PG SEMESTER COURSE STRUCTURE IN MATHEMATICS ALONG WITH DETAILED SYLLABUS w.e.f. 2016-17

- There shall be four semesters in the two-year M.A./M.Sc. course in Mathematics.
- There will be five papers in each semester.
- Each paper will be of 100 marks. Duration for end semester examination of a paper will be 3 hours.

Five papers for Semester 1 are:

1. MAT501 Group Theory
2. MAT503 Complex Analysis
3. MAT505 Point-Set Topology
4. MAT507 Differential Geometry I
5. MAT509 Classical Mechanics

Five papers for Semester 2 are:

1. MAT502 Module Theory
2. MAT504 Measure and Integration
3. MAT506 Partial Differential Equations and Integral Equations
4. MAT508 Mathematical Methods
5. MAT510 Differential Geometry II

Five papers for Semester 3 are:

1. MAT601 Fields and Galois Theory
2. MAT603 Functional Analysis
3. MAT605 Theory of Ordinary Differential Equations
4. MAT607 Fluid Mechanics
5. Any one of (i) MAT651 Riemannian Geometry

   (ii) MAT653 Algebraic Topology

Five papers for Semester 4 are:

1. MAT602 Wavelets

Any FOUR of the following optional papers:

- MAT 652 Advanced Fluid Mechanics
- MAT 654 Advanced Module Theory
- MAT 656 Algebraic Geometry
- MAT 658 Algebraic Number Theory
- MAT 660 Coding Theory

- MAT 662 Complex Manifolds
- MAT 664 Contact manifolds
- MAT 666 Dynamical Systems
- MAT 668 Finsler Geometry
- MAT 670 Hyperbolic Geometry
- MAT 672 Lie Algebras
- MAT 674 Magnetohydrodynamics
- MAT 676 Nonlinear Analysis
- .MAT 678 p-adic Analysis
- MAT 680 Quantum Information and Computation
- MAT 682 Representation Theory of Finite Groups
- MAT 684 Riemann Surfaces
- MAT 686 Stability Theory of Differential Equations and its Applications

Course Outline

1st Semester July to December

Papers	(L-T-P-C)	Marks
MAT501 Group Theory	4 – 0 – 0 – 4	100
MAT503 Complex Analysis	4 – 0 – 0 – 4	100
MAT505 Point-Set Topology	4 – 0 – 0 – 4	100
MAT507 Differential Geometry I	4 – 0 – 0 – 4	100
MAT509 Classical Mechanics	4 – 0 – 0 – 4	100

2nd Semester January to May

Papers	(L-T-P-C)	Marks
MAT502 Module Theory	4 – 0 – 0 – 4	100
MAT504 Measure and Integration	4 – 0 – 0 – 4	100
MAT506 Partial Differential Equations and Integral Equations	4 – 0 – 0 – 4	100
MAT508 Mathematical Methods	4 – 0 – 0 – 4	100
MAT510 Differential Geometry II	4 – 0 – 0 – 4	100

3rd Semester July to December

Papers	(L-T-P-C)	Marks
MAT601 Fields and Galois Theory	4 – 0 – 0 – 4	100
MAT603 Functional Analysis	4 – 0 – 0 – 4	100
MAT605 Theory of Ordinary Differential Equations	4 – 0 – 0 – 4	100
MAT607 Fluid Mechanics	4 – 0 – 0 – 4	100
MAT651 Riemannian Geometry / MAT653 Algebraic Topology	4 – 0 – 0 – 4	100

4th Semester January to May

Papers	(L-T-P-C)	Marks
MAT602 Wavelets	3 – 0 – 2 – 4	100
Optional Paper 1	4 – 0 – 0 – 4	100
Optional Paper 2	4 – 0 – 0 – 4	100
Optional Paper 3	4 – 0 – 0 – 4	100
Optional Paper 4	4 – 0 – 0 – 4	100

L: Lectures per week, T: Tutorials per week, P: Practicals per week, C: Credits

DETAILED PG SYLLABUS

MAT501: GROUP THEORY

Unit I: Isomorphism theorems for groups, Symmetric groups, Alternating groups, Dihedral groups, Matrix groups, Isometry groups of R^2 and R^3, Internal and External direct product and their relationship, Indecomposable groups.

Unit II: Subnormal and normal series, Zassenhaus' lemma, Schreier's refinement theorem, Composition series, Jordan-Hölder's theorem, Chain conditions.

Unit III: Action of a group *G* on a set, Stabilizer subgroups and orbit decomposition, Class equation of an action, Burnside's theorem, Transitive and effective actions, Equivalence of actions, Core of a subgroup.

Unit IV: Sylow subgroups, Sylow's Theorem I, II and III, *p*-groups, Examples and applications, Groups of order *p q*, Direct and inverse images of Sylow subgroups.

Unit V: Commutator subgroup and commutator series of a group, Solvable groups, Solvability of subgroups and factor groups and of finite *p*-groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

Books Recommended:

1. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. J. A. Gallian, Contemporary Abstract Algebra, 4^{th} Edition, Narosa Publ. House, 1998.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. J. B. Fraleigh, A first Course in Abstract Algebra, Pearson Edu. Inc., 2002.
3. Ramji Lal, Algebra, Vols. I & II, Shail Publications, Allahabad, 2002.

MAT503: COMPLEX ANALYSIS

UNIT I: Complex differentiability, Cauchy-Riemann equations, analytic functions, harmonic functions, harmonic conjugates, analyticity of functions defined by power series, the exponential function and its properties.

UNIT II: Branch of logarithm, power of a complex number, basic properties of contour integration, M-L inequality, fundamental theorem of contour integration, Cauchy's integral theorem, Cauchy-Goursat theorem (statement only), Cauchy's integral formula, Cauchy's integral formula for higher derivatives (statement only), Morera's theorem.

UNIT III: Maximum modulus theorem, Schwarz lemma, Taylor's theorem, Cauchy's estimate, zeros of an analytic functions, the identity theorem for analytic functions, Liouville's theorem, the fundamental theorem of algebra, Laurent's theorem.

UNIT IV: Singularities of functions, removable singularity, poles and essential singularities, Casoratti-Weierstrass theorem, residues, Cauchy's residue theorem, evaluation of simple definite integrals using contour integration, meromorphic functions, argument principle, Rouche's theorem, open mapping theorem, singularity and residue at ∞.

UNIT V: Conformality, Möbius transformations, the group of Möbius transformations, cross ratio, invariance of circles, determination of Möbius transformations mapping real line onto itself, upper half plane onto itself, upper half plane onto open disc and an open disc onto an open disc.

Books Recommended:

1. J. B. Conway, Functions of One Complex Variable, Narosa Publ. House, New Delhi, 2002.
2. S. Ponnusamy and H. Silverman, Complex Variables, Birkhäuser, Inc., Boston, MA, 2006.
3. J. Bak, Complex Analysis, Springer, 1996.
4. V. Ahlfors, Complex Analysis (Third Edition), McGraw-Hill, 1979.
5. A. R. Shastri, An Introduction to Complex Analysis, Macmillan India Ltd., 1999.

