}_1 + \lambda\vec{b}_1$, $l_2 : \vec{r} = \vec{a}_2 + \lambda\vec{b}_2$



Let $\vec{a}_1$ and $\vec{a}_2$ be the position vectors of pt.s S and T ~~and~~ on l_1 and l_2 respec. $\vec{PQ}$ is the shortest distance of length 'd' between l_1 and l_2 and is the projection of $\vec{ST}$ along $\vec{PQ}$.

$\vec{PQ}$ is $\perp^{r}$ to both $\vec{b}_1$ and $\vec{b}_2$, $\hat{n}$ being unit vector on $\vec{PQ}$.

$\therefore \vec{PQ} = d\hat{n}$; $\hat{n} = \dfrac{\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}$; $|\vec{PQ}| = d$

Let θ be the angle between $\vec{ST}$ and $\vec{PQ}$; $\vec{ST} = \vec{a}_2 - \vec{a}_1$

$$\cos\theta = \frac{\vec{ST}\cdot\vec{PQ}}{|\vec{ST}|\,|\vec{PQ}|}$$

Also, $\cos\theta = \frac{|\vec{PQ}|}{|\vec{ST}|}$

$$\Rightarrow \frac{|\vec{PQ}|}{|\vec{ST}|} = \frac{\vec{ST}\cdot\vec{PQ}}{|\vec{ST}||\vec{PQ}|}$$

$$d = \frac{(\vec{a}_2-\vec{a}_1)\cdot \cancel{d}\hat{n}}{\cancel{d}}$$

$$d = (\vec{a}_2-\vec{a}_1)\cdot\hat{n}$$

V. form: $\boxed{d = \left|(\vec{a}_2-\vec{a}_1)\cdot\frac{\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}\right|}$

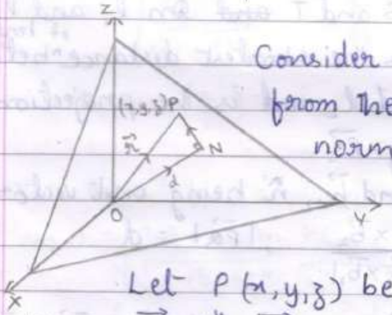
In cartesian form : $l_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

$l_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

Writing in derterminant form we get

$$d = \left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(a_1c_2-a_2c_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

*** IV Equation of a plane in normal form :



Consider a plane whose ⊥ˡʳ distance from the origin is 'd'. If $\vec{ON}$ is the normal with $\hat{n}$ as unit normal vector, then $\boxed{\vec{ON} = d\hat{n}}$

Let P (x, y, z) be any arbitrary pt. on the plane

Now, $\vec{NP}$ ⊥ˡʳ $\vec{ON}$

$$\Rightarrow \vec{NP}\cdot\vec{ON} = 0$$

$$(\vec{OP}-\vec{ON})\cdot(\vec{ON}) = 0$$

$(\vec{r} - d\hat{n}) \cdot d\hat{n} = 0$

$(\vec{r} - d\hat{n}) \cdot \hat{n} = 0$

$\vec{r} \cdot \hat{n} - d\hat{n} \cdot \hat{n} = 0$

$\vec{r} \cdot \hat{n} - d = 0$

V form: $\boxed{\vec{r} \cdot \hat{n} = d}$

Let $\vec{r} = xi + yj + zk$

$\hat{n} = li + mj + nk$;(l, m, n being its DC's)

Substituting these in vector form,

$(xi + yj + zk) \cdot (li + mj + nk) = d$

C form => $\boxed{lx + my + nz = d}$

5] Find the v & c eqs of the planes

(a) passes through $(1, 0, -2)$ and normal to the plane is $i + j - k$.

Sol] Gn pt $A \equiv (1, 0, -2) \Rightarrow \vec{a} = i - 2k$

$\vec{N} = i + j - k \Rightarrow A = 1,\ B = 1,\ C = -1$

V form: $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$[\vec{r} - (i - 2k)] \cdot (i + j - k) = 0$

C-form: $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

$1(x - 1) + 1(y - 0) - 1(z + 2) = 0$

$x - 1 + y - z - 2 = 0$

$\boxed{x + y - z - 3 = 0}$

(b) passing through $(1, 4, 6)$ and the normal vector to the plane is $i - 2j + k$.

