Candidates must write the Code on the title page of the answer-book.

| | NOTE |
|---|---|
| (I) | Please check that this question paper contains 15 printed pages. |
| (II) | Code number given on the right hand side of the question paper should be written on the title page of the answer -book by the candidate. |
| (III) | Please ch eck that this question paper contains 36 questions. |
| (IV) | Please write down the Serial Number of the question in the answer -book before attempting it. |
| (V) | 15 minute t ime has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer -book during this period. |

Time allowed : 3 hours

Maximum Marks : 80

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i)      This question paper comprises   four   Sections A, B, C and D. This question paper carries   36 questions.  All  questions are compulsory.

(ii)     Section A   – Questions no.  1 to 20 comprises of  20 questions of  1 mark each  .

(iii)    Section B   – Questions no.  21 to 26 comprises of  6 questions of  2 marks each  .

(iv)    Section C   – Questions no.  27 to 32 comprises of  6 questions of  4 marks each  .

(v)     Section D   – Questions no.  33 to 36 comprises of  4 questions of  6 marks each  .

(vi)    There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.

(vii)   In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii)  Use of calculators is    not  permitted.

## SECTION A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1.   If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $fog (x)$ for $x < 0$ is

   (A)    4x

   (B)    2x

   (C)    0

   (D)     – 4x

2.   The principal value of $\cot^{-1} (-\sqrt{3})$ is

   (A)    $-\dfrac{\pi}{6}$

   (B)    $\dfrac{\pi}{6}$

   (C)    $\dfrac{2\pi}{3}$

   (D)    $\dfrac{5\pi}{6}$

3. If $A = \begin{bmatrix} - & & 0 \\ 0 & - & 0 \\ 0 & & -20 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

   (A)    64
   (B)    16
   (C)    0
   (D)    $-8$

4. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

   (A)    15
   (B)    12
   (C)    9
   (D)    0

5. $\displaystyle\int \frac{x}{\cos^2 (xe^x)} \, dx$ is equal to

   (A)    $\tan (xe^x) + c$
   (B)    $\cot (xe^x) + c$
   (C)    $\cot (e^x) + c$
   (D)    $\tan [e^x (1 + x)] + c$

6. The degree of the differential equation $x^2 \dfrac{d^2 y}{dx^2} \cdot x \dfrac{dy}{dx} - y^3$ is

   (A)    1
   (B)    2
   (C)    3
   (D)    6

7. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

   (A)    0
   (B)    $\dfrac{1}{\sqrt{3}}$
   (C)    1
   (D)    $\sqrt{3}$

8. The coordinates of the foot of the perpendicular drawn from the point (– 2, 8, 7) on the XZ -plane is
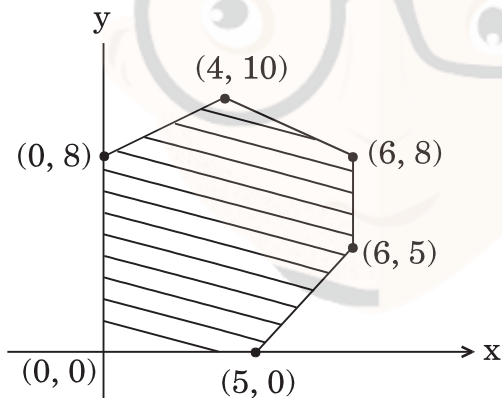
   (A)   (– 2, – 8, 7)

   (B)   (2, 8, – 7)

   (C)   (– 2, 0, 7)

   (D)   (0, 8, 0)

9. The vector equation of XY-plane is

   (A)   $\vec{r} \cdot \hat{k} = 0$

   (B)   $\vec{r} \cdot \hat{j} = 0$

   (C)   $\vec{r} \cdot \hat{i} = 0$

   (D)   $\vec{r} \cdot \vec{n} = 1$

10. The feasible region for an LPP is shown below :
    Let z = 3x – 4y be the objective function. Minimum of z occurs at



   (A)   (0, 0)

   (B)   (0, 8)

   (C)   (5, 0)

   (D)   (4 , 10)

Fill in the blanks in question numbers 11 to 15.

11. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in R$, then $\dfrac{dy}{dx}$ is equal to _____ .

OR

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in Z$, then $\dfrac{dy}{dx}$ is equal to _____ .

12. The value of $\lambda$ so that the function f defined by

$$f(x) = \begin{cases} \lambda x, & \text{if } x < \pi \\ \cos x, & \text{if } x \geq \pi \end{cases}$$

is continuous at $x = \pi$ is _____ .

13. The equation of the tangent to the curve $y = \sec x$ at the point (0, 1) is _____ .

14. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is _____ square units.

OR

The value of $\lambda$ for which the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \lambda\hat{k}$ are orthogonal is _____ .

15. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is _____ .

Question numbers 16 to 20 are very short answer type questions.

16. Construct a $2 \times 2$ matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.

17. Differentiate $\sin^2(\sqrt{x})$ with respect to x.

18. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

19. Evaluate :

$$\int_{-2}^{2} |x|\, dx$$

OR

Find :

$$\int \frac{dx}{x^2}$$

20. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

## SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Solve for x :

$$\sin^{-1} 4x + \sin^{-1} 3x = -\frac{\pi}{2}$$

OR

Express $\tan^{-1} \frac{\cos x}{1 - \sin x}$, $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

22. Express $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

23. If $y^2 \cos \frac{1}{x} = a^2$, then find $\frac{dy}{dx}$.

24. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$,

$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

OR

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle.

25. Find the coordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the $ZX$-plane.

26. If A and B are two events such that $P(A) = 0\cdot4$, $P(B) = 0\cdot3$ and $P(A \cup B) = 0\cdot6$, then find $P(B \mid A)$.

## SECTION C

Question numbers 27 to 32 carry 4 marks each.

27. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

OR

Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation.

28. If $y = x^3 (\cos x)^x + \sin^{-1}\sqrt{x}$, find $\dfrac{dy}{dx}$.

29. Evaluate :
$$\int_{-1}^{5} (|x| + |x + 1| + |x - 5|)\, dx$$

30. Find the general solution of the differential equation
$$x^2 y\, dx - (x^3 + y^3)\, dy = 0.$$

31. Solve the following LPP graphically :

Minimise $z = 5x + 7y$

subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

32. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin ?

OR

The probability distribution of a random variable X, where k is a constant is given below :

$$P(X = x) = \begin{cases} 1 \cdot 0 & \text{if } x = 0 \\ kx^2, & \text{if } x = 1 \\ kx, & \text{if } x = 2 \text{ or } 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine
(a)    the value of k
(b)    $P(x \leq 2)$
(c)    Mean of the variable X.

## SECTION D

Question numbers 33 to 36 carry 6 marks each.

33.    Solve the following system of equations by matrix method :

$$x - y + 2z = 7$$
$$2x - y + 3z = 12$$
$$3x + 2y - z = 5$$

OR

Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -1 & - & 4 \\ & & 2 \end{bmatrix}$$

34.    Find the points on the curve    $9y^2 = x^3$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

35.    Find the area of the following region using integration :

$$\{(x, y) : y \leq |x| + 2, y \geq x^2\}$$

OR

Using integration, find the area of    a triangle whose vertices are (1, 0), (2, 2) and (3, 1).

36.    Show that the lines

$$\frac{x-2}{1} \quad \frac{y-2}{3} \quad \frac{z-3}{1} \text{ and } \frac{x-2}{1} \quad \frac{y-3}{4} \quad \frac{z-4}{2} \text{ intersect.}$$

Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.