MAT505: POINT-SET TOPOLOGY

Unit I: Topological Spaces, metric topology, ordered topology, open sets, closed sets, interior, exterior, boundary and closure of a set, limit points of a set, characterization of closed sets and dense sets, separable spaces, basis and subbasis of a topology, first countable and second countable spaces.

Unit II: Sequences in a metric space, convergence of a sequence, complete metric spaces, nets and filters, continuous maps and their characterization, open maps, closed maps, homeomorphisms, topological invariants.

Unit III: Product topology, Quotient topology and identification spaces (torus, projective spaces $P^{n,}$ Möbius strip and Klein bottle), connected spaces, locally connected spaces, path connected and locally path connected spaces.

Unit IV: Separation Axioms: T_0 spaces, T_1 spaces, T_2 spaces, regular spaces, T_3 spaces, completely regular spaces, normal spaces, Tychonoff spaces, T_4 spaces, characterization of these spaces, Urysohn's lemma, Tietze's extension theorem, Urysohn's embedding and metrization theorem.

Unit V: Compact spaces and their characterizations, compactness in metric spaces and their characterisation (limit point compactness, sequential compactness, complete and total boundesness), locally compact spaces, Tychonoff's theorem, one point compactification.

Books Recommended:

1. J. L. Kelley, General Topology, Van Nostrand, 1995.
2. K. D. Joshi, Introduction to General Topology, Wiley Eastern, 1983.
3. James R. Munkres, Topology, 2nd Edition, Pearson International, 2000.
4. J. Dugundji, Topology, Prentice-Hall of India, 1966.
5. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
6. S. Willard, General Topology, Addison-Wesley, 1970.

MAT507: DIFFERENTIAL GEOMETRY I

UNIT I: Curves in space R^3, parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet-Serret formulae, Frenet approximation of a space curve.

UNIT II: Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of R^3, fundamental theorem of space curves, surfaces in R^3, regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on R^3, surfaces of revolution, tangent vectors, tangent plane, differential of a map.

UNIT III: Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

UNIT IV: Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

UNIT V: Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on a surface of evolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).

Books Recommended:

1. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
2. B. O' Neill, Elementary Differential Geometry, Academic Press, 1997.
3. A. Gray, Differential Geometry of Curves and Surfaces, CRC Press, 1998.
4. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.
5. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.
6. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
7. L. P. Eisenhart, A Treatise on the Differential Geometry of Curves and Surfaces, Ginn and Company, Boston, 1909.

MAT509: CLASSICAL MECHANICS

UNIT I: The momentum of a system of particles, the linear and the angular momentum, rate of change of momentum and the equations of motion for a system of particles, principles of linear and angular momentum, motion of the centre of mass of a system, theorems on the rate of change of angular momentum about different points, with special reference to the centre of mass, the kinetic energy of a system of particles in terms of the motion relative to the centre of mass of the system.
Rigid bodies as systems of particles, general displacement of a rigid body, the displacement of a rigid body about one of its points and the concept of angular velocity, computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately, kinematical examples.

UNIT II: The angular momentum and the kinetic energy of a rigid body in terms of inertia constants, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes.
Euler's dynamical equations of motion, motion under no forces, the invariable line and the invariable cone, Eulerian angles and the geometrical equations of Euler.

UNIT III: Generalized co-ordinates, Geometrical equations, holonomic and non-holonomic systems, Configuration space, Lagrange's equations using D' Alembert's Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time *t* explicitly, Lagrange's multipliers case.

UNIT IV: Deduction of Euler's dynamical equations from Lagrange's equations, Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, Lagrange equations for impulsive motion.

UNIT V: Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, Phase space and Hamilton's variational principle, the principle of least action, Canonical transformations, Poisson-Brackets, Poisson-Jacobi identity, Hamilton-Jacobi theory (outline only).

Books Recommended:

1. E. A. Milne, Vectorial Mechanics, Methuen & Co. Ltd., London, 1965.
2. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.
3. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
4. N. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.

MAT502: MODULE THEORY

UNIT I: Modules over a ring, Endomorphism ring of an abelian group, R-Module structure on an abelian group M as a ring homomorphism from R to End_Z (M), submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Statements of Correspondence theorem and Isomorphism theorems, Hom_R (M, N) as an abelian group and Hom_R (M, M) as a ring, Exact sequences, Five lemma, External and internal direct sums and their universal property.

UNIT II: Free modules, Homomorphism extension property, equivalent characterization as a direct sum of copies of the underlying ring, existence of a basis of a vector space, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules.

UNIT III: Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Divisor chain condition, Unique factorization domains, Examples and counterexamples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Unique factorization in polynomial rings over UFD's.

UNIT IV: Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into T(M) and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT V: Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$-module structure, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Diagonalizable and nilpotent parts of a linear operator, Jordon-Chevalley Theorem

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, N.Y., 1974.
3. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. P. Ribenboim, Rings and Modules, Wiley Interscience, N.Y., 1969.
3. J. Lambek, Lectures on Rings and Modules, Blaisedell, Waltham, 1966.
4. Ramji Lal, Algebra, Vols. II, Shail Publications, Allahabad, 2002.

MAT504: MEASURE AND INTEGRATION

UNIT I: Countable and uncountable sets, cardinality and cardinal arithmetic, Schröder-Bernstein theorem, $a < 2^a, 2^{\aleph_0} = c$, the Cantor's ternary set and its properties.

UNIT II: Semi-algebras, algebras, monotone class, σ-algebras, measure and outer measures, Caratheödory extension process of extending a measure on a semi-algebra to generated σ -algebra, completion of a measure space.

UNIT III: Borel sets, Lebesgue outer measure and Lebesgue measure on $\mathbb{R}$, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets, the Cantor-Lebesgue function.

UNIT IV: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, simple functions and their integrals on $\mathbb{R}$, Littlewood's three principles (statement only), Lebesgue integral on *R* and its properties.

UNIT V: Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, Minkowski's and Hölder's inequalities, Riesz-Fischer theorem (statement only).

Books Recommended:
H. L. Royden and P. M. Fitzpatrick, Real Analysis, (Fourth Edition), Prentice Hall, 2010.

Further Reading:
1. P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
2. E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.
3. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM 33, Hindustan Book Agency, New Delhi, 2005.
4. I. K. Rana, An Introduction to Measure and Integration, (Second Edition), Narosa Publishing House, New Delhi, 2005.

MAT506: PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Unit-I: Formation of P.D.E's, First order P.D.E.'s, Classification of first order P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit-II: Pfaffian differential equations, Compatible systems, Charpit's method, Jacobi's Method.

Unit-III: Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.'s.

