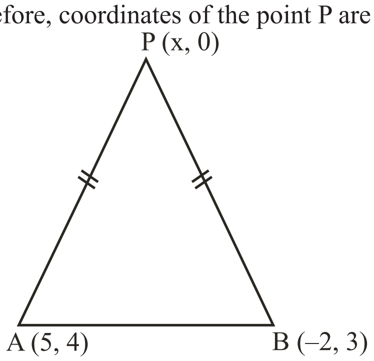
Sample Paper 4

ANSWERKEY																			
1	(b)	2	(a)	3	(b)	4	(b)	5	(d)	6	(d)	7	(c)	8	(a)	9	(c)	10	(b)
11	(a)	12	(c)	13	(c)	14	(c)	15	(c)	16	(c)	17	(b)	18	(b)	19	(c)	20	(a)
21	(a)	22	(d)	23	(d)	24	(a)	25	(b)	26	(d)	27	(a)	28	(b)	29	(a)	30	(c)
31	(d)	32	(b)	33	(b)	34	(b)	35	(d)	36	(c)	37	(b)	38	(c)	39	(a)	40	(b)
41	(a)	42	(d)	43	(a)	44	(b)	45	(c)	46	(a)	47	(c)	48	(b)	49	(d)	50	(a)

SOLUTIONS

1. (b) $x + y = 1$ & $x^3 + y^3 + 3xy$
$= (x + y)^3 - 3xy(x + y) + 3xy = 1$

2. (a) Since, the required point (say P) is on the x-axis, its ordinate will be zero. Let the abscissa of the point be x.
Therefore, coordinates of the point P are (x, 0).



Let A and B denote the points (5, 4) and (–2, 3) respectively.
Given that AP = BP, we have
$AP^2 = BP^2$
i.e. $(x - 5)^2 + (0 - 4)^2$
$= (x + 2)^2 + (0 - 3)^2$
$\Rightarrow x = 2$

3. (b) Here, $x - y = 3$ ...(i)
and $xy = 54$
$\therefore \quad (x + y)^2 = (x - y)^2 + 4xy$
$= (3)^2 + 4(54) = 225$
$\Rightarrow \quad (x + y) = \sqrt{225} = \pm 15$ ...(ii)

Case I :
If $x + y = 15$ and $x - y = 3$
On adding the above two equations
$2x = 18 \Rightarrow x = 9$
$\therefore \quad x + y = 15 \Rightarrow 9 + y = 15 \Rightarrow y = 6$

Case II
If $x + y = -15$ and $x - y = 3$
On adding the above two equations
$2x = -12$
$x = -6$
$\therefore \quad x + y = -15 \Rightarrow -6 + y = -15$
$\Rightarrow \quad y = -15 + 6 \Rightarrow \quad y = -9$

4. (b) Let full fare = ₹ x
and reservation charges = ₹ y
$\therefore$ $x + y = 2125$ ....(i)
Also $(x + y) + \left(\frac{x}{2} + y\right) = 3200$ from (i),

$2125 + \frac{x}{2} + y = 3200, \quad \frac{x}{2} + y = 3200 - 2125$

$\Rightarrow x + 2y = 1075 \Rightarrow x + 2y = 2150$ ....(ii)
Solving (i) and (ii), we get
$-y = -25$ or $y = 25$
Putting the value of y = 25 in (i)
$x + 25 = 2125$
$x = 2125 - 25, \; x = 2100$
full fare = ₹ 2100 and reservation charges = ₹ 25

5. (d) We have, $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta}$

$= \frac{\tan\theta}{\sin\theta\cos\theta} - \frac{\cot\theta}{\sin\theta\cos\theta}$

$= \frac{\sin\theta}{\cos\theta\sin\theta\cos\theta} - \frac{\cos\theta}{\sin\theta\cos\theta\cos\theta}$

$= \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta} = \sec^2\theta - \operatorname{cosec}^2\theta$

$= 1 + \tan^2\theta - 1 - \cot^2\theta = \tan^2\theta - \cot^2\theta$

6. **(d)** L.C.M × H.C.F = First number × second number

Hence, required number = $\frac{36\times 2}{18} = 4$.

