JEE-Main-25-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $x^4+x^3+x^2+x+1=0$ equation has 4 roots $\alpha\beta\gamma\delta$. Then $\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=?$

Options:

(a) 4

(b) 1

(c) -1

(d) 4

Answer: (c)

Solution:

Given, $x^4+x^3+x^2+x+1=0$

$\Rightarrow x^5+x^4+x^3+x^2+x=0$

$\Rightarrow x^5-1=0$

$\Rightarrow x^5=1$

$\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}$

$=\alpha\left(\alpha^5\right)^{400}+\beta\left(\beta^5\right)^{400}+\gamma\left(\gamma^5\right)^{400}+\delta\left(\delta^5\right)^{400}$

$=\alpha+\beta+\gamma+\delta$

$=-1$

Question: A die is rolled twice and let the outcomes be α,β. Then probability such that $x^2+\alpha x+\beta>0$

Options:

(a) $\frac{17}{36}$

(b) $\frac{9}{36}$

(c)

(d)

Answer: (a)

Solution:

$x^2+\alpha x+\beta>0$

$\Rightarrow \alpha^2-4\beta<0$

$\Rightarrow \alpha=1,\ \beta=1,2,3,4,5,6$

$\Rightarrow \alpha=2,\ \beta=2,3,4,5,6$

$\Rightarrow \alpha=3,\ \beta=3,4,5,6$

$\Rightarrow \alpha=4,\ \beta=5,6$

$\Rightarrow \alpha=5,$ not possible

Favourable outcomes $= 6+5+4+2=17$

$\therefore p = \frac{17}{36}$

Question: $\lim_{n\to\infty} \sqrt{n^2-n+1}+n\alpha+\beta=0$ then $8(\alpha+\beta)$

Options:

(a) -8

(b) -4

(c)

(d)

Answer: (b)

Solution:

$$\lim_{n\to\infty} \sqrt{n^2-n+1}+n\alpha+\beta=0$$

$$\lim_{n\to\infty}\left(\frac{\sqrt{n^2-n+1}+n\alpha+\beta}{\sqrt{n^2-n+1}-(n\alpha+\beta)}\right)\left(\sqrt{n^2-n+1}-(n\alpha+\beta)\right)=0$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^2-n+1-(n\alpha+\beta)^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^2(1-\alpha^2)+n(-1-2\alpha\beta)+1-\beta^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$$

$\Rightarrow 1-\alpha^2=0$

$\Rightarrow \alpha=\pm 1$

$-1-2\alpha\beta=0$

$\beta=\frac{-1}{2\alpha}$

$\alpha=1, \beta=\frac{-1}{2}$

$\alpha=-1,\ \beta=\frac{1}{2}$

$8(\alpha+\beta)=\pm 4$

Question: The number of solution of $|\cos x|=\sin x$ such that $-4\pi\le x\le 4\pi$ is

Options:

(a) 4

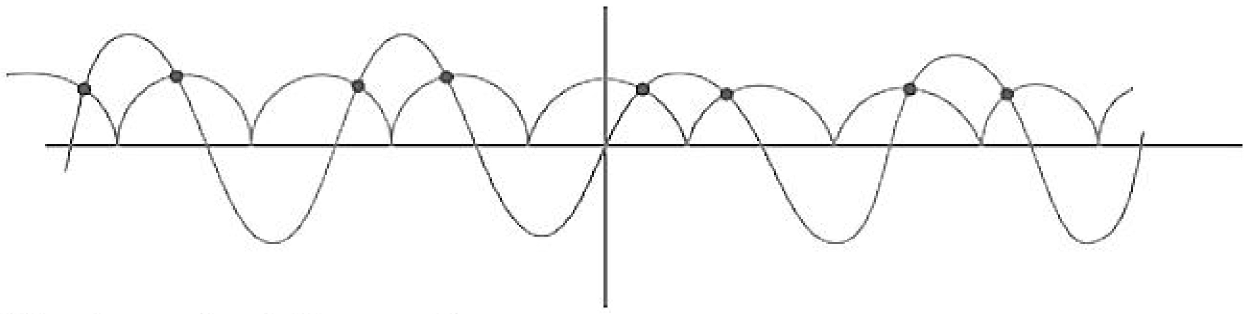
(b) 6

(c) 8

(d) 12

Answer: (c)

Solution:



Number of solutions = 8

Question: Which of the following is a tautology?
Options:
(a) $(\sim p \vee q) \Rightarrow p$
(b) $p \Rightarrow (\sim p \vee q)$
(c) $(\sim p \vee q) \Rightarrow q$
(d) $q \Rightarrow (\sim p \vee q)$
Answer: (d)
Solution:
$q \Rightarrow (\sim p \vee q)$
$\sim q \vee (\sim p \vee q)$
$(\sim q \vee \sim p) \vee (\sim q \vee q)$
$(\sim q \vee \sim p) \vee (T)$
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Question: Set $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$. Find the number of functions from A to B such that $f(1) + f(2) = f(3)$
Answer: 40.00
Solution:
$A = \{1,2,3,4\}$, $B = \{1,2,3,4,5\}$
$f(1) + f(2) = f(3)$
$1+2=3 \rightarrow 2\times 5=10$
$1+3=4 \rightarrow 2\times 5=10$
$1+4=5 \rightarrow 2\times 5=10$
$2+3=5 \rightarrow 2\times 5=10$
Number of functions $=10+10+10+10=40$

Question: If roots of $x^2 - 8ax + 2a = 0$ are $p \& r$, while $q \& s$ are roots of $x^2 + 12bx + 6b = 0$ then find $\frac{1}{a} - \frac{1}{b}$ if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ & $\frac{1}{s}$ are in AP.
Answer:
Solution:
$p + r = 8a,\ pr = 2a$
$q + s = -12b,\ qs = 6b$

$$\Rightarrow \frac{1}{p}+\frac{1}{r}=4, \frac{1}{q}+\frac{1}{s}=-2$$
$$\Rightarrow \frac{1}{p}-\frac{1}{q}+\frac{1}{r}-\frac{1}{s}=6$$
$$\Rightarrow -2d=6$$
$$\Rightarrow d=-3$$
$$\frac{1}{p}+\frac{1}{p}-3=4$$
$$\Rightarrow \frac{2}{p}=7$$
$$\Rightarrow \frac{1}{p}=\frac{2}{7}, \frac{1}{q}=\frac{-19}{7}, \frac{1}{r}=\frac{-40}{7}, \frac{1}{s}=\frac{-61}{7}$$
$$\frac{1}{a}=\frac{2}{pr}=2\cdot\frac{2}{7}\cdot\left(\frac{-40}{7}\right)=\frac{-80}{49}$$
$$\frac{1}{b}=\frac{6}{qs}=6\cdot\left(\frac{-19}{7}\right)\left(\frac{-61}{7}\right)=\frac{6954}{49}$$
$$\frac{1}{a}-\frac{1}{b}=\frac{-80}{49}-\frac{6954}{49}$$
$$=\frac{-7034}{49}$$

Question: If $a_1=b_1=1, a_n=a_{n-1}+2$ & $b_n=a_n+b_{n-1}$, then find $\sum_{n=1}^{15} a_n \times b_n$

Answer: 27560.00
Solution:
$$a_1=b_1=1,\ a_n=a_{n-1}+2$$
$$a_r=1+(r-1)2=2r-1$$
$$b_r=a_r+b_{r-1}=a_r+a_{r-1}+b_{r-2}$$
$$=a_r+a_{r-1}+a_{r-2}+....+a_2+b_1$$
$$=\sum(2r-1)$$
$$=\frac{2(r)(r+1)}{2}-r=r^2$$
$$\sum a_n b_n=\sum(2n-1)n^2$$
$$=\sum 2n^3-\sum n^2$$
$$=2\left(\frac{15\times 16}{2}\right)^2-\frac{15\times 16\times 31}{6}$$
$$=27560$$

Question: A line with slope greater than 1, passes through $A(4,3)$. Line $x-y=2$ intersects former line at B. Find B if $AB=\frac{\sqrt{29}}{3}$.

Answer: $\frac{17}{3},\frac{11}{3}$

Solution:

$$\frac{x-4}{\cos\theta}=\frac{y-3}{\sin\theta}=r$$

$$x=\frac{\sqrt{29}}{3}\cos\theta+4,\ y=\frac{\sqrt{29}}{3}\sin\theta+3$$

$$x-y=2$$

$$\Rightarrow\frac{\sqrt{29}}{3}\cos\theta+4-\frac{\sqrt{29}}{3}\sin\theta-3=2$$

$$\Rightarrow\frac{\sqrt{29}}{3}(\cos\theta-\sin\theta)=1$$

$$\Rightarrow\cos\theta-\sin\theta=\frac{3}{\sqrt{29}}$$

$$\Rightarrow 1-\sin 2\theta=\frac{9}{29}$$

$$\Rightarrow\sin 2\theta=\frac{20}{29}$$

$$\Rightarrow\sin\theta+\cos\theta=\sqrt{1+\left(\frac{20}{29}\right)}$$

$$\Rightarrow\sin\theta+\cos\theta=\frac{7}{\sqrt{29}}$$

$$\sin\theta=\frac{2}{\sqrt{29}},\cos\theta=\frac{5}{\sqrt{29}}$$

$$x=\frac{\sqrt{29}}{3}\times\left(\frac{5}{\sqrt{29}}\right)+4=\frac{5}{3}+4=\frac{17}{3}$$

$$y=\frac{\sqrt{29}}{3}\times\frac{2}{\sqrt{29}}+3=\frac{2}{3}+3=\frac{11}{3}$$

$$B\equiv\left(\frac{17}{3},\frac{11}{3}\right)$$

Question: Find remainder when $(2024)^{2024}$ is divided by 7.

Answer: 1.00

Solution:

$$(2024)^{2024}$$

$$=(289\times+1)^{2024}$$

$= {}^{2024}C_0 + {}^{2024}C_1(289\times 7)+....$

Remainder = 1

Question: $z=1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}$, find arg.

Answer: $\frac{3\pi}{5}$

Solution:

$z=1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}$

$=2\cos^2\frac{6\pi}{10}+2i\sin\frac{6\pi}{10}\cos\frac{6\pi}{10}$

$=2\cos\frac{3\pi}{5}\left(\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}\right)$

$\arg(z)=\frac{3\pi}{5}$

Question: Sum & product of mean & variance of Binomial distribution are 24 & 128 respectively. Find probability of 1 or 2 successes.

Answer:

Solution:

Mean and variance are roots of $x^2-24x+128=0$

$\Rightarrow$ Mean $=np=16$

Variance $=npq=8$

$\Rightarrow p=q=\frac{1}{2},\ n=32$

$p(x=1)+p(x=2)={}^{32}C_1\left(\frac{1}{2}\right)^{32}+{}^{32}C_2\left(\frac{1}{2}\right)^{32}$

$=\left({}^{32}C_1+{}^{32}C_2\right)\left(\frac{1}{2}\right)^{32}$

$={}^{33}C_2\left(\frac{1}{2}\right)^{32}$

Question: Find dictionary rank of MANKIND.

Answer: 1492.00

Solution:

M A N K I N D

$\frac{4\cdot 6!}{2!}+0+\frac{3(4!)}{2!}+2(3!)+2!+1+1$

$=1440+36+12+2+1+1$

$=1492$

Question: If the locus of centre (α, β) of circle that touches both $x^2+(y-1)^2=1$ & x-axis, then find area enclosed by locus & $y=4$

Answer: $\frac{64}{3}$

Solution:

$(x-\alpha)^2+(y-\beta)^2=\beta^2$

$|\beta+1|=\sqrt{(\alpha)^2+(\beta-1)^2}$

$\Rightarrow \alpha^2=4\beta$

$\Rightarrow x^2=4y$

$\Rightarrow y=\frac{x^2}{4}$

Area $=2\int_0^4\left(4-\frac{x^2}{4}\right)=\frac{64}{3}$

Question: $p(x)=x^2+ax^2+bx+c,\ y=p(x)$ touches x-axis at $(-2,0)$ & $p'(0)=3$. Find local maxima.

Answer: 0.00

Solution:

$p(x)=x^3+ax^2+bx+c$

$p(-2)=0$

$0=-8+4a-2b+c$

$p'(-2)=0$

$\Rightarrow 12-4a+b=0$

$p'(0)=3$

$\Rightarrow c=3$

$\Rightarrow b=7$

$\Rightarrow a=\frac{19}{4}$

$p(x)=x^3+\frac{19}{4}x^2+7x+3$

$p'(x)=3x^2+\frac{19}{2}x+7$

$x=\frac{\frac{-19}{2}\pm\sqrt{\left(\frac{19}{2}\right)^2-4\cdot 7\cdot 3}}{6}$

$=\frac{\frac{-19}{2}\pm\frac{5}{2}}{6}=-2,\frac{-7}{6}$

$p''(x) = 6x + \frac{19}{2}$

$p''(-2) = -12 + \frac{19}{2} < 0$

$p''\left(\frac{-7}{6}\right) > 0$

Maxima = $p(-2) = 0$