Unit-IV: Method of separation of variables:- Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations of strings and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

Unit-V: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books recommended:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill Pub.,1957.
2. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Pub. 2005.
3. R.P. Kanwal: Linear Integral Equations, Birkhauser Verlag Pub.,1997.

MAT 508: MATHEMATICAL METHODS

UNIT I:
Boundary-value problems: Orthogonal and Orthonormal sets of functions, Sturm-Liouville (S-L) problems, Eigenvalues and Eigenfunctions of S-L problems , Reality of Eigenvalues and Orthogonality of Eigenfunctions of S-L problems, Singular Sturm-Liouville problems, Mean square Convergence, Completeness of Orthonormal sets, Bessel's inequality, Self-adjoint differential equations. Orthogonal Eigenfunction Expansions, Generalized Fourier series.

UNIT II:
Fourier Series: Periodic functions, Trigonometric series, Fourier series, Euler formulas, A set of sufficient conditions for the convergence of Fourier series of a continuous function of period 2π, Functions of arbitrary periods, Even and Odd functions, Fourier Cosine and Sine series, Half-range expansions, Complex Fourierr series, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error, Bessel's inequality

UNIT III:
From Fourier Series to Fourier Integral, Sufficient conditions for the validity of Fourier integral representation, Fourier Cosine and Sine Integrals, Fourier Cosine and Sine Transforms, Linearity and Fourier Cosine and Sine Transforms of Derivatives, Complex form of Fourier Integral, Fourier Transform and its Inverse, Linearity , Shifting properties, Fourier Transform of Derivatives. Convolution

UNIT IV:
Definition, Linearity and existence of Laplace transform. The inversion formula, Laplace transform of the derivatives and of the integrals of a function, Unit step function Shifting Theorems, Derivatives and Integrals of Laplace Transforms, Convolution products, Application to the Initial Value Problems and System of ODE.

UNIT V:
Calculus of Variations: Functionals and extremals, Variation and its properties, Euler equations, Cases of several dependent and independent variables, Functionals dependent on higher derivatives, Simple applications.

Books Recommended:

1. 1.E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8thEdition, 2001.
2. 2.A. D. Polyanin and A. V. Manzhirov, Handbook of Integral Equations, CRC Press, 2ndEdition, 2008.
3. L. Elsgolts, Differential Equations and Calculus of Variations, Mir Publishers, 1970.
4. A. S. Gupta, Calculus of Variations, Prentice Hall of India, New Delhi, 1999.
5. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004
6. William E. Boyce and Richard C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley & Sons, (Asia), Seventh Edition, 2003.
7. Pipes,Applied Mathematics for Engineers and Physicists, McGraw-Hill International Student Edition, 2nd edition

MAT 510: DIFFERENIIAL GEOMETRY II

Unit I: n-dimensional real vector space, contravariant vectors, dual vector space, covariant vectors, tensor product, second order tensors, tensors of type (r, s), symmetry and skew symmetry of tensors, fundamental algebraic operations, quotient law of tensors.

Unit II: Topological manifolds, compatible charts, smooth manifolds, examples, smooth maps and diffeomorphisms, definition of a Lie group, examples.

Unit III: Tangent and cotangent spaces to a manifold, derivative of a smooth map, immersions and submersions, submanifolds , vector fields, algebra of vector fields, φ-related vector fields, left and right invariant vector fields on Lie groups.

Unit IV: Integral curves of smooth vector fields, complete vector fields, flow of a vector field, distributions, tensor fields on manifolds, r-forms, exterior product, exterior differentiation, pull-back differential forms.

Unit V: Affine connections (covariant differentiation) on a smooth manifold, torsion and curvature tensors of an affine connection, identities satisfied by curvature tensor.

Books Recommended:

1. Kobayashi and Nomizu; Foundations of Differential geometry, Vol-1, Interscience Publishers, 1963.
2. T. J. Willmore; Riemannian geometry, Oxford Science Publication, 1993.
3. S. Kumaresan; A course in Differential Geometry and Lie groups, Hindustan Book Agency, 2002.
4. M. Spivak; A comprehensive Introduction to Differential Geometry, Vols. 1-5, Publish or Perish, Inc., Houston, 1999.
5. W. M. Boothby; An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.
6. U. C. De, A. A. Sheikh; Differential Geometry of Manifolds, Narosa Publishing House, 2007.
7. R. S. Mishra, A course in Tensors with Applications to Riemannian Geometry, Pothishala, Pvt. Ltd., Allahabad, 1965.

MAT601: FIELDS AND GALOIS THEORY

Unit I: Eisenstein's irreducibility criterion, Characteristic of a field, Prime subfields, Field extensions, Finite extensions, Simple extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields.

Unit II: Splitting fields and their uniqueness. Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability.

Unit III: Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, normal closures, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Unit IV: Primitive element theorem, Finite fields, Existence and uniqueness, Subfields of finite fields, Characterization of cyclic Galois groups of finite extensions of finite fields, fundamental theorem of algebra.

Unit V: Cyclotomic extensions and polynomials, Cyclic extensions, Solvability by radicals, Galois' characterization of such solvability, Generic polynomials, Abel-Ruffini theorem, geometrical constructions.

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. T. W. Hungerford, Algebra, Springer (India), Pvt. Ltd., 2004.

MAT603: FUNCTIONAL ANALYSIS

UNIT I: Normed linear spaces, examples and its topological properties, Banach spaces, continuous linear transformations, spaces of continuous linear transformations from a linear space to a Banach space, continuous linear functionals.

UNIT II: Hahn-Banach theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem (or the Uniform boundedness principle).

UNIT III: Conjugate spaces, natural embedding of N in N^{**}, weak and weak*-topology on a conjugate space, conjugate of an operator, simple applications to reflexive separable spaces.

UNIT IV: Hilbert Spaces, Schwarz's inequality, orthogonal complement, Bessel's inequality, orthonormal sets, continuous linear functionals on Hilbert spaces, Riesz- representation theorem, reflexivity of Hilbert Spaces, adjoint of an operator on a Hilbert space.

UNIT V: Self-adjoint and normal operators, unitary operators on a Hilbert space, projections on Hilbert spaces, determinant and the spectrum of an operator, spectral theorem.

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.

MAT605: THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

UNIT I:

Initial and Boundary Value Problems, Picard's Iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Examples of Lipschitzian and Non-Lipschitzian functions, Picard's Theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, examples of problems without solutions and of equations where Picard's iterations do not converge, Differential equations of first order not solved for the derivative, Uniqueness of solutions with a given slope, Singular solutions, *p*- and *c*-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and nonexistence of singular solutions, examples.

UNIT II:

Systems of I order equations arising out of equations of higher order, Norm of Euclidean spaces convenient for analysis of systems of equations, Lipschitz condition for functions from R^{n+1} to R^n, Local existence and uniqueness theorems for systems of I order equations, Gronwall's inequality, Global existence and uniqueness theorems for existence of unique solutions over whole of the given interval and over whole of R, Existence theory for equations of higher order, Conditions for transformability of a system of I order equations into an equation of higher order.