Sol] Gn: pt. $A \equiv (1, 4, 6) \Rightarrow \vec{a} = i + 4j + 6k$

$\vec{N} = i - 2j + k \Rightarrow A = 1,\ B = -2,\ C = 1$

V form: $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$[\vec{r} - (i + 4j + 6k)] \cdot (i - 2j + k) = 0$

C·form : $A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$

$1(x-1) + -2(y-4) + 1(z-6) = 0$

$x - 1 - 2y + 8 + z - 6 = 0$

$\boxed{x - 2y + z + 1 = 0}$

3] Find the C·eq of the following planes.

(a) $\vec{r}\cdot(i + j - k) = 2$

C·form : $x + y - z - 2 = 0$

(b) $\vec{r}\cdot(2i + 3j - 4k) = 1$

C·form : $2x + 3y - 4z - 1 = 0$

(c) $\vec{r}\cdot[(s-2t)i + (3-t)j + (2s+t)k] = 15$

C·form : $(s-2t)x + (3-t)y + (2s+t)z - 15 = 0$

2] Find v·eq of a plane which is at a distance of 7 units from the origin and normal to the vector $3i + 5j - 6k$

Sol] Gn: $\vec{n} = 3i + 5j - 6k \Rightarrow \hat{n} = \dfrac{3i + 5j - 6k}{\sqrt{9 + 25 + 36}} = \dfrac{3i + 5j - 6k}{\sqrt{70}}$

$d = 7$ units

v·form : $\vec{r}\cdot\hat{n} = d$

$\vec{r}\cdot\left(\dfrac{3i + 5j - 6k}{\sqrt{70}}\right) = 7$

1] In each of the following cases, determine the DC's of the normal to the plane and the distance from the origin.

(b) $x + y + z = 1 \Rightarrow a = 1, b = 1, c = 1$

DC's : $\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$; $d = \dfrac{1}{\sqrt{3}}$

(c) $2x + 3y - z = 5$

DC's : $\dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}, \dfrac{-1}{\sqrt{14}}$; $d = \dfrac{5}{\sqrt{14}}$

In the gn·plane

$ax + by + cz = d$

$\Rightarrow$ a, b, c are DR's

$\therefore$ DC's are $\dfrac{a}{\sqrt{a^2+b^2+c^2}}, \dfrac{b}{\cdots}$

$\perp^r$ dist. from origin = $\dfrac{d}{}$

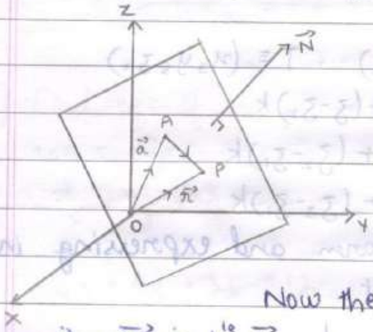
(a) $z = 2$

DC's : $0, 0, 1$; $d = 2$

(d) $5y + 8 = 0 \Rightarrow y = \frac{-8}{5}$

DC's : $0, 1, 0$; $d = \frac{+8}{5}$

$\overline{\text{V}}$ Equation of a plane perpendicular to given vector and passing through a given point.



Consider a plane perpendicular to given vector $\vec{N}$ and passing through the given pt. $A(x_1, y_1, z_1)$.

Let $P(x, y, z)$ be any arbitrary point in the plane with PV $\vec{r}$.

Now the pt. P lies on the plane if and only if $\vec{AP}$ is $\perp^{r}$ $\vec{N}$.

$$\Rightarrow \quad \vec{AP} \cdot \vec{N} = 0$$

$$(\vec{OP} - \vec{OA}) \cdot \vec{N} = 0$$

V. form : $\boxed{(\vec{r} - \vec{a}) \cdot \vec{N} = 0}$

Let $\vec{r} = xi + yj + zk$

$\vec{a} = x_1 i + y_1 j + z_1 k$

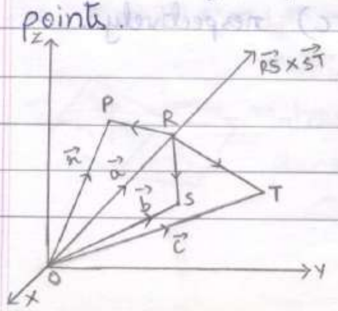
$\vec{N} = Ai + Bj + Ck$ where A, B, C are its DR's

