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MENSURATION

Mensuration is mainly divided into three parts:

One is **Area**, second is **Volume** and third is **Perimeter.**

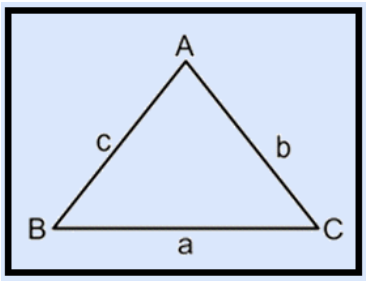
Triangle

Perimeter = a + b + c

Area of Triangle = [½ × base × height] or

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Here, s is a semi perimeter



Q1. Sides of a triangle are 12m, 13m and 11m. Find the area of triangle and height with respect to the side of 12 cm.

Solution:

S= $\frac{12+13+11}{2} = \frac{36}{2} = 18m$

Area of triangle =$\sqrt{s(s-a)(s-b)(s-c)}$

Area= $\sqrt{18\times6\times5\times6}$

Area= $6\sqrt{105}\ m^2$

Side=12m (Base), height=?

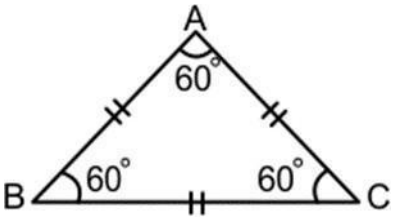
Area of triangle =1/2×base×Height

6√105 = ½ × 12 × height

Height = √105 m

Properties of Equilateral Triangle

- Properties of equilateral triangle
- All three sides are equal.
- All three angles are 60°



Formulae

- Perimeter = 3 × side = 3a
- Area of equilateral triangle = $(\sqrt{3}/4)a^2$
- Height = Median = $(\sqrt{3}/2)a$
- Side of an equilateral triangle = $\frac{The\ sum\ of\ distances\ of\ a\ point\ inside\ a\ triangle\ from\ all\ the\ sides \times 2}{\sqrt{3}}$

Q2. The area of an equilateral triangle is 49√3 m2, find the value of its height and side?

Solution:

The area of an equilateral triangle = $49\sqrt{3}\ m^2$

$(\sqrt{3}/4)a^2 = 49\sqrt{3}$

$a^2 = 49 \times 4$

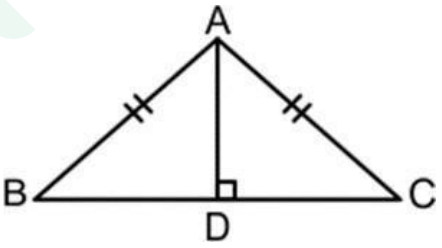
a = 14 m

Height = $(\sqrt{3}/2)a$

Height = $(\sqrt{3}/2) \times 14 = 7\sqrt{3}$ m

Properties of Isosceles Triangle

- Two sides are equal.
- AB = AC



Formula:

Area of Δ = ½ × base × height

Q3. In an isosceles triangle, if the ratio of equal side and unequal side is 5: 8 and its perimeter is 36 cm. Find the area of this triangle.

Solution:

Let the equal and unequal side be 5x and 8x

Perimeter of this triangle = 5x + 5x + 8x

36 = 18x

x = 2

Sides of this triangle are 10 cm, 10 cm and 16 cm

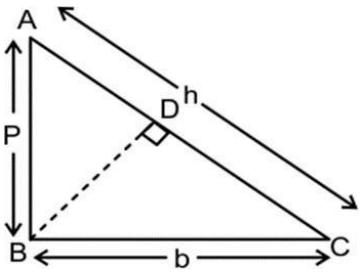
Perpendicular on unequal side = $\sqrt{[10^2 - (16/2)^2]} = \sqrt{[100 - 64]}$

Perpendicular on unequal side = 6 cm

Area of isosceles triangle = ½ × unequal side × perpendicular on unequal side

Area of isosceles triangle = ½ × 16 × 6 = 48 cm^2

Properties of Right-Angle Triangle



- Hypotenuse^2 = (base)^2 + (height)^2
- Area = ½ × base × height = ½ × b × p
- Length of Perpendicular at hypotenuse = $\frac{Height \times Base}{Hypotenuse}$

Q4. The area of a right-angle triangle is 30 m and base is 7 m more than height, find the length of perpendicular on hypotenuse.