UNIT III:

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

UNIT IV:

Convergence of real power series, Radius and interval of convergence, Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points.

UNIT V:

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Orthogonality relations, Fourier-Legendre expansion, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Sturm comparison theorem, Zeros of Bessel functions, Orthogonality relations, Generating function, Fourier-Bessel expansions.

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.

MAT 607: FLUID MECHANICS

UNIT I:

Review of basic concepts, Real and ideal fluids, Newton's law of viscosity, Convective transport of scalar and vector quantities, Differentiation following the motion and acceleration, The equation of Continuity, Velocity Potential , Body forces, surface forces, Stress vector at a point, Nature of stresses, Stress on an Arbitrary Plane: Cauchy's Stress formula, State of stress at a point, Stress tensor, Isotropic Law of Pressure, Principal stresses and Principal Directions, Stress invariants, General displacement of a fluid element.

UNIT II:

Nature of strains, Rates of strain components, Relation between stress and rates of strain, Transformation of Stress- Components, Transformation of Rates of Strain, Navier- Stokes equation, Euler's Equation, Energy Dissipation due to Viscosity, Diffusion of Vorticity,

UNIT III:

Stream tube and Vortex tube, Helmholtz's vorticity theorem, Kelvin's Circulation theorem, Energy Flux, Mean Potential over a spherical surface in a simply connected region, Kinetic Energy in Irrotational Flow, Kelvin's Minimum kinetic energy Theorem, Uniqueness of the Irrotational motion

UNIT IV:

Two dimensional irrotational motion, The stream function, The Complex potential for two dimensional irrotational motion, Concept of line- sources, sinks, doublets and vortices, Superposition of solutions, The concept of Images, The Vortex pair, Vortex rows: Single infinite row of Line Vortices,the Karman vortex street, Milne-Thomson Circle Theorem, Blasius Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder, Conformal transformation: Uniform line distributions (source, vortex and doublet) under conformal transformation.

UNIT V:

Three dimensional irrotational flow, Concept of Sources, Sinks and Doublets, Axisymmetric flows, Stokes stream function, Statements of Weiss's and Butler's sphere theorems and their applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at rest at infinity,
Gravity waves – Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
4. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
5. F. Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi, 1985

MAT 651: RIEMANNIAN GEOMETRY

Unit I: Riemannian metrics, Riemannian manifolds, examples, Levi-Civita connection, fundamental theorem of Riemannian geometry, Curvature tensors- Riemannian curvature tensor, sectional curvature, Schur's Theorem, Ricci curvature, scalar curvature, Einstein manifolds.

Unit II : Gradient vector fields, divergence of a vector field, Covariant derivative along a curve, parallel transport, length of a curve. Distance function, geodesics, Exponential map,

Unit III: Jacobi fields, Gauss Lemma, complete Riemannian manifolds, Hopf –Rinow Theorem, The theorem of Hadamard, Riemannian immersions, second fundamental form, Gauss equation, Model spaces of constant curvature.

Unit IV: Lie derivative, Lie derivatives of scalars, vectors, tensors and linear connections, commutation formula for Lie differential operator and covariant differential operator.

Unit V: Motion, Affine motion, projective motion in a Riemannian space, curvature collineation, conformal and homothetic transformations.

Books Recommended:

1. M. P. do Carmo; Riemannian Geometry, Berkhauser, 1992.
2. P. Peterson; Riemannian Geometry, Springer, 2006.
3. J. Jost; Riemannian Geometry and Geometric Analysis, Springer, (6^{th} edition), 2011.
4. J. M. Lee; Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
5. S. Gallot, D. Hullin. J. Lafontaine; Riemannian Geometry, Springer,3^{rd} edition, 2004.
6. K. Yano; The Theory of Lie derivatives and its Applications, North Holland Publishing Company, Amsterdom, 1957.

MAT 653: ALGEBRAIC TOPOLOGY

UNIT I:
Homotopy of paths, fundamental group of a topological space, fundamental group functor, homotopy of maps of topological spaces; homotopy equivalence; contractible and simply connected spaces.

UNIT II:
Fundamental group of the circle, Calculation of fundamental groups of S^n $(n > 1)$, $RP^{2,}$ torus and dunce cap , Brouwer's fixed- point theorem for the disc, fundamental theorem of algebra, vector fields, Borsuk- Ulam theorem for S^2 .

UNIT III:
Covering spaces, unique lifting theorem, path-lifting theorem, covering homotopy theorem, criterion of lifting of maps in terms of fundamental groups, universal covering space, covering transformations, orbit spaces.

UNIT IV:
Singular complex of a topological space , singular homology groups and their funcoriality, homotoy invariance of homology, Eilenberg-Steenrod axioms (without proof), abelianization of the fundamental group, relative homology.

UNIT V:
Calculations of homology of S^n , Brouwer's fixed point theorem for $f : D^n \to D^n$ $(n > 2)$ and its applications to spheres and vector fields, Meyer-Vietoris sequence and its Applications, Jordan-Brouwer separation theorem, invariance of domain.

Books Recommended:

1. 1. J. R. Munkres , Topology, Prentice-Hall of India, 2000.
2. M. J. Greenberg and J. R. Harper, Algebraic topology, a first course, Addison-Wesley Publishing co., 1997.
3. S. Deo, Algebraic Topology, A Primer, Hindustan Book Agency, 2006.
4. J. W. Vick, Homology Theory , An introduction to Algebraic Topology, Springer-Verlag, 1994.

MAT602: WAVELETS

Theory: 75%

UNIT I: The discrete Fourier transform and the inverse discrete Fourier transform, their basic properties and computations, The fast Fourier transform, The discrete cosine transform and the fast cosine transform.

UNIT II: Construction of wavelets on $\mathbb{Z}_N$, First stage and by iteration, The Haar system, Shannon wavelets, Daubechies' D6 wavelets on $\mathbb{Z}_N$, Description of $l^2(\mathbb{Z})$, $L^2[-\pi,\pi)$, $L^2(\mathbb{R})$, their orthonormal bases, Fourier transform and convolution on $l^2(\mathbb{Z})$, wavelets on $\mathbb{Z}$ Haar wavelets on $\mathbb{Z}$, Daubechies' D6 wavelets for $l^2(\mathbb{Z})$.

UNIT III: Orthonormal bases generated by a single function in $L^2(\mathbb{R})$, Fourier transform and inverse Fourier transform of a function f in $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, Parseval's relation, Plancherel's formula, Orthonormal wavelets in $L^2(\mathbb{R})$, Balian-Low theorem.

UNIT IV: Multi-resolution analysis and MRA wavelets, certain function in $L^2(\mathbb{R})$ for which $\{\psi_{j,k}\}$ does not form an orthonormal system, compactly supported wavelets, band-limited wavelets.