Substituting these in the vector form,

$$[(x - x_1) i + (y - y_1) j + (z - z_1) k] \cdot (Ai + Bj + Ck) = 0$$

$$\Rightarrow \boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$$

$\overline{\text{VI}}$ Equation of a plane passing through 3 non-collinear points.



Let $\vec{a}, \vec{b}, \vec{c}$ be the PV's of the gn. 3 non-collinear pt.s R, S, T resp.

Let $P(x, y, z)$ be any arbitrary pt. in the plane with PV $\vec{r}$.

The vectors $\vec{RS}$ and $\vec{RT}$ are in the gn. plane.

$\therefore$ $\vec{RS} \times \vec{RT}$ is $\perp^{r}$ to ~~both~~ the plane.

Eq. of the ~~line~~ plane passing through pt. R and $\perp^{r}$ to $\vec{RS} \times \vec{RT}$ is

$$(\vec{r}-\vec{a})\cdot(\vec{RS}\times\vec{RT})=0$$

$$(\vec{r}-\vec{a})\cdot[(\vec{OS}-\vec{OR})\times(\vec{OT}-\vec{OR})]=0$$

V. form: $\boxed{(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0}$

$R \equiv (x_1, y_1, z_1)$, $S \equiv (x_2, y_2, z_2)$, $T \equiv (x_3, y_3, z_3)$

$\vec{RP} = (x-x_1)i + (y-y_1)j + (z-z_1)k$

$\vec{RS} = (x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k$

$\vec{RT} = (x_3-x_1)i + (y_3-y_1)j + (z_3-z_1)k$

Substituting these in V. form and expressing in the determinant, we get

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

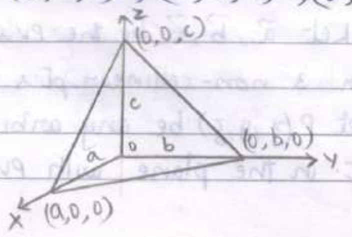
NOTE: If $\Delta = 0$, then the gn. 3 pt.s are collinear and infinite no. of planes pass through the gn. 3 colliner pt.s.

VII Intercept form of the equation of a plane.

Consider a plane $Ax + By + Cz + D = 0 \rightarrow$ ①

Let the plane make intercepts a, b, c on the coordinate axes respectively.

Then plane passes through the pt.s $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ respectively.



$\Rightarrow Aa + D = 0 \quad | \quad Bb + D = 0 \quad | \quad Cc + D = 0$

$A = \frac{-D}{a} \quad | \quad B = \frac{-D}{b} \quad | \quad C = \frac{-D}{c}$

Substituting these in eq ①

$$-\frac{Dx}{a} - \frac{Dy}{b} - \frac{Dz}{c} = -D$$

$\div$ by $-D$

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

Eg 14] Find eq. of the plane with intercepts 2, 3, 4 on coordinate axis resp.

Sol] $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

$6x + 4y + 3z = 12$

Eg 15] Find the distance of the plane $2x+3y+4z-6=0$ from the origin.

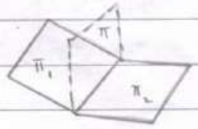
Sol] Consider $\sqrt{(2)^2+(3)^2+(4)^2} = \sqrt{4+9+16} = \sqrt{29}$

$\therefore$ DC's are $\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$

$\perp^{le}$ distance from origin $= d = \frac{6}{\sqrt{29}}$

viii Equation of a plane passing through the intersection of two planes.

