$\Rightarrow f$ is invertible and $f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$

Linear Programming Problems: (LPP) :1 + 6 (part E: 5Q(6) - graph) definition

* LP is a collection of procedures for maximizing or minimizing linear functions (objective functions), subjected to certain constraints in the form of linear inequalities.
* Eg: To determine the best distribution system that will minimize the transportation cost from production place to the market locations. (transportation problem).

Eg2: To determine the best way to maximize the profit in the usage of machines and labours required in production (Production problem

Eg3: To determine the best diet in minimizing cholestrol, with the usage the minerals and vitamins (diet problem)

*** (IMP) Definitions:

i] Feasible region – The common region determined by all constraints, including non negative contraints of an LPP.

ii] Optimal solution – Any feasible solution of an LPP which maximises or minimises the objective function is called an optimal solution.

iii] Objective function – A linear function of the involved variables which is to be maximised or minimised is called an objective funⁿ.
If Z is the objective funⁿ then $Z = ax + by$.

iv] Decision variables – The variables in an LPP is a set of quantities to be determined in the problem of optimization, denoted by x and y. These variables are called decision variable.

v] Constraints – The limitations on the resources often expressed in the form of linear inequalities are called linear constraints.
$x \geq 0$, $y \geq 0$ are called non negative constraints

vi] Constrained set – The set of points satisfying the constraints of an LPP is said to be constrained set.

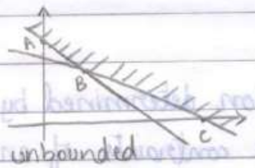
vii] Feasible solution – Any point of the constrained set is called a feasible solution of the LPP.

viii] Infeasible solution – Any point lying outside the feasible region of an LPP is called infeasible solution.

ix] Bounded and unbounded sets – A subset S of a plane is said to be bounded if there exists a boundary such that S lies completely inside the boundary, else S is said to be unbounded.

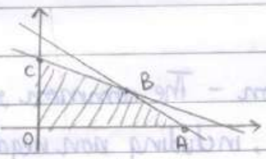
x] Convex set – A subset S of a plane is said to be convex if the line segment joining any two points of S lies entirely inside S.

xi] Corner point – The vertices of convex polygon (feasible region) are called corner points.



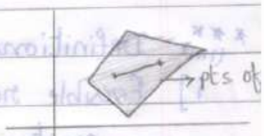
unbounded feasible region

pt A, B, C are corner points

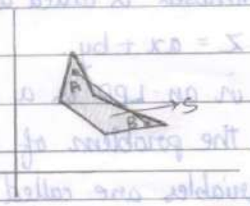


bounded feasible region

points O, A, B, C are corner points



convex set

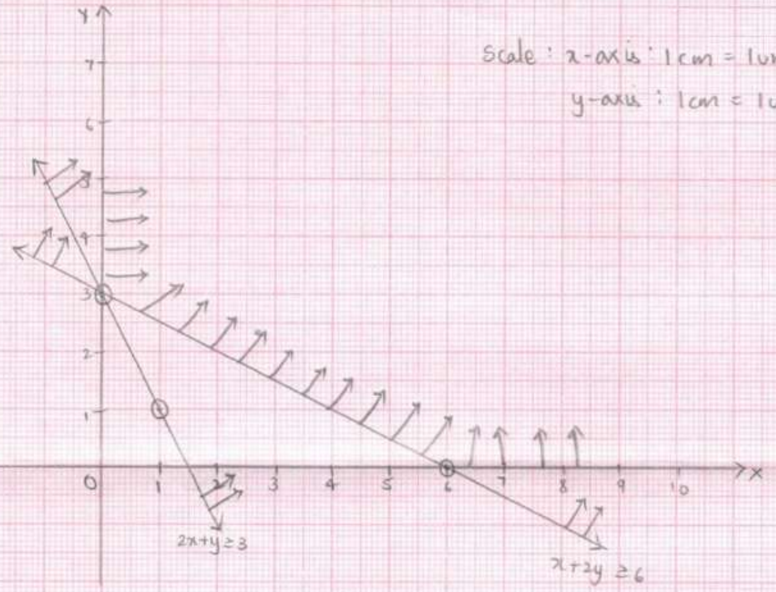


not convex

DATE:

EXERCISE 12.1

6] Minimise $Z = x + 2y$ subjected to constraints $2x + y \geq 3$, $x + 2y \geq 6$ and $x, y \geq 0$.



Objective function :