UNIT V: Franklin wavelets on $\mathbb{R}$, Dimension function, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journe's wavelet, Decomposition and reconstruction algorithms of Wavelets.

Lab-work: 25%

The following lab work is recommended:

1. To plot a member in $l^2(\mathbb{Z}_N)$, its Fourier transform and its inverse Fourier transform.
2. To verify various identities relating Fourier transform, inverse Fourier transform, inner product, norm and convolution.
3. Computing Fourier coefficients of an element of $l^2(\mathbb{Z}_N)$, with respect to a given wavelet (Haar, Daubechies' D6) at a certain level such as $\langle z, \psi_{-2,k} \rangle$ etc.

Books Recommended:

1. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer-Verlag, 1999.
2. Eugenio Hernández and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.

Further Reading:

1. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
2. Ingrid Daubechies, Ten Lectures on Wavelets, CBS-NFS Regional Conferences.

MAT652: ADVANCED FLUID MECHANICS

UNIT-I: Stress Principle of Cauchy, Equations for conservation of linear and angular Momentum, Constitutive equations for Newtonian fluids; Navier- Stokes equations in Vector and Tensor forms, Navier- Stokes equations in orthogonal coordinate systems (particularly in Cartesian, cylindrical and spherical coordinate systems).

UNIT-II: Vorticity equations; Energy dissipation due to viscosity, Dynamical similarity and dimensionless numbers and their significance in the fluid dynamics, Some Exact solutions – Fully developed plane Poiseuille and Couette flows between parallel plates, Steady flow between pipes of uniform cross-section.

UNIT-III: Couette flow between coaxial rotating cylinders, Small Reynolds number flow – Flow between steadily rotating spheres, Stokes equations, Dynamic equation satisfied by stream function, Relation between pressure and stream function; General stream function solution of Stokes equations in spherical polar coordinates; Steady flow past a sphere, Drag on a body.

UNIT-IV: Flow past a circular cylinder, Stokes paradox, Oseen's equations, Elementary ideas about perturbation and cell methods as applied to slow flow problems, Boundary layer concept.

Unit-V: Two-dimensional boundary layer equations, separation phenomena, Boundary layer on a semi-infinite plane, Blasius equation and solution, Karman's Integral method, Displacement thickness, Momentum thickness and Energy thickness.

Reference books:

1. Z.U.A. Warsi, Fluid Dynamics, CRC Press (2005)
2. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, Kluwer Academic Publishers group, (1983)
3. W.E. Langlois, Slow Viscous flow, Macmillan, (1964)
4. T.C. Papanastasiou, G.C.Georgiou, A.N.Alexandrou, Viscous Fluid Flow; CRC Press (2000).
5. N. Curle &H.J. Davies, Modern Fluid Dynamics (Vol.-I), D.Van Nostrand Comp. Ltd. (London), (1964)

MAT 654: ADVANCED MODULE THEORY

UNIT I: Modules over rings, modular law, annihilators, factor theorem, projections, idempotent endomorphisms.

UNIT II: Chain conditions on modules, Noetherian modules and rings, artinian modules and rings, equivalent characterizations, composition series of modules, Jordan–Hölder theorem and Hilbert basis theorem (statement only).

UNIT III: Fitting lemma**,** Jacobson radical, Jacobson semi-simple ring, nilpotent and nil ideals, Hopkins Levitzki theorem, Nakayama's Lemma.

UNIT IV: Injective modules and divisible modules, embedding theorem for modules, essential extension, injective envelope of a module.

UNIT V: Small submodules, projective modules and projective covers, Jacobson radical of a projective module.

References:

1. F. W. Anderson and K. R. Fuller, Rings and Category of Modules, Graduate texts in mathematics, Vol 13, Springer-Verlag Inc., New York, 1974.
2. Paul E. Bland, Rings and their Modules, Walter de Gruyter GmbH & Co. KG, Berlin/New York, 2011.
3. T. Y. Lam, A First Course in Noncommutative Rings, Graduate texts in mathematics: 131, Springer-Verlag New York, Inc., 1991.
4. D. W. Sharpe and P. Vamos, Injective Modules, Cambridge University Press, 1972.

Further Readings:

1. O. Zariski and P. Samuel, Commutative Algebra, Volume I, D. Van Nostrand Company, Inc., 1958.
2. T. Y. Lam, Lectures on Modules and Rings, Graduate texts in mathematics: 189, Springer-Verlag New York, Inc., 1999.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Third Edition, John Willy and Sons, Inc., 2004.

MAT 656: ALGEBRAIC GEOMETRY

UNIT I:

Affine algebraic sets , Zariski topology, algebraic set and ideal correspondence, Hilbert's nullstellensatz, affine varieties.

UNIT II:

Polynomial maps, the coordinate ring functor, rational maps and birational equivalence, dimension and product of affine varieties.

UNIT III:

Projective algebraic sets and projective varieties, projective closures, rational functions and morphisms, Segre embedding and Veronese embedding.

UNIT IV:

Tangent spaces, smooth and singular points, algebraic characterizations of the dimension of a variety, blowing-up a point on a variety.

UNIT V:

Plane curves, rational curves, multiple points, intersection numbers, Bezout's theorem, Max Noether's fundamental theorem.

Books Recommended:

1. C. Musli, Algebraic Geometry for Beginners, TRIM-20, Hindustan Book Agency, 2001.
2. W. Fulton, Algebraic Curves, An Introduction to Algebraic Geometry, W.A. Benjamin, 1969.
3. K. Hulek, Elementary Algebraic Geometry , SML, vol 20, American Mathematical Society, 2003.
4. M. Ried, Undergraduate Algebraic Geometry, LMS Student texts 12, Crambridge University Press, 1988.

MAT 658: ALGEBRAIC NUMBER THEORY

UNIT I: Number fields, the ring of algebraic integers, calculation for quadratic, cubic and cyclotomic case, norms and traces, integral bases and discriminants.

UNIT II: Dedekind domains, unique factorization of ideals, norm of ideals, factorization of prime ideals in extensions.

UNIT III: The ideal class group, lattices in R^n, Minkowski's theorem, finiteness of the class number and its consequences , some class number computations.

UNIT IV: Dirichlet unit theorem, units in real quadratic fields, some Diophantine equations.

UNIT V: Cubic residue symbol, Jacobi sums, Cubic reciprocity law, biquadratic reciprocity law and Eisenstein reciprocity law.

Books Recommended:

1. J. Esmonde and M. Ram Murty, Problems in Algebraic Number Theory, GTM-190, Springer-Verlag, 1999.
2. R.A. Mollin, Algebraic Number Theory, CRC Press, 2011.
3. D. A. Marcus, Number Fields, Springer-Verlag, 1977.
4. S. Alaca and K. S. Williams, Introductory Algebraic Number theory. Cambridge University Press, 2004.