7. **(c)** Let BD = x cm

Since AC = BC, therefore ΔABC is an isoscele triangle.

$\Rightarrow$ $\angle B = \angle CAB = 72°$

Since AD bisects ∠A

$\therefore$ ∠DAB = 36° so, In ΔADB, ∠ADB = 72°

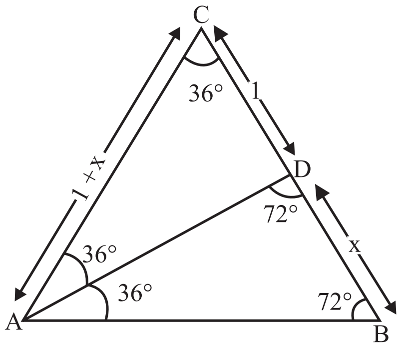
$\Rightarrow$ ΔADB is an isoscele triangle

$\therefore$ AB = AD = 1cm

$\Rightarrow$ AB = 1 cm

Similarly, ΔADC is also an isoscele triangle.

$\therefore$ AD = CD ⇒ AD = 1 cm



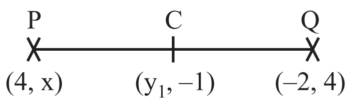
Now $\frac{AC}{AB} = \frac{CD}{BD}$

$\Rightarrow \frac{1+x}{1} = \frac{1}{x} \quad \Rightarrow \quad x + x^2 - 1 = 0$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2} = \frac{-1\pm\sqrt{5}}{2}$

BD = $\frac{\sqrt{5}-1}{2}$

8. **(a)** Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.



We have, $\frac{4-2}{2} = y$ and $\frac{4+x}{2} = -1$

$\therefore$ $y = 1$ and $x = -6$

9. **(c)** Required probability = $\frac{5}{25} = \frac{1}{5}$.

10. **(b)** Value of $n = 2$.

11. **(a)** Let the radii of the outer and inner circles be r_1 and r_2 respectively; we have

Area $= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$

$= \pi(r_1 - r_2)(r_1 + r_2)$

$= \pi(5.7 - 4.3)(5.7 + 4.3) = \pi \times 1.4 \times 10$ sq. cm

$=$ 3.1416 × 14sq. cm. = 43.98 sq. cms.

12. **(c)** Given, area of two similar triangles,

$A_1 = 81\text{cm}^2$, $A_2 = 49\text{ cm}^2$

Ratio of corresponding medians $= \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{81}{49}} = \frac{9}{7}$

13. **(c)** We have, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$\Rightarrow \cos\theta\left(\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}\right) = 4$

$\Rightarrow \frac{2\cos\theta}{\cos^2\theta} = 4 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

14. **(c)** Let the required ratio be $k : 1$

Then, $2 = \frac{6k-4(1)}{k+1}$ or $k = \frac{3}{2}$

$\therefore$ The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $y = \frac{3(3)+2(3)}{3+2} = 3$

15. **(c)** Let unit's digit : x, tens digit : y

then $x = 2y$, number $= 10y + x$

Also $10y + x + 36 = 10x + y$

$\therefore$ $9x - 9y = 36$ or $x - y = 4$

Solve, $x = 2y$, $x - y = 4$

Substitute $x = 2y$ in $x - y = 4$

we get, $2y - y = 4 \Rightarrow y = 4$

and $x = 8$

So, the number $= 10y + x = 48$

16. **(c)** Total outcomes = HH, HT, TH, TT

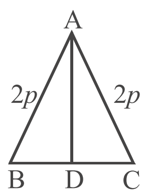
Favourable outcomes = HT, TH, TT

P(at most one head) = $\frac{3}{4}$.

17. **(b)** Given an equilateral triangle ABC in which

$AB = BC = CA = 2p$

and $AD \perp BC$.