$2x + y = 3$

x	0	1
y	3	1

$x + 2y = 6$

$\frac{x}{6} + \frac{y}{3} = 1$

$(6,0), (0,3)$

Corner points : $Z = x + 2y$

$(0,3)$: $0 + 6 = 6$

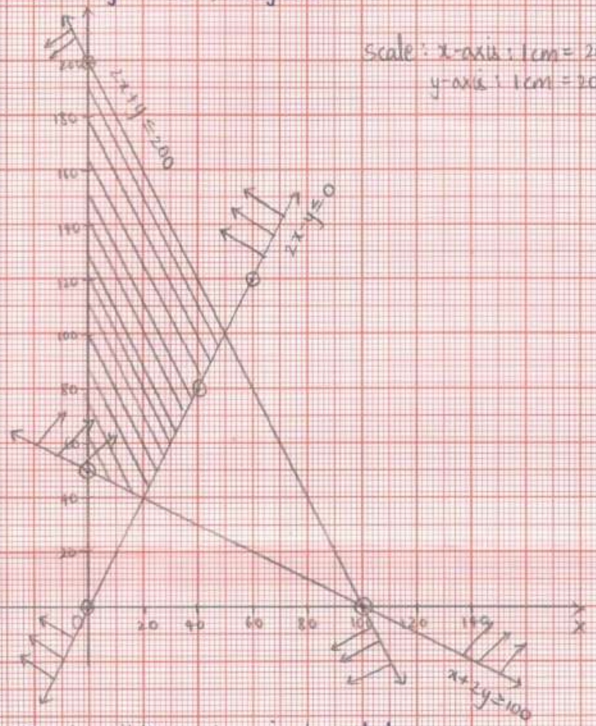
$(6,0)$: $6 + 0 = 6$

The feasible region is unbounded.

Minimum value 6 occurs at 2 points $(0,3)$ and $(6,0)$

EXERCISE 12.1 DATE :

8] Minimise and maximise $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.



Objective function:

$x + 2y = 100$

$\frac{x}{100} + \frac{y}{50} = 1$

(100, 0) , (0, 50)

$2x - y = 0$

$y = 2x$

x	0	40	60
y	0	80	120

$2x + y = 200$

$\frac{x}{100} + \frac{y}{200} = 1$

(100, 0) , (0, 200)

Corner points:

(0, 200)	$Z = 0 + 400 = 400$
(50, 100)	$Z = 50 + 200 = 250$
(20, 40)	$Z = 20 + 80 = 100$
(0, 50)	$Z = 0 + 100 = 100$

* The feasible region is bounded.
* Min value 100 occurs at pts (0, 50) and (20, 40)
* Max value 400 occurs at pt. (0, 200)

Chandra's

EXERCISE 12.1

DATE :

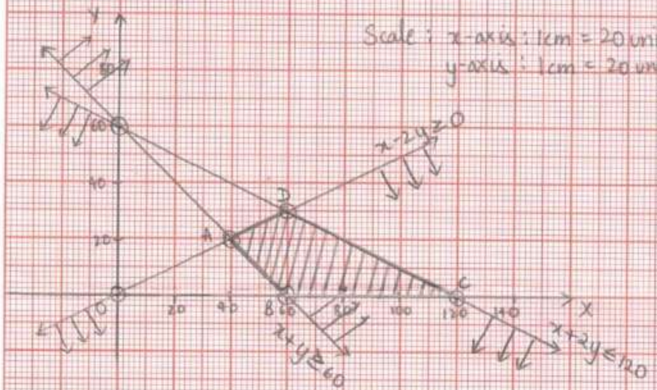
7] Maximise & minimise $Z = 5x + 10y$ subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$

Gn: Objective funⁿ : $Z = 5x + 10y$

Constraints :

$x + 2y = 120$	$x + y = 60$	$x - 2y = 0$
$\frac{x}{120} + \frac{y}{60} = 1$	$\frac{x}{60} + \frac{y}{60} = 1$	$x = 2y$
(120,0), (0,60)	(60,0), (0,60)	

x	0	40	60
y	0	20	30



Corner pts	$Z = 5x + 10y$	
A (40, 20)	200 + 200 = 400	
B (60, 0)	300 + 0 = 300	min
C (120, 0)	600 + 0 = 600	max
D (60, 30)	300 + 300 = 600	max

The given objective function obtains its maximum value 600 at pts C(120,0) and D(60,30).

It obtains its minimum value 300 at pt B(60,0).

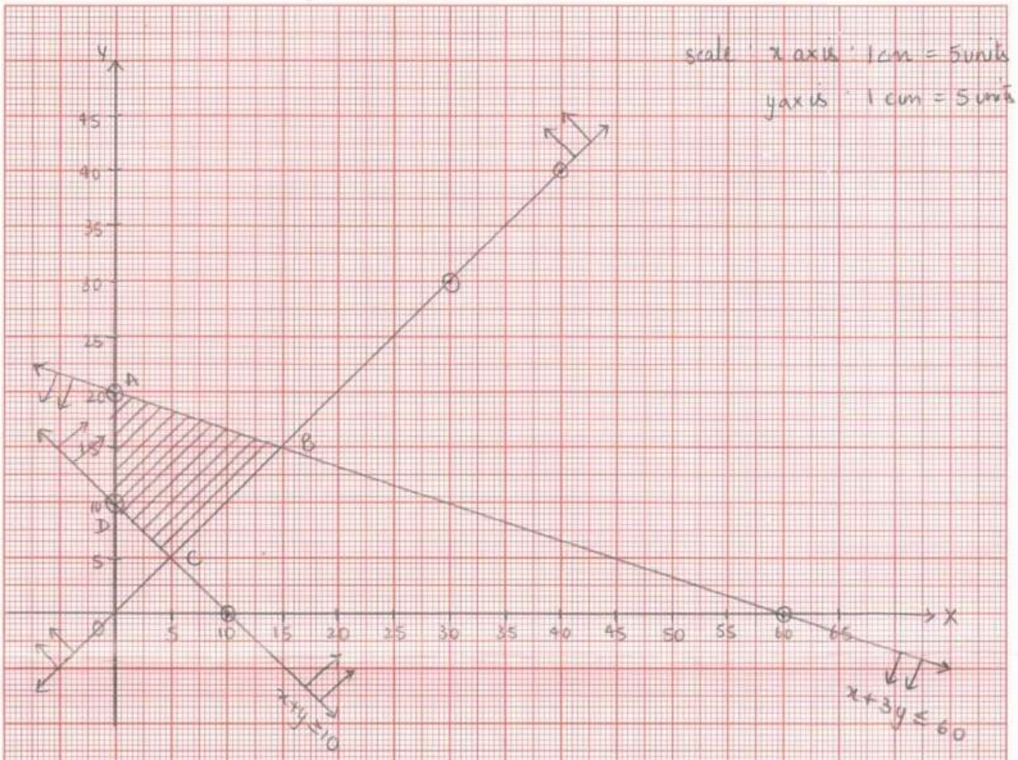
Eg 3] Solve the following problem graphically, minimise and maximise $Z = 3x + 9y$ subject to $x + 3y \le 60$, $x + y \ge 10$, $x \le y$, $x, y \ge 0$

