JEE-Main-26-06-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: Normal to $y^2 = 6x$ at P, passes through (5, -8). Find ordinate of point of intersection of directrix & tangent at P.

Answer: $\frac{-9}{4}$

Solution:

Given, $y^2 = 6x$

So, $a = \frac{3}{2}$

Thus equation of directrix is $x = -\frac{3}{2}$

Now, equation of normal be

$tx + y = 2at + at^3$

It pass through (5, -8)

Then $5t - 8 = 2\left(\frac{3}{2}\right)t + \frac{3}{2}t^3$

$5t - 8 = 3t + \frac{3}{2}t^3$

$10t - 16 = 6t + 3t^3$

$\Rightarrow 3t^3 - 4t + 16 = 0$

$\Rightarrow 3t^2(t+2) - 6t(t+2) + 8(t+2) = 0$

$(t+2)(3t^2 - 6t + 8) = 0$

$\therefore t = -2$

Equation of tangent

$ty = x + at^2$

$-2y = -x + \left(\frac{3}{2}\right)(4)$

$-2y = x + 6$

$\therefore$ Intersecting point $-2y = -\frac{3}{2} + 6$

$y = \frac{3}{4} - 3$

$= \frac{-9}{4}$

Question: A biased coin is tossed 5 times, probability of 4 heads = probability of 5 heads then probability of atmost 2 heads.

Answer: $\frac{46}{6^4}$

Solution:

Let p be the probability of getting head

And q be the probability of getting tail

i.e., such that $p+q=1$

Now, according to question

$^5C_4 p^4 q^1 = {}^5C_5 p^5$

$\Rightarrow 5(1-p) = p$

$p = \frac{5}{6}$

Now, probability of atleast 2 heads

$= {}^5C_0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$

$= \left(\frac{1}{6}\right)^5 + 5\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^4 + 10\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$

$= \frac{1}{65}\left(1+5^2+250\right)$

$= \frac{276}{6^5} = \frac{46}{6^4}$

Question: If $\frac{x}{a}+\frac{y}{b}=1$ is tangent to $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=1$ then $n \in$ ___

Answer: ()

Solution:

Given, $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=1$

$\Rightarrow n\left(\frac{x}{a}\right)^{n-1}\frac{1}{a}+n\left(\frac{y}{b}\right)^{n-1}\frac{1}{b}\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-b}{a}\left(\frac{xb}{ya}\right)^{n-1}$

At (a,b)

$\frac{dy}{dx}=-\frac{b}{a}$

Equation of tangent

$$(y-b)=\frac{-b}{a}(x-a)$$

$$\frac{y}{b}-1=-\frac{x}{a}+1$$

$$\frac{x}{a}+\frac{y}{b}=2$$

Thus, for number of value of 'n', $\frac{x}{a}+\frac{y}{b}=1$ is tangent.

Question: If $A=\{x:\text{HCF}\{x,45\}=1\}$ & $B=\{x=2k;1\le k\le 100\}$ then $A\cap B=$

Answer: 53.00

Solution:

$B=\{x=2k;1\le k\le 100\}$

Thus, $x\in\{2,4,6,8,10,\ldots 100\}$

$B=\{2,4,6,8,10,12,\ldots 200\}$

Thus, $n(B)=100$

Now, $A=\{x:\text{HCF}\{x,45\}=1\}$

Thus, multiple of 5 and 3 should not be there

Thus, $A=\{1,2,4,7,8,11,13,4,\ldots\}$

Thus, $A\cap B$ will contain elements in B which are not multiple of 2 and 5, i.e., 10, 20, ... 200

Thus, total 20

And not multiple of 3 and 2 i.e., 6, 12, 18, ..., 198

Thus, total $33-6=27$

Thus, $n(A\cap B)=100-47=53$

Question: Find remainder when 2021^{2023} is divided by 7.

Answer: 5.00

Solution:

Given, 2021^{2023}

$\Rightarrow(2016+5)^{2023}$

$\Rightarrow {}^{2023}C_0 2016^{2023}+\ldots.+{}^{2023}C_{2022}(2016)(5)^{2022}+5^{2023}$

$\Rightarrow 7(\text{Integer})+5^{2023}$

Now, $5^{2023}=5\left(5^2\right)^{674}$

$=5(126-1)^{674}$

$=5\left[{}^{674}C_0(126)^{674}+\ldots.+1\right]$

$=5[7\text{ Integer}+1]$

$= 7(\text{Integer}) + 5$

Thus remainder is 5

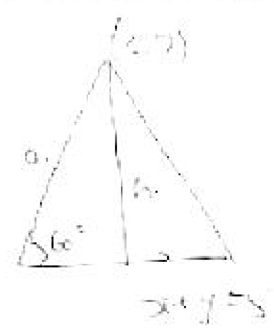
Question: The point P(3, 7) is one of the vertices of an equilateral ΔPQR & $x + y = 5$ is the equation of QR, then area of ΔPQR is:

Answer: $\frac{25}{2\sqrt{3}}$

Solution:

Given, one vertex (3, 7)

$\therefore h = \left|\frac{10-5}{\sqrt{2}}\right| = \frac{5}{\sqrt{2}}$



Also, $\sin 60° = \frac{4}{9}$

$a = \frac{h}{\sin 60°} = \frac{5}{\sqrt{2}} \frac{2}{\sqrt{3}} = \frac{5\sqrt{2}}{\sqrt{3}}$

$\therefore$ Area of $\Delta = \frac{\sqrt{3}}{4}\left(\frac{5\sqrt{2}}{\sqrt{3}}\right)^2$

$= \frac{25}{2\sqrt{3}}$

Question: $\lim_{x \to \sqrt{2}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)} = ?$

Answer: $-\frac{1}{\sqrt{2}}$

Solution:

Given, $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$.