Consider two planes π_1 and π_2 given by $\pi_1 : \vec{r} \cdot \vec{n_1} = d_1$ and $\pi_2 : \vec{r} \cdot \vec{n_2} = d_2$



Considering the P.V on the line of intersection π_1 and π_2, equation of the plane π is given by $\pi_1 + \lambda \pi_2 = 0$

$$(\vec{r}\cdot\vec{n}_1 - d_1) + \lambda(\vec{r}\cdot\vec{n}_2 - d_2) = 0$$

$$\vec{r}\cdot(\vec{n}_1 + \lambda\vec{n}_2) - (d_1 + \lambda d_2) = 0$$

V form $\therefore$ $\boxed{\vec{r}\cdot(\vec{n}_1 + \lambda\vec{n}_2) = (d_1 + \lambda d_2)}$

Let $\vec{r} = xi + yj + zk$; $\vec{n}_1 = A_1 i + B_1 j + C_1 k$; $\vec{n}_2 = A_2 i + B_2 j + C_2 k$

Substituting these in the vector form, we get

$$x(A_1 + \lambda A_2) + y(B_1 + \lambda B_2) + z(C_1 + \lambda C_2) = d_1 + \lambda d_2$$

C form $\Rightarrow \boxed{(A_1 x + B_1 y + C_1 z - d_1) + \lambda(A_2 x + B_2 y + C_2 z - d_2) = 0}$

<u>MISCELENEOUS</u> <u>EXERCISE</u> :

1] ST the line joining the origin to the pt. (2,1,1) is $\perp^{lr}$ to the line determined by the pt's (3,5,-1), (4,3,-1).

Sol] L_1 : (0,0,0), (2,1,1) $\Rightarrow$ DR's : $\underset{a_1}{2}, \underset{b_1}{1}, \underset{c_1}{1}$

L_2 : (3,5,-1), (4,3,-1) $\Rightarrow$ DR's : $\underset{a_2}{1}, \underset{b_2}{-2}, \underset{c_2}{0}$

Consider $a_1 a_2 + b_1 b_2 + c_1 c_2 = 2 - 2 + 0 = 0$

$\Rightarrow L_1$ is $\perp^{lr}$ L_2.

3] Find the angle b/w the lines whose DR's are a, b, c and b-c, c-a, a-b

Sol] L_1 : DR's : $\underset{a_1}{a}, \underset{b_1}{b}, \underset{c_1}{c}$

L_2 : DR's $\underset{a_2}{b-c}, \underset{b_2}{c-a}, \underset{c_2}{a-b}$

Consider $a_1 a_2 + b_1 b_2 + c_1 c_2 = a(b-c) + b(c-a) + c(a-b)$

$= ab - \cancel{ac} + \cancel{bc} - ab + \cancel{ac} - \cancel{bc} = 0$

$\Rightarrow L_1$ is $\perp^{lr}$ L_2 $\Rightarrow \theta = 90°$.

4] Find the eq. of a line $\parallel^{le}$ to x-axis and passing through origin.

Sol] Gn: $\parallel^{le}$ vector $= x$-axis $\Rightarrow$ DR's : $\underset{a}{1}, \underset{b}{0}, \underset{c}{0}$

$pt. \equiv (\underset{x_1}{0}, \underset{y_1}{0}, \underset{z_1}{0})$

Eq. of line : V.form : $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = (0i + 0j + 0k) + \lambda(i + 0j + 0k)$

C.form : $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$

5] $A \equiv (1,2,3)$, $B \equiv (4,5,7)$, $C \equiv (-4,3,-6)$, $D \equiv (2,9,2)$.

Find the $\angle^{le}$ b/w $\vec{AB}$ and $\vec{CD}$

Sol] DR's of $\vec{AB}$: 3, 3, 4 DR's of $\vec{CD}$: 6, 6, 8

Clearly $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

$\Rightarrow \vec{AB}$ is $\parallel^{le}$ $\vec{CD}$ $\Rightarrow \theta = 0°$

6] If the lines $\frac{x-1}{\underset{a_1}{-3}} = \frac{y-2}{\underset{b_1}{2K}} = \frac{z-3}{\underset{c_1}{2}}$ and $\frac{x-1}{\underset{a_2}{3k}} = \frac{y-1}{\underset{b_2}{1}} = \frac{z-6}{\underset{c_2}{-5}}$

are $\perp^{le}$. find the value of k.