Gn: Objective funⁿ : $Z = 3x + 9y$

Constraints :

$x + 3y = 60$	$x + y = 10$	$x = y$
$\frac{x}{60} + \frac{y}{20} = 1$	$\frac{x}{10} + \frac{y}{10} = 1$	(30,30), (40,40)
(60,0), (0,20)	(10,0), (0,10)	

DATE :



Corner pts : $Z = 3x + 9y$

A (0, 20)	0 + 180 = 180	max
B (15, 15)	45 + 135 = 180	max
C (5, 5)	15 + 45 = 60	min
D (0, 10)	0 + 90 = 90	

The given objective function obtains it maximum value 180 at points A (0, 20) and B (15, 15).

It obtains its minimum value 60 at C (5, 5).

EXERCISE 12.2 DATE :

4] A manufacturer produces nuts and bolts. He takes 1 hr of work on machine A and 3 hr on machine B to produce a pack of nuts. He takes 3 hr on A and 1 hr on B to produce a pack of bolts. He earns a profit of ₹ 17.50 per pack on nuts and ₹ 7 per pack on bolts. How many packs of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hr a day.

Sol] Objective function : $Z = 17.50x + 7y$

	x (nuts)	y (bolts)	hrs
A	1	3	≤ 12
B	3	1	≤ 12

Constraints :
(i) $x \geq 0$, $y \geq 0$ (non -ve constraint)
(ii) $x + 3y \leq 12$
(iii) $3x + y \leq 12$

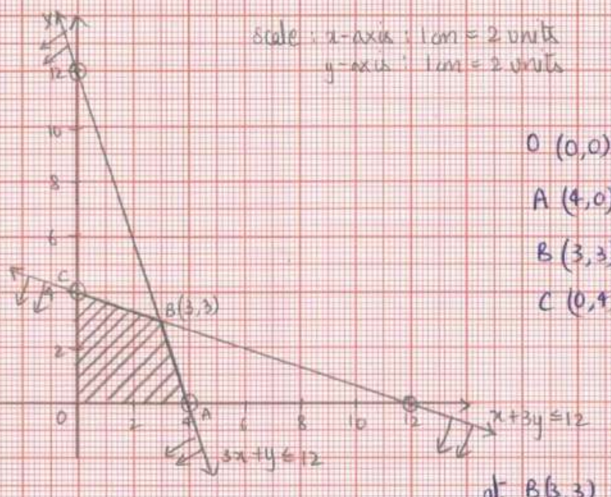
Consider :

$x + 3y = 12$
$\frac{x}{12} + \frac{y}{4} = 1$
$(12,0), (0,4)$

$3x + y = 12$
$\frac{x}{4} + \frac{y}{12} = 1$
$(4,0)$ $(0,12)$

$3x + 9y = 36$
$3x + y = 12$
$8y = 24$
$\boxed{y = 3}$ $\boxed{x = 3}$

Scale : x-axis : 1 cm = 2 units
y-axis : 1 cm = 2 units



Corner points :

	$Z = 17.50x + 7y$	
O (0,0)	0 + 0	= 0
A (4,0)	70 + 0	= 70
B (3,3)	52.5 + 21	= 73.5 } max profit
C (0,4)	0 + 28	= 28

The manufacturer obtains the maximum profit of ₹ 73.5 at B(3,3) i.e when he produces 3 packs of nuts and 3 packs of bolts per day.

Chandra's

EXERCISE 12.2

DATE :

9] A diet is to contain atleast 80 units of Vitamin A and 100 units of Minerals. Two foods F_1, F_2 are available. F_1 costs ₹4 per unit and F_2 costs ₹6 per unit. 1 unit of F_1 contains 3 units of Vitamin and 4 units of mineral, F_2 - 6 V & 3 M. Find the minimum cost for diet that contains a mixture of these two foods meeting minimum nutritional requirements.