$\therefore$ In ΔADB,

$AB^2 = AD^2 + BD^2$

(By Pythagoras theorem)

$\Rightarrow (2p)^2 = AD^2 + p^2 \quad \Rightarrow \quad AD^2 = \sqrt{3}\,p$.

18. **(b)** $x^2 + 4x + 2 = (x^2 + 4x + 2) - 2 = (x + 2)^2 - 2$

Lowest value = – 2 when $x + 2 = 0$

19. **(c)** $-\dfrac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\dfrac{4}{-9} = \dfrac{4}{9}$

20. **(a)** Required area $= \left(7^2 - \dfrac{1}{4}\times\dfrac{22}{7}\times 7^2\right)$ cm^2

$= (49 - 38.5)$ cm^2 = 10.5 cm^2

21. **(a)** Quadratic polynomial $p(x) = k(x + 1)^2$

$p(-2) = k(-2 + 1)^2 = 2$

$k = 2$

$p(x) = 2(x + 1)^2$

$p(2) = 2(2 + 1)^2 = 2 \times 3 \times 3 = 18$

22. **(d)** All the statements given in option (a, b, c) are correct.

23. **(d)** We have, $\sin 5\theta = \cos 4\theta$

$\Rightarrow 5\theta + 4\theta = 90°$

[$\because \sin\alpha = \cos\beta$, than $\alpha + \beta = 90°$]

$\Rightarrow 9\theta = 90° \Rightarrow \theta = 10°$

Now, $2\sin 3\theta - \sqrt{3}\tan 3\theta$

$= 2\sin 30° - \sqrt{3}\tan 30°$

$= 2\times\dfrac{1}{2} - \sqrt{3}\times\dfrac{1}{\sqrt{3}} = 1 - 1 = 0$

24. **(a)** Given equations are :

$7x - y = 5$ and $21x - 3y = k$

Here $a_1 = 7, b_1 = -1, c_1 = 5$

$a_2 = 21, b_2 = -3, c_2 = k$

We know that the equations are consistent with unique solution

if $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Also, the equations are consistent with many solutions

if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\therefore \dfrac{7}{21} = \dfrac{-1}{-3} = \dfrac{5}{k} \Rightarrow \dfrac{1}{3} = \dfrac{5}{k} \Rightarrow k = 15$

Hence, for $k = 15$, the system becomes consistent.

25. **(b)** $\alpha + \beta = 5$ ...(i)

$\alpha\beta = k$ ...(ii)

$\alpha - \beta = 1$ ...(iii)

Solving (i) and (iii), we get $\alpha = 3$ and $\beta = 2$.

Putting the value of α and β in (ii), we get

26. **(d)** We have, $\dfrac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{2\times 3}{\sqrt{3}\times 4} = \frac{\sqrt{3}}{2}$$

Alternate method:

$\left(\text{Using identity}, \sin 2A = \dfrac{2\tan A}{1+\tan^2 A}\right)$

$\sin 60° = \dfrac{2\tan 30°}{1+\tan^2 30°} = \dfrac{\sqrt{3}}{2}$

27. **(a)** The largest number of four digits is 9999. Least number divisible by 12, 15, 18, 27 is 540.

On dividing 9999 by 540, we get 279 as remainder.

Required number = (9999 – 279) = 9720.

28. **(b)** Let $P(x, 0)$ be a point on X-axis such that $AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x + 2)^2 + (0 - 3)^2 = (x - 5)^2 + (0 + 4)^2$

$\Rightarrow x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$

$\Rightarrow 14x = 28 \Rightarrow x = 2$

Hence, required point is (2, 0).

29. **(a)** Let side of a square = x cm

$\therefore$ By Pythagoras theorem, $x^2 + x^2 = (16)^2 = 256$

$\Rightarrow 2x^2 = 256 \Rightarrow x^2 = 128 \Rightarrow x = 8\sqrt{2}$ cm.