Sd] Gn: $L_1 \perp^{le} L_2 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

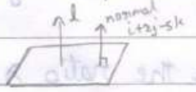
$-9k + 2k - 10 = 0$

$-7k - 10 = 10$

$\boxed{k = \frac{-10}{7}}$

**7] Find the v.eq. of the line passing through (1,2,3) and $\perp^{le}$ to the plane ~~$\vec{r}.(i+2j-5k+9) = 0$~~ $\vec{r}\cdot(i + 2j - 5k + 9) = 0$

Sol]



l is $\perp^{le}$ to plane

$\Rightarrow l$ is $\parallel^{le}$ $\vec{n}$ $\Rightarrow$ same DR's

$\therefore a = 1, b = 2, c = -5$

For l : pt $= (1,2,3) \Rightarrow \vec{a} = i+2j+3k$

$\vec{b} = i+2j-5k$

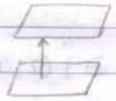
Eq. of l: v.form : $\vec{r} = \vec{a} + \lambda\vec{b}$

$= (i+2j+3k) + \lambda(i+2j-5k)$

c.form : $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{-5}$

8] Find the eq. of the plane passing through pt (a,b,c) and $\parallel^{le}$ to the plane $\vec{r}\cdot(i+j+k) = 2$

Sol]



~~Kd~~ Gn : π passes through (a,b,c)

but π is $\parallel^{le}$ to the gn. plane.

$\therefore \pi : \vec{r}\cdot(i+j+k) = p \longrightarrow ①$

① passes through (a,b,c)

$\therefore (ai+bj+ck)\cdot(i+j+k) = p$

$a+b+c = p$

$\therefore \parallel^{le}\ \pi$: $\boxed{\vec{r}\cdot(i+j+k) = a+b+c}$

9] Find the SD b/w the lines $\vec{r} = 6i+\overset{a_1}{2j}+2k+\lambda(i-\overset{b_1}{2j}+2k)$ and $\vec{r} = -\overset{a_2}{4i}-k+\mu(3i-\overset{b_2}{2j}-2k)$

Sol] $d = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$= \left|\frac{(-10i-2j-3k)\cdot(8i+8j+4k)}{\sqrt{64+64+16}}\right|$