Sol) Objective function : $Z = 4x + 6y$

	(x) F_1	(y) F_2	Total
V	3	6	≥ 80
M	4	3	≥ 100

Constraints are :

(i) $x \geq 0, y \geq 0$ (non -ve constraints)

(ii) $3x + 6y \geq 80$

(iii) $4x + 3y \geq 100$

$3x + 6y = 80$

$\frac{3x}{80} + \frac{6y}{80} = 1$

$\frac{x}{26.6} + \frac{y}{13.3} = 1$

$(26.6, 0)$, $(0, 13.3)$

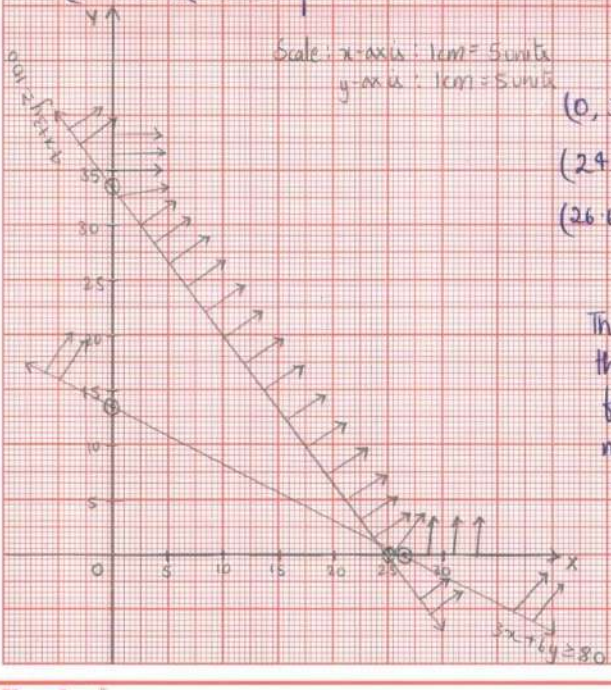
$4x + 3y = 100$

$\frac{x}{\frac{100}{4}} + \frac{y}{\frac{100}{3}} = 1$

$(25, 0)$, $(0, 33.3)$

$$\begin{aligned} 3x + 6y &= 80 \\ 8x + 6y &= 200 \\ \hline -5x &= -120 \end{aligned}$$

$\boxed{x = 24}$ $\boxed{y = 1.33}$



Corner points : $Z = 4x + 6y$

$(0, 33.3)$	$0 + 199.8 = 199.8$
$(24, 1.33)$	$96 + 7.98 = 103.98$ } min cost
$(26.6, 0)$	$106.4 + 0 = 106.4$

The minimum cost for diet that contains a mixture of two foods meeting minimum nutritional requirements is ₹ 103.98 at $(24, 1.33)$.

Chandra's

A manufacturing company makes 2 models A and B of a product. Each piece of A requires 9 labours for fabricating and 1 labour for finishing. Model B requires 12 and 3 hours for fabrication and finishing. For fab and fin maximum labours available are 180 and 30 hrs respe.. The company makes a profit of ₹8000 on each piece of A and ₹12000 on each of B. How many pieces of A and B should be manufactured per week to get maximum profit. Formulate LPP and solve graphically.

Sol] Objective funⁿ, $Z = 8000x + 12000y$

	A(x)	B(y)	Total
Fab	9	12	≤ 180
Fin	1	3	≤ 30

Constraints:

(i) $x \ge 0, y \ge 0$ (non -ve constraint)

(ii) $9x + 12y \le 180$

$3x + 4y \le 60$

(iii) $x + 3y \le 30$

$3x + 4y = 60$

$\frac{x}{20} + \frac{y}{15} = 1$

$(20, 0), (0, 15)$

$x + 3y \le 30$

$\frac{x}{30} + \frac{y}{10} \le 1$

$(30, 0), (0, 10)$

$3x + 4y = 60$

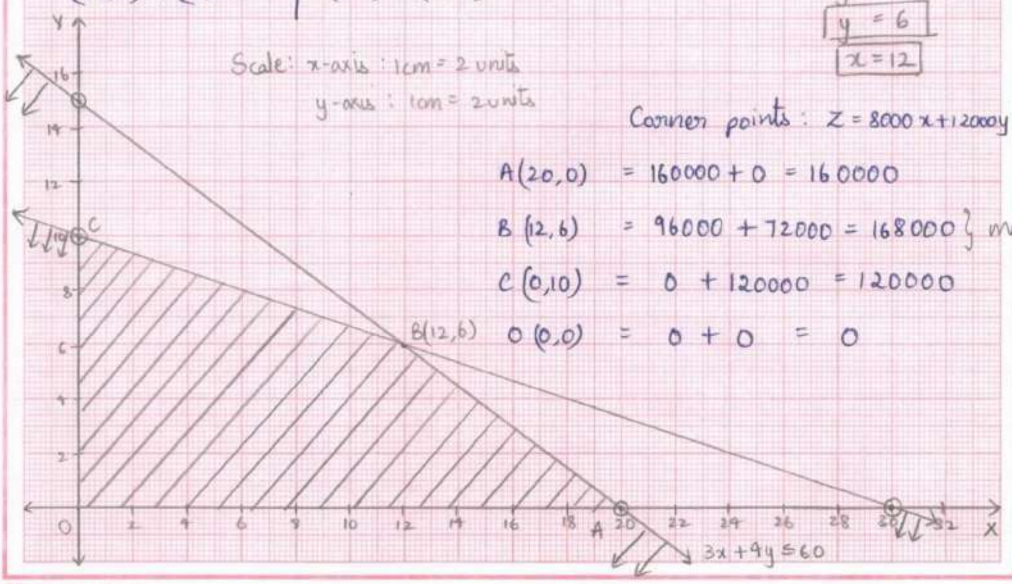
$3x + 9y = 90$

$\ominus \quad \ominus \quad \ominus$

$-5y = -30$

$\boxed{y = 6}$

$\boxed{x = 12}$



Corner points : $Z = 8000x + 12000y$

$A(20, 0) = 160000 + 0 = 160000$

$B(12, 6) = 96000 + 72000 = 168000$ } max profit

$C(0, 10) = 0 + 120000 = 120000$

$O(0, 0) = 0 + 0 = 0$

∴ The company has to produce 12 pieces of model A and 6 pieces of B per week to obtain the maximum profit ₹168000.