Solution:

Let the height of the triangle be x

⇒ Base = x + 7

⇒ Area = ½ × base × height

⇒ 30= ½ × (x + 7) × x

⇒ 60 = x^2 + 7x

⇒ x^2 + 7x – 60 = 0

⇒ (x + 12) (x – 5) = 0

⇒ x = 5 m

⇒ Base = x + 7 = 12 m

⇒ Hypotenuse^2 = (base)^2 + (height)^2

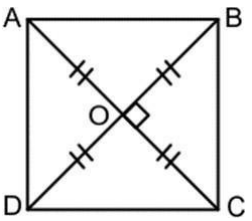
⇒ Hypotenuse^2 = 144 + 25 = 169

⇒ Hypotenuse = 13 m

The length of perpendicular on hypotenuse is (60/13) m.

Square

Properties of square



- Opposite sides of a square are both parallel and equal in length.
- The diagonal of a square is equal. AC = BD
- The diagonals of a square bisect each other and meet at 90°. AO = OC, BO = OD and ∠BOC = 90°
- All four angles of a square are 90°.

Formula:

- Perimeter = 4 × side
- Area = $(side)^2$ = ½ × (diameter)
- Diameter = √2 side

Q5. Area of a square is 1352 m2. Find the perimeter and the length of the diagonal.

Solution:

Area of a square = 1352

$(Side)^2$ = 1352

Side = 26√2 m

Perimeter = 4 × side

Perimeter = 4 × 26√2 = 104√2 m

Length of diagonal = √2 × side

Length of diagonal = √2 × 26√2 = 52 m

Rectangle

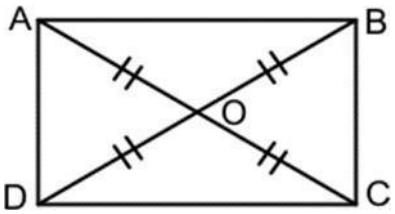
Properties of rectangle

- Opposite sides of a rectangle are equal in length.

  AB = DC, AD = BC
- Opposite sides of a rectangle are parallel.

  AB ‖ CD, AD ‖ BC
- The diagonals of a rectangle bisect each other.

  AO = OC, BO = OD
- All four angles of a square are 90°.



Formulae:

Area = l × b

Perimeter = 2(l + b)

Diameter = √ (l^2 + b^2)

Here, Length = l
Breadth = b

Q6. In a rectangle diagonal is 9 times of its length. Find the ratio of length and breath.

Solution:

Let the length of rectangle be x

Diagonal = 9x

Diameter = $\sqrt{(l^2 + b^2)}$

$9x = \sqrt{(x^2 + b^2)}$

$x^2 + b^2 = 81\ x^2$

$80\ x^2 = b^2$

$b = (4\sqrt{5})x$

Length: breath = x: $(4\sqrt{5})x$

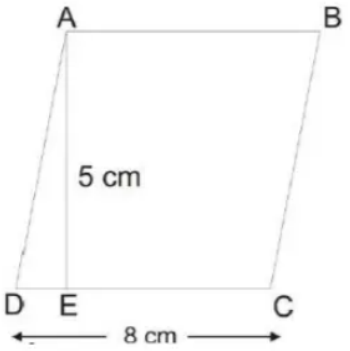
Length: breath = 1: $4\sqrt{5}$

Parallelogram

In a parallelogram, the opposite sides are parallel and equal. But all 4 sides are not equal. Also, the angle of the corners is not at 90 degrees. Whereas in square and rectangle, the angle of corners is 90 degrees.

- Area of parallelogram = l × h
- Perimeter of parallelogram = 2(l+b)

Here, l = length or base of parallelogram. h = height of parallelogram.



