MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$. There is a $\vec{u}$ such that $\vec{u} \times \vec{a} = \vec{b} \times \vec{c}$ & $\vec{u} \cdot \vec{a} = 0$. Find $25|\vec{u}|^2$

(1) 560 (2) $\frac{925}{7}$

(3) 446 (4) 330

Answer (2)

Sol. $(\vec{u} \times \vec{a})^2 + (\vec{u} \cdot \vec{a})^2 = |\vec{u}|^2 |\vec{a}|^2$

$|\vec{b} \times \vec{c}|^2 + 0 = |\vec{u}|^2 \cdot 14$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 1 & 2 & 2 \end{vmatrix}$$

$= \hat{i}(-8) - \hat{j}(-1) + \hat{k}(3)$

$= -8\hat{i} + \hat{j} + 3\hat{k}$

$|\vec{b} \times \vec{c}| = \sqrt{74}$

$74 + 0 = 14|u|^2$

$\Rightarrow \; 25|u|^2 = \frac{74}{14} \cdot 25$

$= \frac{925}{7}$

2. The range of $y = \frac{x^2 + 2x + 1}{x^2 + 8x + 1}$ is $(x \in R)$

(1) $\left(-\infty, -\frac{2}{3}\right] \cup [2, \infty)$ (2) $(-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

(3) $(-\infty, \infty)$ (4) $\left(-\infty, \frac{-2}{5}\right] \cup [1, \infty)$

Answer (2)

Sol. $y = \frac{x^2 + 2x + 1}{x^2 + 8x + 1}$

$\Rightarrow \; x^2(y - 1) + x(8y - 2) + y - 1 = 0, \; x \in R$

If $y \neq 1$

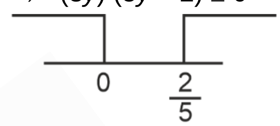
$D \geq 0$

$4(4y - 1)^2 - 4(y - 1)(y - 1) \geq 0$

$\Rightarrow \; (4y - 1)^2 - (y - 1)^2 \geq 0$

$\Rightarrow \; (4y - 1 - (y - 1))(4y - 1 + y - 1) \geq 0$

$\Rightarrow \; (3y)(5y - 2) \geq 0$



$y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right) - \{1\}$

If $y = 1$

$6x = 0 \Rightarrow x = 0$

$\therefore \; y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

3. If $a, b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$, then

(1) R_1 is symmetric but R_2 is not

(2) R_2 is symmetric but R_1 is not

(3) R_1 and R_2 are both symmetric

(4) R_1 and R_2 are both transitive

Answer (1)

Sol. $R_1 \to a^2 - b^2 \in I$

as $a, b \in Z$ if $a^2 - b^2 \in Z$ then $b^2 - a^2 \in Z$

Also $a^2 - b^2 \in Z$ & $b^2 - c^2 \in Z \;\Rightarrow\; a^2 - c^2 \in Z$

$\therefore$ R_1 is symmetric as well as transitive.

$R_2 \to 2 + \frac{a}{b} > 0 \;\Rightarrow\; \frac{a}{b} > -2$

then $2 + \frac{b}{a} > 0$, then it is not necessary $\frac{a}{b} > -2$

$\therefore$ R_2 is not symmetric.

Now if $2 + \frac{a}{b} > 0$ & $2 + \frac{b}{c} > 0$

then $2 + \frac{a}{c}$ can be positive or negative.

4. If $\int \frac{xdx}{x^2+x+2}; Af(x)+Bg(x)+C$ where C is constant of integration, then $A + 2B$ is equal to

(1) 1
(2) 0
(3) –1
(4) –2

Answer (2)

Sol. Let $x = \alpha(2x+1)+\beta$

$$\alpha = \frac{1}{2}, \beta = \frac{-1}{2}$$

$$I = \int \frac{xdx}{\sqrt{x^2+x+2}} = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+2}} - \frac{1}{2}\int \frac{1}{\sqrt{x^2+x+2}}$$

$$= \frac{1}{2}\times 2\sqrt{x^2+x+2} - \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}} + C$$

$$= \sqrt{x^2+x+2} - \frac{1}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+2}\right| + C$$

$$I = Af(x)+Bg(x)+C$$

$$\Rightarrow \quad A = 1, B = \frac{-1}{2}$$

5. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$ is equal to

(1) 27
(2) $\frac{27}{2}$
(3) 18
(4) 6

Answer (3)