MAT 660: CODING THEORY

UNIT I: Block codes, linear codes, minimum distance, generator and parity check matrices, Hamming codes, Nearest neighbour decoding for linear codes, syndrome decoding, weight enumerators.

UNIT II: Singleton and sphere packing bounds, MDS codes and perfect codes, Gilbert–Varshamov bound, Griesmer bound, Hadamard codes, , binary and ternary Golay codes.

UNIT III: Constructions of linear codes**,** Reed–Muller codes, Subfield codes, Cyclic codes, generator and parity-check polynomials, BCH codes,

UNIT IV: Reed Solomon codes, quadratic residue codes, binary cyclic codes of Length 2n (n odd), generalized Reed-Muller codes.

UNIT V: Quaternary codes, binary codes derived from codes over $\mathbb{Z}_4$, Galois ring over $\mathbb{Z}_4$, cyclic and quadratic residue codes over $\mathbb{Z}_4$, self dual codes over $\mathbb{Z}_4$.

Books Recommended:

1. J. H. Van Lint, Introduction to Coding Theory, 3rd ed., Graduate Text in Mathematics, 86, Springer-Verlag, 1999.
2. W. C. Huffman and V. Pless, Fundamentals of Error-correcting Codes, Cambridge University Press, 2003.
3. S. Ling and C. Xing, Coding Theory, A First Course, Cambridge University Press, 2004.

MAT 662: COMPLEX MANIFOLDS

Unit I: Complexification of a real vector space, complex structure, relation between complexification and complex structure, conjugate complex structure, complexification of the dual space, expressions in terms of bases, orientations, complex structures, necessary conditions for a complex structure to exist, examples of complex manifolds.

Unit II: The tangent space and the cotangent space, complexified tangent space, complex structure on the tangent space, complex structure on the cotangent space, relation between the canonical complex structure and the manifold complex structure, vectors and tensors, real tensors, vectors and one-forms of type (1,0) and type (0,1), complex tensors and complex manifolds, tensor fields.

Unit III: Almost complex structure, conditions for existence of an almost complex structure, almost complex structure on a complex manifold, the Nijenhuis tensor, vanishing of the Nijenhuis tensor as necessary and sufficient condition for integrability.

Unit IV: Hermitian structures on vector spaces, Hermitian manifolds, curvature tensor on a Hermitian manifold, holomorphic sectional curvature, Kaehlerian manifolds, curvature on Kaehlerian manifolds, complex space forms.

Unit V: Nearly Kaehler and para Kaehler Manifolds, projective correspondence between two nearly Kaehler manifolds, conformal flatness of a para Kaehler manifold, curvature identities.

Books Recommended:

1. S.S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geomerty, World Scientific, 2000.
2. E.J. Flaherty, Hermitian and Kaehlerian Geometry in Relativity, LNP 46, Springer, 1976.
3. T. J. Wilmore, Riemannian Geometry, Oxford Science Publications, 1993.
4. Kobayashi and Nomizu, Foundations of Differential geometry, Vol-II, Interscience Publishers, 1963.
5. K. Yano & M. Kon, Structures, on Manifolds, World Scientific, 1984.

MAT 664: CONTACT MANIFOLDS

Unit I: Contact manifolds. Definition, Examples, Almost contact manifolds, Killing vector field, The tensor field h, Curvature properties of contact metric manifolds

Unit II: K-contact Manifolds, Characterizations of K- contact manifolds, Curvature properties, Sectional curvature, Locally symmetric K- contact manifolds.

Unit III: Sasakian manifolds, Curvature properties, ϕ- sectional curvature of a Sasakian manifold, cosymplectic manifolds, Sasakian space forms.

Unit IV: Cosymplectic manifolds, Nearly cosymplectic manifolds, cosymplectic space forms, trans- Sasakian manifolds, , 3- dimensional trans- Sasakian manifolds.

Unit V: Para contact manifolds, Examples, Torsion tensor fields, Integrability of almost para contact structure, Para- Sasakian manifolds, LP- Sasakian manifolds.

Books Recommended:

1. D. E. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Birkhauser, 2010.
2. K. Yano & M. Kon, Structures on Manifolds, World Scientific, 1984.

MAT 666: DYNAMICAL SYSTEMS

UNIT I: Dynamical systems, Iterates of a function, trajectories and orbits, recursion equations, phase portraits, the logistic function.

UNIT II: Review of metric spaces, topology of IR and analysis of real functions, fixed (or equilibrium) points, periodic points, asymptotic points, stable sets, graphical analysis.

UNIT III: Sarkovskii's ordering, Sarkovskii's theorem, sufficient conditions for a function on a closed interval to have a unique fixed point, dynamical information from a differentiable function, hyperbolic and nonhyperbolic periodic points, attracting periodic points.

UNIT IV: σ-algebras and subalgebras, measure algebras, atoms of a measure algebra and the Caratheodory's theorem, symbol space, shift maps, topologically transitive functions, sensitive dependence, chaotic functions, topological conjugates.

UNIT V: Measure preserving transformations, definitions and examples, construction of a new measure preserving transformation from given ones, homomorphisms, isomorphisms, recurrence property of a measure preserving transformation, Poincare's recurrence theorem, introduction to ergodicity.

Books Recommended:

1. R. A. Holmgren, A First Course in Discrete Dynamical Systems, Springer-Verlag, 1994.
2. H. L. Roydenand P. M. Fitpatrick, Real Analysis (Fourth Edition), Prentice Hall, 2010.
3. P. Walters, An Introduction to Ergodic Theory, Springer, 1982.

Further Reading:

1. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Second Edition, Addison- Wesley, 1989.
2. K. R. Parthsarthy, Introduction to Probability and Measure, Hindustan Book Agenc, 2005.

MAT668: FINSLER GEOMETRY

UNIT I: Basic Concepts of a Finsler space

Line elements, Finsler space, Minkowskian space, Tangent space, Indicatrix, Metric Tensor, Dual tangent space, Hamiltonian function, Angle between two vectors, Generalized Christoffel symbols, Geodesics.

UNIT II: Covariant Differentiation

δ-derivative, Partial δ-derivative, Fundamental postulates of E. Cartan, Different deductions, Cartan's two processes of covariant differentiation, Berwald connection parameters, Berwald's covariant differentiation.

UNIT III: Theory of Curvature

Commutation formulae resulting from Cartan's covariant differentiation, Cartan curvature tensor, Commutation formulae resulting from Berwald's covariant differentiation, Berwald curvature tensor, Generalizations of Bianchi identities, Space of scalar curvature, Space of constant curvature, Generalization of Schur's theorem, Recurrent spaces, Symmetric spaces.

UNIT IV: Projective Change

Projective change, Projective invariants, Projective change of Berwald's connection parameters, Projective deviation tensor, Generalized Weyl's projective curvature tensor, Projective connection parameters, Projectively flat spaces, Szabó Theorem.