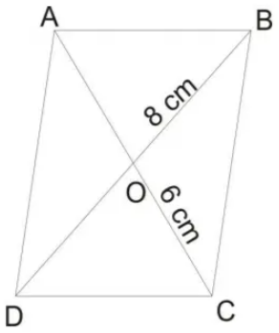
Rhombus

Rhombus is also a parallelogram. The main factor is that all 4 sides are equal in the rhombus. The mensuration formula chart for the rhombus is,

- Area of rhombus = d1 × d2 / 2

- The perimeter of rhombus = 4l

Here, d1 and d2 are the lengths of the diagonals. l is the side of the rhombus.

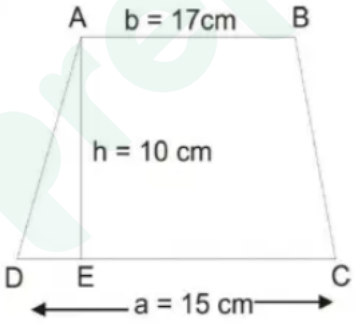


Trapezium

In trapezium, one set of opposite sides are parallel and unequal. The other set of opposite sides are not parallel.

- Area = 1 / 2 h(a+b)
- Perimeter = Sum of all sides

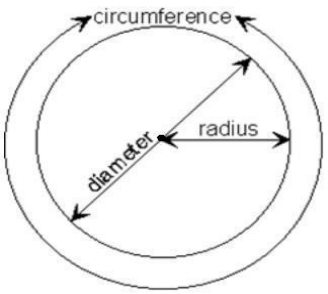
Here a and b are the top and bottom sides of the trapezium. h is the height of the trapezium.



Circle

- Circumference of a circle = π x diameter
- Diameter of circle = 2r
- Area of a circle = π x r x r

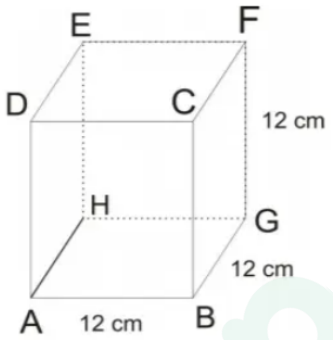
Here r is the radius of the circle.



Cube

Cube is a 3d shape of a square. In the cube all the length, breadth, and height are equal. The mensuration formula for bank exams also includes all 3d shapes. So, refer to the mensuration formulas for the cube here.

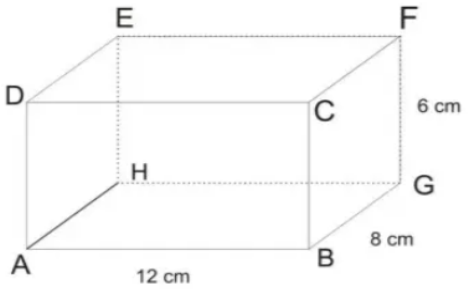
- Volume of a cube = (side)3
- Total surface area of a cube = 6 × (side)2
- Diagonal of cube = √3 x (side)



Cuboid

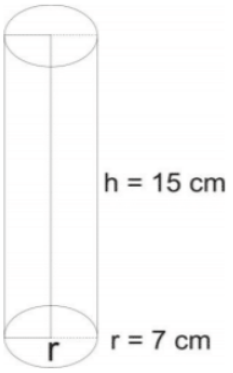
It is a 3d shape of a rectangle. The mensuration formulas for the cuboid are,

- Volume of a cuboid = (length × breadth × height) = lbh
- Total surface area of cuboid = 2(lb + bh + hl)



Cylinder

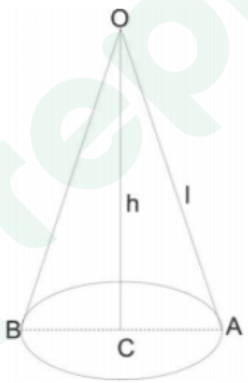
- Area of curved surface = (perimeter of base) x height = 2πrh
- Total surface area = 2πr (r + h)
- Volume = π x r x r x h



Cone

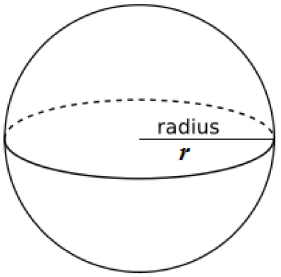
- Curved surface area = πrl
- Total surface area = πr(r + l)
- Volume of cone = (1/3) x π x r x r x h

Here l is the slant height of the cone.