Sol. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$

$$= \lim_{x\to\infty} \frac{2\cdot\left(\sqrt{3}\right)^6 x^6 + \text{.......(lower power of } x)}{2x^6 + \text{........(lower power of } x)}$$

= 27

6. Foot of perpendicular from origin to a plane which cuts the coordinate axes at A, B, C is (2, a, 4). Area of tetrahedron $OABC$ is 144 m^2. Which of the following points does not lie on plane?

(1) (2, 2, 4)
(2) (0, 3, 4)
(3) (1, 1, 5)
(4) (5, 5, 1)

Answer (2)

Sol. Equation of required plane:

$2(x – 2) + a(y – a) + 4(z – 4) = 0$

$\Rightarrow \quad 2x + ay + 4z = 20 + a^2$

$$A\left(10+\frac{a^2}{2},0,0\right), B\left(0,\frac{20+a^2}{a},0\right), C\left(0,0,\frac{20+a^2}{4}\right)$$

Area of tetrahedron $= \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 144$

$$\Rightarrow \quad \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$$

$\Rightarrow \quad (20 + a^2)^3 = 144 \times 48a$

$\Rightarrow \quad a = 2$

$\therefore \quad$ Equation of plane: $2x + 2y + 4z = 24$

$\Rightarrow \quad x + y + 2z = 12$

(0, 3, 4) does not lie on plane

7. If $z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$, then z is

(1) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(2) $\frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(4) $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

Answer (3)

Sol. $z = \dfrac{i-1}{\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}} = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = (i-1)\cdot e^{-\frac{\pi}{3}}$

$\Rightarrow \quad z = (i-1)\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$

$= (i-1)\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)$

$= \frac{1}{2}\left(i + \sqrt{3} - 1 + \sqrt{3}i\right)$

$\Rightarrow \quad \frac{\sqrt{3}-1}{2} + i\left(\frac{\sqrt{3}+1}{2}\right)$

$\therefore \quad \arg(z) = \frac{5\pi}{12}$ & $|z| = \sqrt{2}$

$\therefore \quad z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

8. Given that $\theta \in [0, 2\pi]$, the largest interval of values of θ which satisfy the inequation $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) \geq 0$ is

(1) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(3) $[0, \pi]$ (4) $\left[\frac{\pi}{2}, \frac{5\pi}{4}\right]$

Answer (1)

Sol. $\sin^{-1}(\sin\theta) - \left(\frac{\pi}{2} - \sin^{-1}\sin\theta\right) \geq 0$

$\Rightarrow \quad \sin^{-1}\sin\theta \geq \frac{\pi}{4}$

$\frac{1}{\sqrt{2}} \leq \sin\theta \leq 1$

$\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

9. If $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, where $r \in \{p, q, \sim p, \sim q\}$, then the number of values of r is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (2)

Sol. $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$

$\Rightarrow \quad ((\sim p \vee \sim q) \vee (r \vee q)) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad ((\sim p \vee r \vee (q \vee \sim q)) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad T \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad \sim p \vee \sim r \vee q$

For the above statement to be tautology r can be $\sim p$ or q

$\therefore$ Two values of r are possible

10. If $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$, Given that $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$. Find $\dfrac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}}$

(1) 3 (2) $-\sqrt{3}$

(3) 1 (4) –3

Answer (2)

Sol. $\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{0}$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2\left(4|b|^2 + 9|c|^2 + 12\vec{b} \cdot \vec{c}\right)$

$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b} + 3\vec{c})$

$\dfrac{|\vec{a} \times \vec{c}|}{|\vec{a} \cdot \vec{b}|} = \dfrac{2|\vec{b} \times \vec{c}|}{2\vec{b} \cdot \vec{b} + 3\vec{c} \cdot \vec{b}}$