$= \left|\frac{-80-16-12}{12}\right| = \frac{108}{12} = 9$ units

$\vec{a}_2 - \vec{a}_2 = -10i-2j-3k$

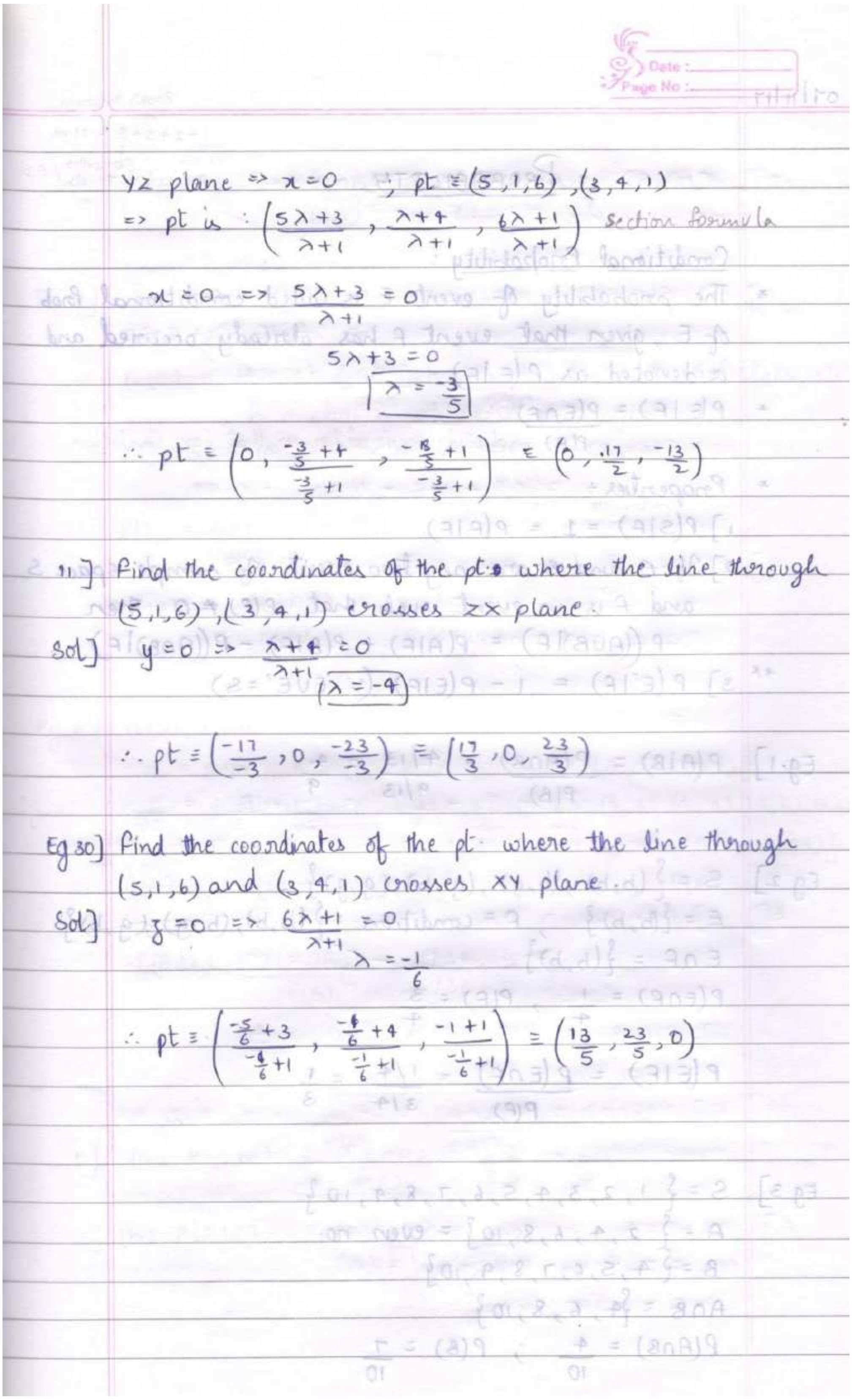
$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= i(8) - j(-8) + k(4)$

$= 8i+8j+4k$

10] Find the coordinates of the pt where the line through $(5,1,6)$ and $(3,4,1)$ crosses the YZ plane.

Sol] Let the pt be R dividing YZ plane in the ratio $\lambda : 1$



yz plane ⇒ $x = 0$; pt ≡ $(5,1,6)$, $(3,4,1)$

⇒ pt is : $\left(\frac{5\lambda+3}{\lambda+1}, \frac{\lambda+4}{\lambda+1}, \frac{6\lambda+1}{\lambda+1}\right)$ section formula

$x = 0 \Rightarrow \frac{5\lambda+3}{\lambda+1} = 0$

$5\lambda+3 = 0$

$\boxed{\lambda = \frac{-3}{5}}$

$\therefore$ pt ≡ $\left(0, \frac{\frac{-3}{5}+4}{\frac{-3}{5}+1}, \frac{-\frac{18}{5}+1}{-\frac{3}{5}+1}\right) \equiv \left(0, \frac{17}{2}, \frac{-13}{2}\right)$

11] Find the coordinates of the pt where the line through $(5,1,6)$, $(3,4,1)$ crosses zx plane.

Sol] $y = 0 \Rightarrow \frac{\lambda+4}{\lambda+1} = 0$

$\boxed{\lambda = -4}$

$\therefore$ pt ≡ $\left(\frac{-17}{-3}, 0, \frac{-23}{-3}\right) \equiv \left(\frac{17}{3}, 0, \frac{23}{3}\right)$

Eg 30] Find the coordinates of the pt where the line through $(5,1,6)$ and $(3,4,1)$ crosses XY plane.

Sol] $z = 0 \Rightarrow \frac{6\lambda+1}{\lambda+1} = 0$

$\lambda = \frac{-1}{6}$

$\therefore$ pt ≡ $\left(\frac{\frac{-5}{6}+3}{\frac{-1}{6}+1}, \frac{\frac{-1}{6}+4}{\frac{-1}{6}+1}, \frac{-1+1}{\frac{-1}{6}+1}\right) \equiv \left(\frac{13}{5}, \frac{23}{5}, 0\right)$