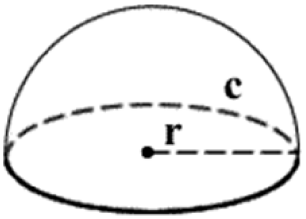
Sphere

- r = radius
- Volume: V = $\frac{4}{3}$ πr^3
- Curved Surface Area = Total Surface Area = 4π^2



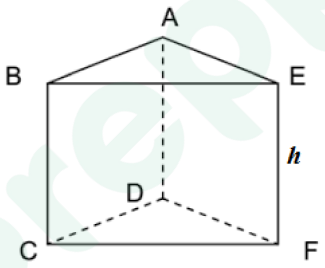
Hemisphere

- Volume = $\frac{2}{3}\pi r^3$
- Curved surface area = $2\pi r^2$
- Total surface area = $3\pi r^2$



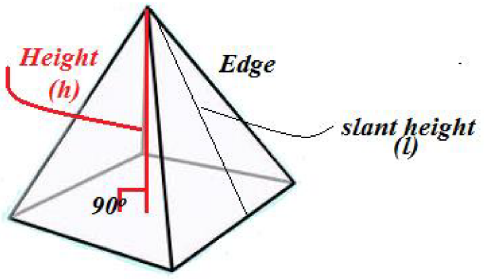
Prism

- Volume = Base area x height
- Lateral Surface area = perimeter of the base x height
- Total Surface area = LSA + 2(Area of base)



Pyramid

- Volume of a right pyramid = $\frac{1}{3}$ X area of the base height X height
- Area of the lateral faces of a right pyramid = $\frac{1}{2}$ x perimeter of the base x slant height
- Area of whole surface of a right pyramid = Area of the lateral faces + Area of the base.



Practice Questions:

Q1. Three cubes of metal whose edges are in ratio of 3:4:5, are melted and one new cube is formed. If the diagonal of the cube is 18√3 cm, then find the edge of the largest among three cubes.

(A) 18 cm
(B) 24 cm
(C) 15 cm
(D) 12 cm

Q2. What is the area of the largest triangle that can be fitted into a rectangle of length 'a' units and width 'b' units?

(A) unit2
(B) unit2
(C) unit2
(D) unit2

Q3. Find the perimeter and area of an isosceles triangle whose equal sides are 5 cm and the height is 4 cm.

(A) 24 cm2 , 13 cm
(B) 18 cm2 , 16 cm
(C) 12 cm2 , 13 cm
(D) 12 cm2 , 16 cm

Q4. A parallelogram has area A sq. mts. A second parallelogram is formed by joining the mid-points of its sides. A third parallelogram is formed by joining the mid-poinds of the sides of the second parallelogram. This process is continued upto infinite. What is the sum of area (in sq. mts) of all the parallelograms so formed?

(A) A
(B) $\frac{3A}{2}$
(C) 2A
(D) $\frac{A}{2}$

Q5. The radius of a cylindrical milk container is half its height and surface area of the inner part is 616 sq.cm. The amount of milk that the container can hold, approximately, is
[Use: √5= 2.23 and p = 22/7]

(A) 1.42 litres
(B) 1.53 litres
(C) 1.71 litres
(D) 1.82 litres

Q6. From the four corners of a rectangular sheet of dimensions 25 cm x 20 cm, square of side 2 cm is cut off from four corners and a box is made. The volume of the box is-

(A) 828 cm3
(B) 672 cm3
(C) 500 cm3

(D) 1000cm3

Q7. A rectangular paper sheet of dimensions 22 cm × 12 cm is folder in the form of a cylinder along its length. What will be the volume of this cylinder?