$|\vec{b} \times \vec{c}|^2 = |b|^2 |c|^2 - (\vec{b} \cdot \vec{c})^2$

$= \frac{1}{4} \cdot 4 - \left(1\left(-\frac{1}{2}\right)\right)^2$

$= 1 - \frac{1}{4} = \frac{3}{4}$

$\dfrac{|\vec{a} \times \vec{c}|}{|\vec{a} \cdot \vec{b}|} = \dfrac{\sqrt{3}}{2 \cdot \frac{1}{4} - \frac{3}{2}} = \dfrac{\sqrt{3}}{-1}$

11. Number of 7 digit odd numbers formed using 7 digits 1, 2, 2, 2, 3, 3, 5 will be

(1) 80 (2) 420

(3) 240 (4) 140

Answer (3)

Sol. Even numbers formed

_ _ _ _ _ _ _ 2

Number of ways $= \dfrac{6!}{2!2!} = 180$

Total numbers $= \dfrac{7!}{3!2!} = \dfrac{720 \times 7}{12} = 420$

Odd numbers = 420 – 180

= 240

12. The minimum value of the function

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[x]$ denotes greatest integer function, is

(1) $\frac{3}{4}$ (2) $\frac{5}{4}$

(3) $\frac{1}{4}$ (4) 0

Answer (1)

Sol. $x^2 - x + 1 = g(x)$ attains minimum value

when $x = \frac{1}{2}$

So, minimum value of $f(x)$ will be at $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{3}{4} + 0$

$= \frac{3}{4}$

13. If for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $(\pm 4, 0)$ are foci and $e = \sqrt{3}$. Then length of latus rectum is

(1) 8 (2) $\frac{16}{\sqrt{3}}$

(3) 4 (4) $2\sqrt{3}$

Answer (2)

Sol. $ae = 4$

$a = \frac{4}{\sqrt{3}}$

$LR = \frac{2b^2}{a}$

$= \frac{2}{a}a^2(e^2 - 1)$

$= 2a(e^2 - 1)$

$= \frac{8}{\sqrt{3}}(3 - 1)$

$= \frac{16}{\sqrt{3}}$

14. If $[\alpha\ \beta\ \gamma]\begin{bmatrix} 5 & 6 & 8 \\ 6 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix} = [0\ 0\ 0]$

Where (α, β, γ) be a point on $2x + 5y + 3z = 5$ then $6\alpha + 5\beta + 9\gamma = ?$

(1) 20 (2) $\frac{20}{3}$

(3) 21 (4) 7

Answer (2)

Sol. $5\alpha + 6\beta - \gamma = 0$

$6\alpha + 3\beta + 3\gamma = 0$

$8\alpha + 8\beta = 0 \Rightarrow \boxed{\alpha = -\beta}$

& $\boxed{\beta = \gamma}$

$\alpha = k, \beta = k, \gamma = -k$

$2(k) + 5(-k) + 3(-k) = 5$

$k = -\frac{5}{6}$

$\alpha = -\frac{5}{6}, \beta = \frac{5}{6}, \gamma = \frac{5}{6}$

$6\alpha + 5\beta + 9\gamma = -5 + \frac{25}{6} + \frac{45}{6}$

$= \frac{40}{6} = \frac{20}{3}$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Coefficient of x^{-6} in expansion of $\left(\frac{4x}{5} - \frac{5}{2x^2}\right)^9$ is

Answer (–5040)

Sol. $T_{r+1} = {}^9C_r\left(\frac{4x}{5}\right)^{9-r}\left(\frac{-5}{2x^2}\right)^r$

$9 - 3r = -6$

$\boxed{r = 5}$

Coefficient of $x^{-6} = {}^9C_5\left(\frac{4}{5}\right)^4\left(-\frac{5}{2}\right)^5$

$= \frac{9!}{5!4!}\frac{4^4}{5^4}\left(\frac{-5^5}{2^5}\right)$

$= 6 \cdot 7 \cdot 3 \cdot 8(-5)$

$= -5040$

22. The value of sum

$1.1^2 - 2.3^2 + 3.5^2 - 4.7^2 \dots + 15.(29)^2$ is

Answer (6952)