Applying L-Hospital rule

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\cos(\cos^{-1} x)\left(\frac{-1}{\sqrt{1-x^2}}\right) - 1}{-\sec^2 x(\cos^{-1} x)} = \frac{\frac{1}{\sqrt{2}}(-\sqrt{2}) - 1}{2(-\sqrt{2})}$$

$= \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

Question: If $\bar{a}.b = 1, \bar{b}.\bar{c} = 2$ & $\bar{c}.\bar{a} = 3$ then $\left[\bar{a}\times\left(\bar{b}\times\bar{c}\right), \bar{b}\times\left(\bar{c}\times\bar{a}\right), \bar{c}\times\left(\bar{b}\times\bar{a}\right)\right] = ?$

Answer: 0.00

Solution:

Given, $\left[a\times(b\times c) \quad b\times(c\times a) \quad c\times(b\times a)\right]$

$\Rightarrow \left\{\left(a\times(b\times c)\right)\times\left(b\times(c\times a)\right)\right\}\cdot\left(c\times(b\times a)\right)$

$\Rightarrow \left\{\left((\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}\right)\times\left(\bar{b}.\bar{a}\right)\bar{c} - \left(\bar{b}.\bar{c}\right)\bar{a}\right\}\cdot\left((\bar{c}.\bar{a})\bar{b} - (\bar{c}.\bar{b})\bar{a}\right)$

$\Rightarrow \left\{\left(3\bar{b} - \bar{c}\right)\times\left(\bar{c} - 2\bar{a}\right)\right\}\cdot\left(3\bar{b} - 2\bar{a}\right)$

$\Rightarrow \left\{3(b\times c) - 6\left(\bar{b}\times\bar{a}\right) - 0 + 2\left(\bar{c}\times\bar{a}\right)\right\}\cdot\left(3\bar{b} - 2\bar{a}\right)$

$\Rightarrow 6(c\times a)\cdot b - 6(b\times c)\cdot a$

$\Rightarrow 6[a \quad b \quad c] - 6[a \quad b \quad c] = 0$

Question: From 10 Boys & 5 Girls, in how many we can select 3 boys & 3 girls such that B_1 & B_2 are not together in a group.

Answer: 1120.00

Solution:

8 Boys	2 Boys	5 Girls
3		3
2	1	3

Number of ways $= {}^8C_3 \times {}^5C_3 + {}^8C_2 \times {}^2C_1 \times {}^5C_3$

$= 560 + 560$

$= 1120$

Question: If $\sin^2 10\times\sin 20\times\sin 40\times\sin 50\times\sin 70 = \alpha - \frac{\sin 10}{16}$. Find α.

Answer: $\frac{1}{64}$

Solution:

Given, $\sin^2 10\times\sin 20\times\sin 40\times\sin 50\times\sin 70$

$\Rightarrow \sin 10\times\cos 160\times\cos 140\times\cos 40\times\cos 20\times\cos 80$

$\sin 10[\cos 20.\cos 40.\cos 80.\cos 160]\sin 40$

$\sin 10\left[\frac{\sin 2^4 10}{2^4 \sin 10}\right]\sin 40$

$\sin 10\left[\frac{\sin 160}{16\sin 10}\right]\sin 40$

$\frac{\sin 10 \sin 20 \sin 40}{16\sin 10}$

$\Rightarrow \frac{1}{32}(2\sin 20 \sin 40)$

$\Rightarrow \frac{1}{32}(\cos(40-20)-\cos(40-20))$

$\Rightarrow \frac{1}{32}(\cos 20-\cos 60)$

$\frac{\cos 20}{32}-\frac{1}{64}$

$\frac{102\sin^2 10}{32}-\frac{1}{64}$

$\frac{1}{32}-\frac{1}{64}-\frac{1}{16}(\sin^2 10)$

$\frac{1}{64}-\frac{\sin^2 10}{16}$

Thus, $\alpha=\frac{1}{64}$

Question: If mean of $a,b,8,5,10$ is 6 & variance is 6.8 then find 25M, where M is mean deviation about mean.

Answer: 60.00

Solution:

Given, $\overline{x}=\frac{a+b+8+5+10}{5}=6$

$a+b+23=30$

$a+b=7 \qquad ...(1)$

Variance $=\frac{\sum(x_i)^2}{5}-(\overline{x})^2$

$6.8=\frac{a^2+b^2+64+25+100}{5}-36$

$(42\cdot 8)5=a^2+b^2+189$

$a^2+b^2=25 \qquad ....(2)$

Thus, $a=4$ and $b=3$

Thus, mean deviation $=\frac{\sum|x_i-\overline{x}|}{5}$

$=\frac{2+3+2+1+4}{5}$

$M = \frac{12}{5}$

$\therefore 25M = 25\left(\frac{12}{5}\right) = 60$

Question: If $|adj\,24A| = |adj\,3(adj(2A))|$ & A is 3×3, then $|A|^2$ is ___

Answer: 64.00

Solution:

$|adj\;A| = |A|^{n-1}$ & $|kA| = k^n|A|$

$\Rightarrow |24A|^{3-1} = |3\;adj\,2A|^{3-1}$

$\Rightarrow (24^3)^2|A|^2 = (3^3)^2|adj\;2A|^2$

$24^6|A|^2 = 3^6\left(|2A|^2\right)^2$

$24^6|A|^2 = 3^6(2^3)^4|A|^4$

$24^6|A|^2 = 3^6\cdot8^4|A|^4$

$3^6\times8^6|A|^2 = 3^68^4|A|^4$

$|A|^2 = 8^2 = 64$