(A) 460 cm^3

(B) 462 cm^3

(C) 624 cm^3

(D) 400 cm^3

Q8. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diameter and 18 m length. The radius (in cm) of the wire is:

(A) $\frac{2}{15}$

(B) $\frac{1}{15}$

(C) $\frac{1}{30}$

(D) 15

Q9. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8 gm/cm3, then the weight of the pipe is:

(A) 3.696 kg

(B) 3.6 kg

(C) 36 kg

(D) 36.9 kg

Q10. Water flows at the rate of 10 meters per minute from cylindrical pipe 5 mm in diameter how long it will take to fill up a conical versel whose diameter at the base is 30 cm and depth 24 cm?

(A) 28 minutes 48 seconds

(B) 51 minutes 12 seconds

(C) 51 minutes 24 seconds

(D) 28 minutes 36 seconds

Solution:

Q1. (C)

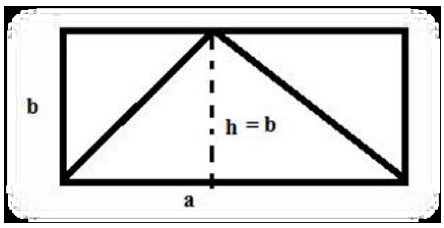
Total volume = (3x)3+(4x)3+(5x)3 = 216x3

Then side of new cube/= 6x

Then diagonal of new cube = $6x\sqrt{3} \rightarrow 18\sqrt{3}$ x = 3

Then side of larger cube = 5×3 = 15

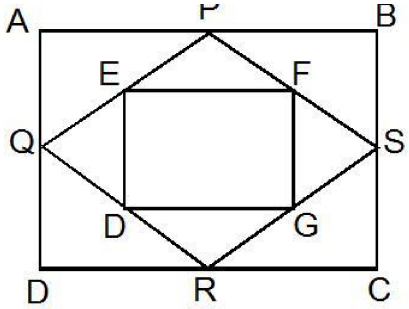
Q2. (B)



Area of triangle = (1/2) base × height
= (1/2) ab unit2

Q3. (D)
12 cm^2, 16cm

Q4. (C)



Sum of areas:

$$= A + \frac{1}{2}A + \frac{1}{4}A + - - - \infty$$

$$= \frac{A}{1-\frac{1}{2}} = 2A$$

Q5. (B)
Let radius of the container is x cm.
Then height of the container is 2x cm.
According to question 2πr(2r) + πr2 = 616
5πr2 = 616

$$r^2 = \frac{616 \times 7}{5 \times 22}$$

$$r = \frac{14}{\sqrt{5}}$$

$$V = \frac{22}{7} \times \frac{14}{\sqrt{5}} \times \frac{14}{\sqrt{5}} \times \frac{28}{\sqrt{5}}$$

= 1546.90 ml.
= 1.54 lit.

Q6. (B)
Volume of the box = 16×21×2 = 672 cm^3

Q7. (B)

$2\pi r = 22$

$R = \frac{7}{2}$ cm

$v = \pi R^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$

$= 462\ cm^3$

Q8. (C)

$\pi r^2 h = \pi R^2 h = \frac{1}{4} \times 8 = 2$

$r^2 = \frac{2}{1800} = \frac{1}{900}$

$r = \frac{1}{30}$ cm.

Q9. (A)

$r = \pi\left(R^2 - r^2\right) \times h$

$= \frac{22}{7} \times \left(4^2 - 3^2\right) \times 21 = 462\ cm^3$

$\therefore 1cm^3 = 8\,g$

$462\ cm^3 = 462 \times 8g$

$= 2696\ g = 3.696\ kg.$

Q10. (A)

Volume of water flowing from the pipe in 1 minute

$\pi \times .25 \times .25 \times 1000$

$r = \frac{1}{3}\pi \times 15 \times 15 \times 24$

$T = \frac{1}{3} \times \frac{\pi \times 15 \times 15 \times 24}{\pi \times .25 \times .25 \times 1000} = 28\frac{4}{5}$

28 minutes 48 seconds