DATE:

** One kind of cake requires 200g of flour and 25g of fat, another kind of cake requires 100 g of flour and 50g of fat. Find the max no. of cakes which can be made from 5 kg of flour and 1kg of fat assuming there is no shortage.

Sol) Objective function : $Z = x + y$

	Cake 1 (x)	Cake (2)	Total
Flour	200	100	$\le 5\text{kg} = 5000\text{g}$
Fat	25	50	$\le 1\text{kg} = 1000\text{g}$

Constraints :

(i) $x \ge 0,\ y \ge 0$

(ii) $200x + 100y \le 5000$

$2x + y \le 50$

(iii) $25x + 50y \le 1000\text{ g}$

$x + 2y \le 40$

$2x + y \le 50$

$\frac{x}{25} + \frac{y}{50} = 1$

$(25, 0)\ (0, 50)$

$x + 2y \le 40$

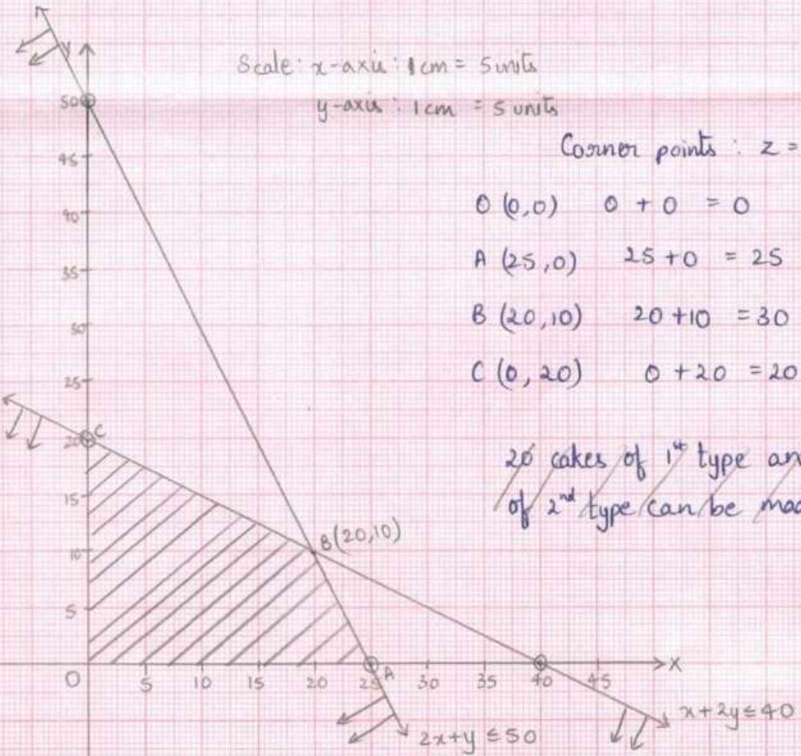
$\frac{x}{40} + \frac{y}{20} = 1$

$(40, 0), (0, 20)$

$$\begin{aligned} 2x + y &= 50 \\ 2x + 4y &= 80 \\ \hline 3y &= 30 \end{aligned}$$

$\boxed{y = 10}$

$\boxed{x = 20}$



Corner points : $Z = x + y$

O (0,0)	0 + 0 = 0	
A (25,0)	25 + 0 = 25	
B (20,10)	20 + 10 = 30	max
C (0,20)	0 + 20 = 20	

~~20 cakes of 1st type and 10 cakes of 2nd type can be made.~~

Vaishnav's SPL

A max of 30 cakes should be made, 20 peices of cake 1 and 10 pieces of cake 2 from 5kg of flour and 1kg of fat.

DATE:

Mis Ex 3]

A dietician wishes to mix together foods x and y such that the mix contains atleast 10 units of Vit A and 12 of Vit B and 8 of Vit C. The Vit contents of 1 kg of food is given below.

	Vit A	Vit B	Vit C
x	1	2	3
y	2	2	1
	≥10	≥12	≥8

1 kg food of x costs ₹ 16 and that of y ₹ 70. Find the least cost of the mix to produce the required diet.