30. **(c)** $\dfrac{3x+4y}{x+2y} = \dfrac{9}{4}$

$\Rightarrow 4(3x + 4y) = 9(x + 2y)$

Hence, $12x + 16y = 9x + 18y$ or $3x = 2y$

$\therefore x = \dfrac{2}{3}y$.

Substitute $x = \dfrac{2}{3}y$ in the required expression.

i.e. $3x + 5y : 3x - y$

$= 3\left(\dfrac{2}{3}y\right) + 5y : 3\left(\dfrac{2}{3}y\right) - y$

$= 2y + 5y : 2y - y = 7y : y = 7 : 1$

31. **(d)**

32. **(b)** $10x = 7.\overline{7}$ or $x = 0.\overline{7}$

Subtracting, $9x = 7 \quad \therefore \quad x = \dfrac{7}{9}$

$2x = \dfrac{14}{9} = 1.555........ = 1.\overline{5}$

33. **(b)**

34. **(b)** The numbers that can be formed are xy and yx. Hence $(10x + y) + (10y + x) = 11(x + y)$. If this is a perfect square then $x + y = 11$.

35. **(d)** Since, $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$\Rightarrow QR^2 = \frac{16 \times (4.5)^2}{9} \Rightarrow QR = 6 \text{ cm}$$

36. **(c)**

37. **(b)** Centroid is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

i.e. $\left(\frac{3 + (-8) + 5}{3}, \frac{-7 + 6 + 10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right) = (0, 3)$

38. **(c)** Required probability $= \frac{1+2+1}{11} = \frac{4}{11}$.

39. **(a)** **Given:** The natural number, when divided by 13 leaves remainder 3

The natural number, when divided by 21 leaves remainder 11

So, 13 – 3 = 21 – 11 = 10 = k

Now, LCM (13, 21) = 273

But the number lies between 500 and 600

∴ 2 LCM (13, 21) – k = 546 – 10 = 536

536 = 19 × 8 + 4 ∴ remainder = 4

40. **(b)** Since, $\Delta ABC \sim \Delta APQ$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \frac{BC^2}{PQ^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{4 \cdot ar(\Delta ABC)} = \frac{BC^2}{PQ^2} \Rightarrow \left(\frac{BC}{PQ}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \frac{BC}{PQ} = \frac{1}{2}$$

41. **(a)** Area of ΔABC $= \frac{\sqrt{3}}{4} a^2$

$17320.5 = \frac{\sqrt{3}}{4} a^2$

$a^2 = \frac{17320.5 \times 4}{1.73205} = 40000$

a = 200 cm

42. **(d)** Radius of circle $= \frac{200}{2}$

= 100 cm

43. **(a)** Area of each sector

$= \frac{60}{360} \times \pi r^2$

$= \frac{1}{6} \times 3.14 \times 10000 = 5233.3 \text{ cm}^2$

44. **(b)** Area of the shaded region

= Area of ΔABC – 3 × Area of each sector

$= 17320.5 - 3 \times \frac{31400}{6} = 1620.5 \text{ cm}^2$

45. **(c)** Perimeter of ΔABC = 3 × 200 = 600 cm

46. **(a)** Radius of inner semicircular end

$= \frac{60}{2} = 30$ m

47. **(c)** Radius of outer semicircular end

= 30 + 10 = 40 m

48. **(b)** The distance arounded the track along its inner edge

= 106 × 2 + 2 × πr

$= 212 + 2 \times \frac{22}{7} \times 30 = 212 + 188.57$

= 400.57 m

49. **(d)** The distance arounded the track along its outer edge

= 106 × 2 + 2 × πr

$= 212 + 2 \times \frac{22}{7} \times 40 = 212 + 251.43$

= 463.43 m

50. **(a)** The area of the track

= 2 × Area of ractangle + 2 × Area of semicircular ring.

$= 2(10 \times 106) + 2 \times \frac{1}{2} \times \frac{22}{7} \times (40^2 - 30^2)$

$= 2120 + 2200 = 4320 \text{ m}^2$