Sol. Separating odd placed and even placed terms we get

$S = (1.1^2 + 3.5^2 + \dots 15.(29)^2) - (2.3^2 + 4.7^2 + \dots + 14.(27)^2)$

$$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2$$

Applying summation formula we get

$= 29856 - 22904 = 6952$

23. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2 × 2 matrix such that $a, b, c, d \in \{0, 1, 2, 3, 4, \}$. The number of matrices A such that sum of elements of A is a prime number lying between 2 and 13 is

Answer (204)

Sol. As given $a + b + c + d = 3$ or 5 or 7 or 11

if sum = 3

$(1 + x + x^2 + \dots + x^4)^4 \longrightarrow x^3$

$(1 - x^5)^4 (1 - x)^{-4} \longrightarrow x^3$

$\therefore\ 4 + {}^{3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1 - 4x^5)(1 - x)^{-4} \longrightarrow x^5$

$\Rightarrow\ {}^{4+5-1}C_5 - 4.{}^{4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^7$

$\Rightarrow\ {}^{4+7-1}C_7 - 4.\ {}^{4+2-1}C_2 = {}^{10}C_7 - 4.\ {}^5C_2 = 80$

If sum = 11

$(1 - 4x^5 + 6x^{10})(1 - x)^{-4} \rightarrow x^{11}$

$\Rightarrow\ {}^{4+11-1}C_{11} - 4.{}^{4+6-1}C_6 + 6.{}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4.{}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

24. If $\dfrac{{}^{2n+1}P_{n-1}}{{}^{2n+1}P_n} = \dfrac{11}{21}$, then $n^2 + n + 15$ equals

Answer (45)

Sol. $\dfrac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \dfrac{11}{21}$

$\Rightarrow\ \dfrac{(2n+1)(2n)}{(n+2)(n+1)n} = \dfrac{11}{21}$

$\Rightarrow\ \dfrac{2n+1}{(n+1)(n+2)} = \dfrac{11}{42}$

$\Rightarrow\ n = 5$

$\Rightarrow\ n^2 + n + 15 = 25 + 5 + 15 = 45$

25. $\displaystyle\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$

then α is equal to

Answer (02.00)

Sol. $\displaystyle\int_0^{\alpha} \frac{x}{\alpha}\left(\sqrt{x+\alpha}+\sqrt{x}\right)$

$\displaystyle\int_0^{\alpha} \frac{1}{\alpha}\left[(x+\alpha)^{3/2} - \alpha(x+\alpha)^{1/2} + x^{3/2}\right]$

$\displaystyle\frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{5/2} - \alpha\frac{2}{3}(x+\alpha)^{3/2} + \frac{2}{5}x^{5/2}\right]_0^{\alpha}$

$\displaystyle= \frac{1}{\alpha}\left(\frac{2}{5}(2\alpha)^{5/2} - \frac{2\alpha}{3}(2\alpha)^{3/2} + \frac{2}{5}\alpha^{5/2} - \frac{2}{5}\alpha^{5/2} + \frac{2}{3}\alpha^{5/2}\right)$

$\displaystyle= \frac{1}{\alpha}\left(\frac{2^{7/2}\alpha^{5/2}}{5} - \frac{2^{5/2}\alpha^{5/2}}{3} + \frac{2}{3}\alpha^{5/2}\right)$

$\displaystyle= \alpha^{3/2}\left(\frac{2^{7/2}}{5} - \frac{2^{5/2}}{3} + \frac{2}{3}\right)$

$\displaystyle= \frac{\alpha^{3/2}}{15}\left(3\cdot 2^{7/2} - 5\cdot 2^{5/2} + 10\right)$

$\displaystyle= \frac{\alpha^{3/2}}{15}\left(24\sqrt{2} - 20\sqrt{2} + 10\right) = \frac{\alpha^{3/2}}{15}\left(4\sqrt{2} + 10\right)$

$\displaystyle= \frac{16+20\sqrt{2}}{15}$

$\Rightarrow\ \alpha = 2$

26.

27.

28.

29.

30.

❑ ❑ ❑