Sol] Objective function : $16x + 70y = Z$

$x + 2y = 10$	$x + y = 6$	$3x + y = 8$
$\frac{x}{10} + \frac{y}{5} = 1$	$\frac{x}{6} + \frac{y}{6} = 1$	$\frac{x}{\frac{8}{3}} + \frac{y}{8} = 1$
(10,0) (0,5)	(6,0) (0,6)	$(\frac{8}{3},0)$ (0,8)

Constraints :

(i) $x \geq 0,\ y \geq 0$

(ii) $x + 2y \geq 10$

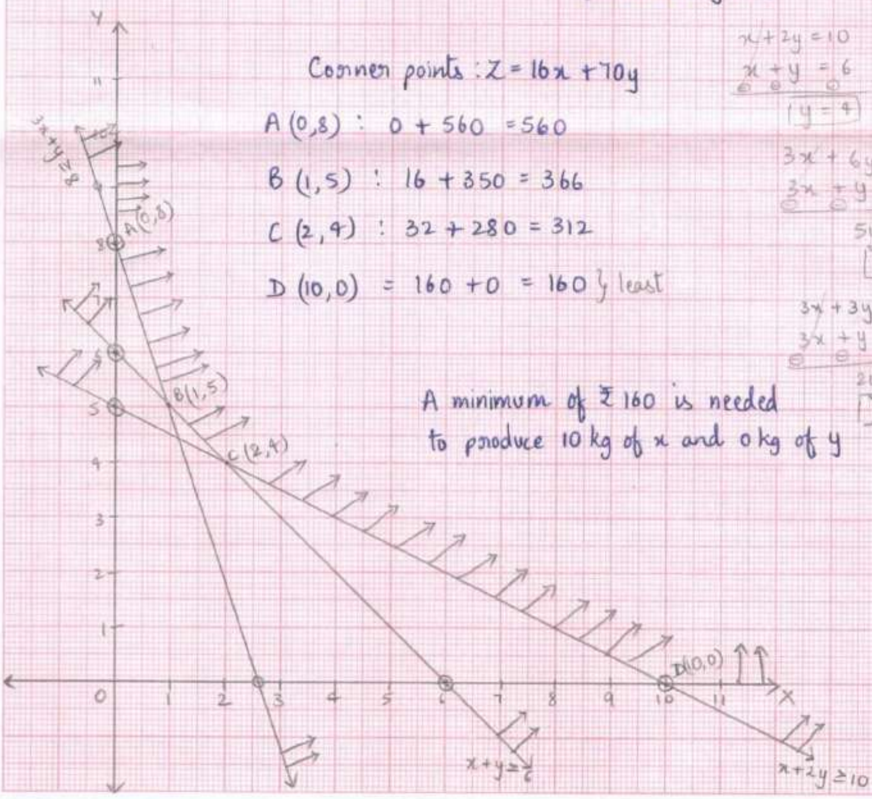
(iii) $2x + 2y \geq 12 \Rightarrow x + y \geq 6$

(iv) $3x + y \geq 8$

$x + 2y = 10$
$x + y = 6$
$\boxed{y = 4}$ $\boxed{x = 2}$

$3x + 6y \geq 30$
$3x + y \geq 8$
$5y = 22$
$\boxed{y = 4.4}$ $\boxed{x = 1.2}$

$3x + 3y \geq 18$
$3x + y \geq 8$
$2y = 10$
$\boxed{y = 5}$ $\boxed{x = 1}$



Corner points : $Z = 16x + 70y$

A (0,8) : 0 + 560 = 560

B (1,5) : 16 + 350 = 366

C (2,4) : 32 + 280 = 312

D (10,0) = 160 + 0 = 160 } least

A minimum of ₹ 160 is needed to produce 10 kg of x and 0 kg of y

DATE:

Mis Ex 4]

A manufacturer makes 2 types of toys A and B. 3 machines are needed and the time required for each toy on machines is given below.

		I	II	III
(x)	A	12	18	6
(y)	B	6	0	9
		(≤ 6	≤ 6	≤ 6)

Each machine is available for a max of 6 hr. per day. If the profit of type A is ₹ 7.50 and that on B is ₹ 5. Show that 15 toys of type A and 30 of type B should be manufactured per day to get max profit.

Sol] Objective fun^n : $Z = 7.50x + 5y$

Constraints:

(i) $x \geq 0,\ y \geq 0$

(ii) $12x + 6y \leq 6$

$\Rightarrow 2x + y \leq 1$

(iii) $18x + 0y \leq 6$

$3x \leq 1$

(iv) $6x + 9y \leq 6$

$2x + 3y \leq 2$

$2x + y = 1$	$x = \frac{1}{3}$	$2x + 3y = 2$
$\frac{x}{\frac{1}{2}} + \frac{y}{1} = 1$	$x \approx 0.33$	$\frac{x}{1} + \frac{y}{2/3} = 1$
$(\frac{1}{2}, 0), (0, 1)$	line II^{l} to y-axis	$(1, 0), (0, 0.66)$