UNIT V: Lie derivatives and their applications

Infinitesimal transformations, Lie derivative of scalars, vectors and tensors, Lie derivative of connection parameters of Cartan and Berwald, Motion, Affine motion and Projective motion.

Books Recommended:

1. H. Rund, The Differential Geometry of Finsler Spaces, Springer-Verlag, Berlin, 1959.
2. M. Matsumoto, Foundations of Finsler Geometry and Special Finsler Spaces, Kaisheisha Press, Otsu, 1986.
3. P. L. Antonelli (ed.), Handbook of Finsler Geometry, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003

MAT 670: HYPERBOLIC GEOMETRY

UNIT I:

Euclid's parallel postulate and its independence, conformal disk, upperhalf plane and hyperboloid model of the hyperbolic palne, geodesics and arc lenths in Euclidean n-space and Spherical n-space.

UNIT II:

Lorentzian n-space, Lorentz group and their actions, hyperbolic n-space, arc length, angle and geodesics an hyperbolic n-space.

UNIT III:

Different models of hyperbolic n-space, classicication of Mobius transformations, basic theory of topological groups.

UNIT IV:

Group of isometries of hyperbolic n-space, discrete groups, discrete Euclidean groups, elementary groups.

UNIT V:

Geometry of discrete groups, geometry of conves polyhedra, fundamental domains, tessellations, reflection and crystallographic groups.

Books Recommended:

1. J. Ratcliffe, Foundations of Hyperbolic Manifolds (Second Edition),GTM-149, Springer, 2005.
2. B. Iversion, Hyperbolic Geometry, LMS Students Texts no. 25, Cabbridge University Press, 1992.
3. J. W. Anderson, Hyperbolic Geometry, (Second Edition), Springer, 2005.

MAT672: LIE ALGEBRAS

UNIT I: Definition and examples of Lie algebras, classical Lie algebras, derivations of a Lie algebra, abelian Lie algebra, Lie subalgebras, ideals and homomorphisms, normalizers and centralizers of a Lie subalgebra, representations of lie algebras (definition and some examples), automorphisms of a Lie algebra.

UNIT II: Solvable algebra, Solvable radical, nilpotent algebra, Engel's Theorem, semi-simple Lie algebra, Lie's Theorem, Jordan-Chevalley decomposition (existence and uniqueness), Cartan trace criterion for solvability, Killing form and criterion for semisimplicity.

UNIT III: Simple ideals, inner derivations, abstract Jordan-Chevalley decomposition, Lie algebra modules, Schur's Lemma, Casimir elements of a representation, Weyl's Theorem for preservation of Jordan-decomposition.

UNIT IV: Representation of sl(2, **C**): weights, highest weight, maximal vectors, classification of irreducible modules, toral and maximal toral subalgebra, root space decomposition and properties of roots.

UNIT V: Abstract root systems, Weyl group, root strings, bases and their existence, Weyl chambers, Classification of rank 2 root systems.

Books Recommended:

1. J, E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, Lie Algebras, Wiley-Interscience, New York, 1962.
3. J. P. Serre, Lie Algebras and Lie Groups, Benjamin, New York, 1965.

MAT 674: MAGNETOHYDRODYNAMICS

Unit I:

Maxwell's equations, Conservation of energy, Poynting vector, Conservation of momentum and Maxwell's stress tensor, Electromagnetic momentum density.

Unit II:

Nature of Magnetohydrodynamics, Main assumptions of MHD, Electromagnetic fields in a conductor at rest, a uniformly moving rigid conductor and a deformable conductor. Basic equations of non- viscous and viscous magnetohydrodynamics: mass, momentum and energy conservation laws.

Unit III:

Basic Properties of the magnetic field and MHD terms: Magnetic Reynolds number, magnetic viscosity, magnetic pressure, magnetic diffusion and frozen- in- effect. Magnetohydrodynamic boundary conditions.

UNIT IV:

Magnetohydrodynamic Flows, Formulation and solution of Linear flow, Flow between parallel plates Hartmann flow, Couette flow.

Unit V:

Magnetohydrodynamic Waves, Linearized equations, MHD waves in a perfectly conducting fluid, Alfven waves and magnetosonic waves.

Books Recommended:

1. J.D.Jackson, Classical Electrodynamics, Wile Eastwern Limited, New Delhi, 1990.
2. L. D. Landau and E. M. Lifshitz, Classical Electrodynamics, Butterworth-Heinemann, 2nd Edition, 1984.
3. A. Jaffery, Magnetohydrodynamics, Oliver and Boyd, N.Y. 1966.

MAT676: NONLINEAR ANALYSIS

UNIT I: Compactness in Metric spaces, Measure of Noncompactness, Normed spaces, Banach spaces, Hilbert spaces, Uniformly convex, strictly convex and reflexive Banach spaces, Lipschitzian and contraction mapping, Banach's contraction principle, Application to Volterra and Fredholm integral equations.

UNIT II: Nonexpansive, asymptotically nonexpansive, accretive and quasinonexpansive mappings, Fixed point theorems for nonexpansive mappings, Nonexpansive operators in Banach spaces satisfying Opial's conditions, The demiclosedness principle.

UNIT III: Schauder's fixed point theorem. Condensing maps. Fixed points for condensing maps, The modulus of convexity and normal structure, radial retraction, Sadovskii's fixed point theorem, Set-valued mappings.

UNIT IV: Fixed point iteration procedures, The Mann Iteration, Lipschitzian and Pseudocontractive operators in Hilbert spaces, Strongly pseudocontractive operators in Banach spaces, The Ishikawa iteration, Stability of fixed point iteration procedures.

UNIT V: Iterative solution of Nonlinear operator equations in arbitrary and smooth Banach spaces, Nonlinear *m*-accretive operator, Equations in reflexive Banach spaces.

Books Recommended:

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.
2. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, New York, 2001.
3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.
4. V. I. Istratescu, Fixed Point Theory, An Introduction, D. Reidel Publishing Co., 1981.
5. K. Goebel and W. A. Kirk, Topic in Metric Fixed Point Theory, Cambridge University Press, 1990.

MAT 678: *p*-ADIC ANALYSIS

UNIT I:

Norms on a field, Archimedean and non-Archimedean norm s, *p*-adic norm on rationals, metric induced by a norm, isosceles triangle principle, equivalent norm Ostrowski's theorem.

UNIT II:

Completion Q of with respect to the *p*-adic norm, *p*-adic numbers and *p*-adic Integers, canonical expansion of *p*-adic numbers, arithmetic in , Hensel's lemma, algebraic properties of p-adic integers.

UNIT III:

Topology of Q_p and its comparison with topology of R, sequences and series in $Q_{p,}$ properties of p-adic exponential and logarithm , zeros os p-adic power series.

UNIT IV:

p-adic functions, locally constant functions, continuity and differentiability of p-adic functions, isometiers of $Q_{p,}$ interpolation.