$2x + 3y = 2$

$2x + y = 1$

$2y = 1$

$\boxed{y = \frac{1}{2}}$

$\boxed{x = \frac{1}{4}}$

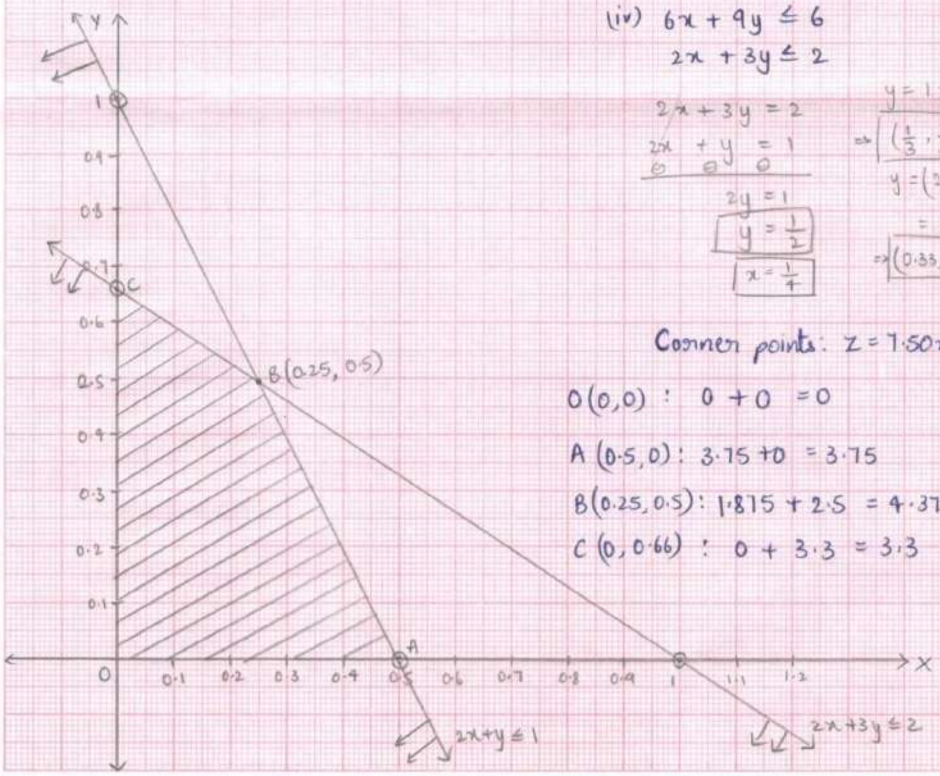
$y = 1 - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \boxed{(\frac{1}{3}, \frac{1}{3})} \Rightarrow (0.33, 0.33)$

$y = (2 - \frac{2}{3})\frac{1}{3}$

$= \frac{4}{9} = 0.44$

$\Rightarrow \boxed{(0.33, 0.44)}$



Corner points: $Z = 7.50x + 5y$

$O(0,0)$: $0 + 0 = 0$

$A(0.5, 0)$: $3.75 + 0 = 3.75$

$B(0.25, 0.5)$: $1.875 + 2.5 = 4.375$

$C(0, 0.66)$: $0 + 3.3 = 3.3$

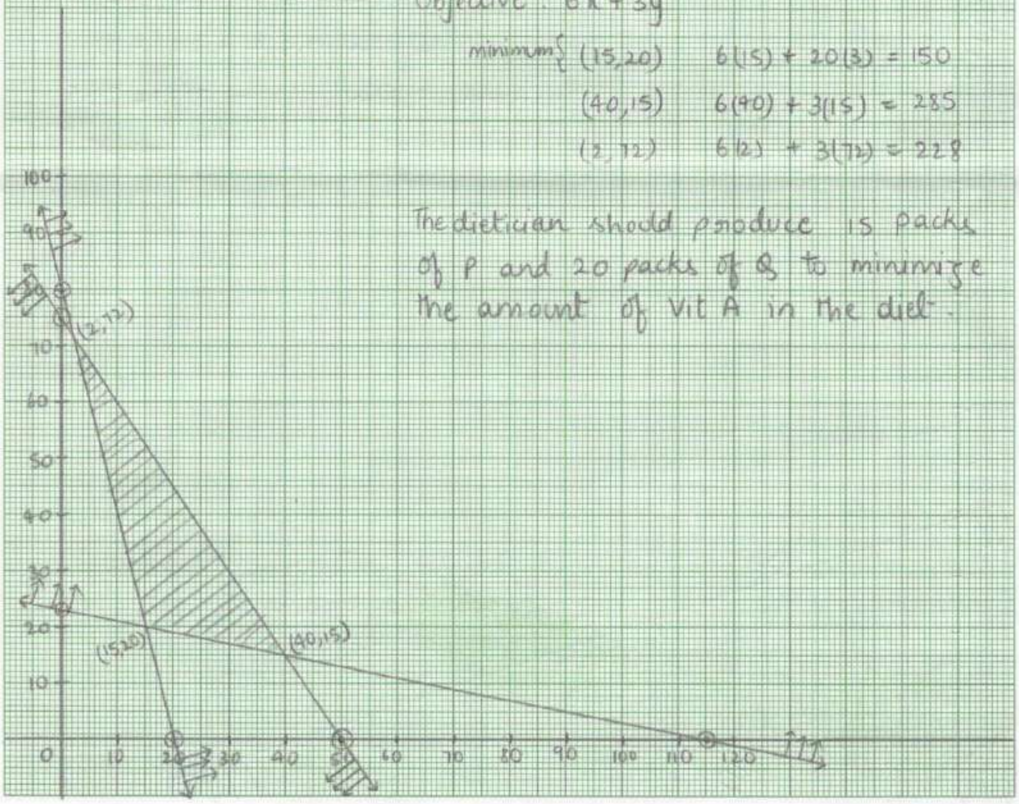
Batch No. :- .... Date :

A dietician wishes to mix 2 types of foods ~~in such a way that vitamin~~ P and Q such that P contains 12 units of Ca, 4 units of Fe, 6 units of cholestrol and 6 units of VA. Each packet of same quantity of Q contains 3 units Ca, 20 Units Fe, 4 units of cholestrol, 3 units of VA. Diet requires atleast 240 units of Ca, atleast 460 unit of Fe and atmost 300 unit of cholestrol. How many packs of each food should be used to minimize the amount of VA in the diet.