UNIT V:

p-adic interpolation of function $f(s)=a^s$, p-adic distributions, Bernoulli's distribution, p-adic zeta function, the algebraic closure of $Q_{p.}$

Books Recommended:

1. *p*-adic Number, *p*-adic Analysis, and Zeta-Functions – Neal Koblitz, Graduate Text in Mathematics, vol 58, Springer, 1977.
2. 2.S. Katok, p-adic Analysis Compared with Reals, SML-37, American Math. Society, 2007.
3. A. M. Roberts, A Course in p-adic Analysis, GTM, Springer, 1998.

MAT 680: QUANTUM INFORMATION AND COMPUTATION

Unit I: Basic concepts of Probability and Classical Information Theory: Random Experiments, Random Events, Axioms of Probability. Multiplication rule of probabilities. Conditional Probability, Bayes' Theorem, and Independence of events.
Uncertainty and Shannon entropy. Axiomatic characterization and properties of Shannon entropy. Joint and conditional entropies. Relative entropy and mutual information.

Unit II: Inadequacy of classical mechanics, Stern-Gerlach experiment. Basic postulates of Quantum mechanics: System, states, observables, measurements, unitary evolution of quantum states. Mixed states and density operators. Composite systems, sub-systems, partial trace and partial transpose.

Unit III: Theory of entanglement: Separability and inseparability, Schmidt decomposition. Separability and PPT criteria, Local operation and LOCC. POVM and its projective extension on a larger space. Von-Neumann entropy and is properties (subadditivity, concavity, etc.).
EPR paradox, Hidden variables and Bell's inequalities.

Unit IV: Quantum teleportation, basics idea of classical cryptography, limits of classical cryptography, quantum key distribution, BB84 and Ekert91 protocols, quantum dense coding.

Hadamard and controlled-NOT gates. Universality of quantum gates.

Unit V: Deutsch's algorithm, Deutsch-Jozsa algorithm, Grover's algorithm, Shor's factorization algorithm. Basics of linear error correcting codes. Quantum error correction and simple examples. Shor code, CSS codes, stabilizer code formalism.

Books Recommended:

1. M.A. Nielsen and I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.

Further Reading:

1. J. Gruska, Quantum Computing, McGraw Hill Companies, 1999.
2. G. Benenti, G. Casati and Giuliano Strini, Principles of Quantum Computation and Information, Vol. 1: Basic Concepts, Vol II: Basic Tools and Special Topics, World Scientific, Singapore, 2004.
3. Asher Peres, Quantum Theory: Concepts and Methods, Kluwer Academic Publ., Dordrecht, 1995.
4. J. Preskill, Lecture notes on Quantum Computation, Physic 219/Comp. Sc.219 Course, available in http://theory.caltech.edu/people/preskill/ph229.
5. H. K. Lo, S. Popescu and T. Spiller, Introduction to Quantum Computation and Information, World Scientific, Singapore, 1998.
6. N. D. Mermin, Quantum Computer Science (Cambridge, 2007).
7. M. Le Bellac (Cambridge University Press, 2006).
8. M. Hayashi, Quantum Information, Springr Verlag, Berlin, 2006.
9. J. J. Sakurai, Modern Quantum Mechanics, 2nd eds., Addison-Wesley, ISE Reprint, 1999.

MAT682: REPRESENTATION THEORY OF FINITE GROUPS

UNIT I: Irreducible and completely reducible modules, Schur's Lemma, Jacobson density Theorem, Wedderburn Structure theorem for semisimple modules and rings. Group Algebra Maschke's Theorem:

UNIT II: Representations of a group on a vector space, matrix representation of a group, equivalent and non-equivalent representations, Decomposition of regular representation, Number of irreducible representations.

UNIT III: Characters, irreducible characters, Orthogonality relations, Integrality properties of characters, character ring, Burnside's $p^a q^b$ Theorem.

UNIT IV: Representations of direct product of two groups, Induced representations, The character of an induced representation, Frobenius reciprocity Theorem. Construction of irreducible representations of Dihedral group D_n, Alternating group A_4, Symmetric groups S_4 and S_5 .

UNIT V: Mackey's irreducibility criterion, Clifford's Theorem, Statement of Brauer and Artin's Theorems.

Books Recommended:

1. M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.
2. L. Dornhoff, Group Representation Theory, Part A, Marcel Dekker, Inc., New York, 1971.
3. N. Jacobson, Basic Algebra-II, Hindustan Publishing Corporation, New Delhi, 1983.
4. S. Lang, Algebra, 3rd ed., Springer, 2004.
5. J. P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.

MAT 684: RIEMANN SURFACES

UNIT I:

Riemann surfaces, Genus of a compact Riemann surfaces, Complex Tori, Riemann surfaces and algebraic curves.

UNIT II:

Functions on Riemann surfaces, meromorphic functions on Riemann spheres, projective lines and complex torus, global properties of holomorphic maps.

UNIT III:

Group actions on Riemann surfaces, Ramification of quotient map, Hurwitz's theorem on automorphisms.

UNIT IV:

Monodromy of a finite covering and a holomorphic maps, differential forms, integration on Riemann surfaces.

UNIT V:

Divisors, linear equivalence of divisors, space of forms associated to a divisor, Riemann-Roch theorem and the field of meromorphic function on a compact Riemann surface.

Books Recommended:

1. R. Miranda, Algebraic Curves and Riemann Surfaces, GSM-5, American Math. Society, 1995.
2. H. M. Farkas and I. Kra, Riemann Surfaces, GTM-71, Springer, 1980.
3. Riemann Surfaces, Mathematical Phamphlets no. 1, TIFR.

MAT 686: STABILITY THEORY OF DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

UNIT I: Uncoupled and coupled linear Systems, Reduction of coupled linear system to uncoupled linear system, Exponentials of operators, Fundamental theorem for linear systems, Non-homogeneous linear systems.

UNIT II: Non-linear Autonomous system, Linearization, The phase plane & its phenomena, Critical points, Types of critical points, Phase plane analysis, Conservative systems.

UNIT III: Variational matrix, Stability analysis of linear and nonlinear systems using variational matrix, Liapunov Function, Stability by Liapunov's Direct Method.

UNIT IV: Mathematical model, Formulation of mathematical models, Classification of mathematical models, Malthusian growth model, Logistic growth model, Regrowth Model, Delayed differential models.

UNIT V: Lotka-Volterra predation model, Rosenzweig-MacArthur model, Lotka-Volterra competition model, Lotka-Volterra models of mutualism, obligate and non-obligate mutualism, effect of mutualism on predator-prey and competitive systems

Books Recommended

1. Lawrence Perko, Differential Equations and Dynamical Systems, Springer-Verlag, New York, Inc., 2001.
2. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata-McGraw Hill, 1991.
3. H. I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel Dekker, New York, 1980.