	P	Q	
Ca	12	3	≥ 240
Fe	4	20	≥ 460
Choles	6	4	≤ 300
Vit A	6	3	$= Z$

Objective $Z = 6x + 3y$

$12x + 3y \geq 240$	$4x + 20y \geq 460$	$6x + 4y \leq 300$
$(20,0), (0,80)$	$(115,0), (0,23)$	$(50,0), (0,75)$



Objective : $6x + 3y$

minimum $\{$ $(15,20)$ $6(15) + 20(3) = 150$

$(40,15)$ $6(40) + 3(15) = 285$

$(2,72)$ $6(2) + 3(72) = 228$

The dietician should produce 15 packs of P and 20 packs of Q to minimize the amount of Vit A in the diet.

Batch No. :- .... Date : ....

Manufacturer produce nuts and bolts. It takes 1 hour of work on mach A and 3 hours on Mach B to produce a pack of nuts, 3 hour on mach A and 1 hour on Mach B for a pack of bolts. He earns Rs 17.50 on pack of nuts and Rs 7 for pack of bolts. How many packs of each should he produce each day so as to maximise his profit, if he operates his machines for atmost 12 hr per day.

	Nuts	Bolts	
MA	1	3	≤ 12
MB	3	1	≤ 12

$x + 3y \le 12$ | $3x + y \le 12$

$(12, 0), (0, 4)$ | $(4, 0), (0, 12)$

Objective : $17.5x + 7y$

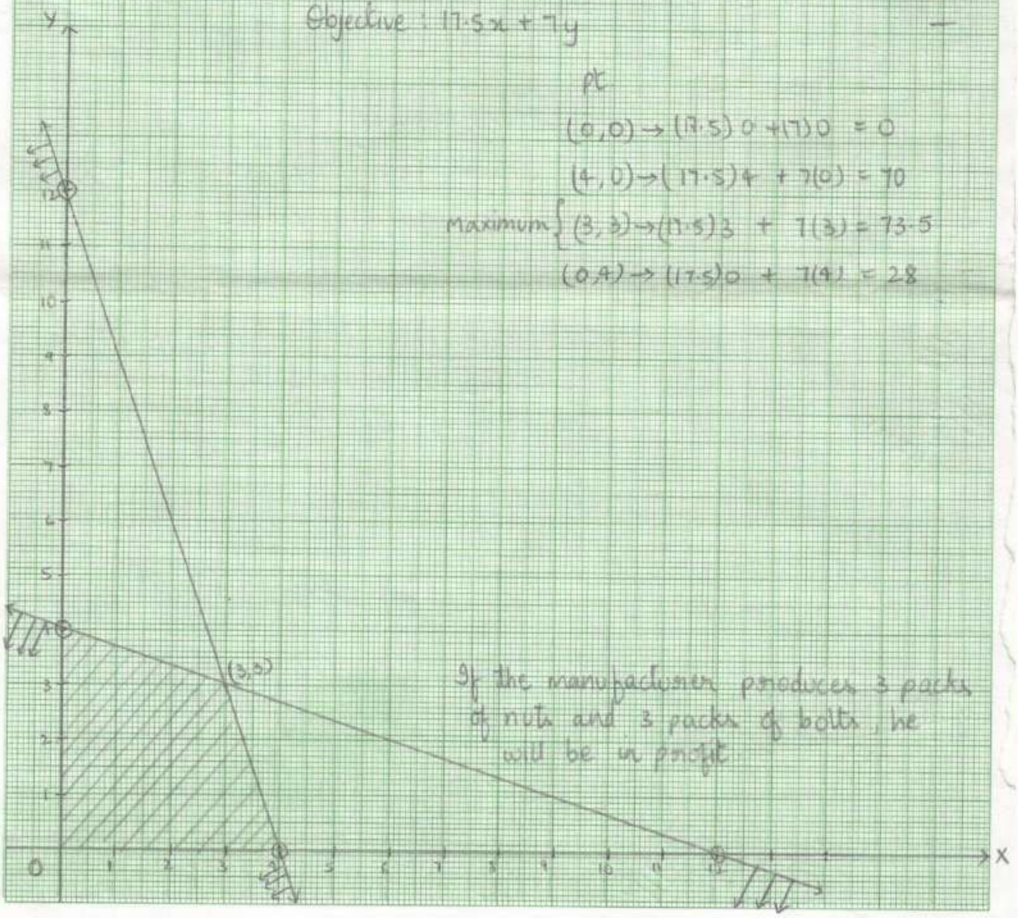
pt

$(0,0) \to (17.5)0 + (7)0 = 0$

$(4,0) \to (17.5)4 + 7(0) = 70$

Maximum $\{ (3,3) \to (17.5)3 + 7(3) = 73.5$

$(0,4) \to (17.5)0 + 7(4) = 28$



If the manufacturer produces 3 packs of nuts and 3 packs of bolts, he will be in profit.