JEE-Main-29-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}$ is equal to:

Options:

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(2)-\frac{\pi}{4}$

(c) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$

(d) $\frac{\pi}{3}-\tan^{-1}(2)$

Answer: (b)

Solution:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x} = \int_0^{\frac{\pi}{2}} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}=t \Rightarrow \sec^2\frac{x}{2}dx = 2dt$

$$= \int_0^{\frac{\pi}{2}} \frac{2dt}{2t^2+4t+4} = \int_0^{\frac{\pi}{2}} \frac{dt}{(t+1)^2+1}$$

$$\Rightarrow \left[\tan^{-1}(t+1)\right]_0^1 = \tan^{-1}(2)-\tan^{-1}(1)$$

$$= \tan^{-1}(2)-\frac{\pi}{4}$$

Question: Let $z=2+3i$, then value of $(z)^5+(\overline{z})^5$ is:

Options:

(a) 246

(b) 244

(c) 248

(d) 234

Answer: (b)

Solution:

$$(z)^5+(\overline{z})^5=(2+3i)^5+(2-3i)^5$$

$= 2\left[{}^5C_0 \cdot 2^5 + {}^5C_2 \cdot 2^3 (3i)^2 + {}^5C_4 \cdot 2^1 \cdot (3i)^4 \right]$

$= 2[32 - 720 + 810]$

$= 244$

Question: Let $\vec{a} = 3\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b}$ is non parallel to $\vec{c}$, then value of λ is:

Options:

(a) 5

(b) –5

(c) 1

(d) –1

Answer: (b)

Solution:

Given, $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

Also,

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} + \lambda\vec{c}$

$\therefore \lambda = -(\vec{a} \cdot \vec{b}) = -(2+3) = -5$

Question: If $\lim_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$, then which of the following option is incorrect?

Options:

(a) $\alpha^2 + \beta^2 + \gamma^2 = 1$

(b) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(c) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(d) $\alpha^2 - \beta^2 + \gamma^2 + 4 = 0$

Answer: (b)

Solution:

$$\lim_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$$

For indeterminacy, $\alpha + \beta = 0$ ....(i)

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x^3} \times \frac{x^2}{\sin^2 x} = \frac{2}{3}$$

Apply L-Hospital rule,

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^x - \beta e^{-x} + \gamma \cos x}{3x^2} = \frac{2}{3}$$

$\because \alpha - \beta + \gamma = 0$ ...(ii)

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^{x} + \beta e^{-x} - \gamma \sin x}{6x} = \frac{2}{3}$$

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^{x} - \beta e^{-x} - \gamma \cos x}{6} = \frac{2}{3}$$

$$\Rightarrow \alpha - \beta - \gamma = 4 \qquad ....(\text{iii})$$

$$\Rightarrow \beta = -1, \alpha = 1, \gamma = -2$$

Question: If $A = \{1, 2, ..., 60\}$ and B is relation on A defined as $B = \{(x, y) : y = pq$ where p and q are primes $\geq 3\}$ then number of elements in B is:

Options:

(a) 720

(b) 660

(c) 540

(d) 600

Answer: (b)

Solution:

Given $y = pq$ { p, q are prime numbers ≥ 3 }

$\therefore$ y can be generated from

$3\times3, 3\times5, 3\times7, 3\times11, 3\times13, 3\times17, 3\times19, 5\times5, 5\times7, 5\times11, 7\times7$

$\Rightarrow$ Total 11 possibilties

x can be $\{1, 2, ..., 60\}$

Number of relations $= 60 \times 11 = 660$

Question: If $f(x) = 3^{(x^2-2)^3} + 4$ and

$P: f(x)$ attains maximum value at $x = 0$.

$Q: f(x)$ have point of inflection at $x = \sqrt{2}$.

$R: f(x)$ is increasing for $x > \sqrt{2}$, then which of the following statement are correct?

Options:

(a) P and R

(b) Q and R

(c) P and Q

(d) P, Q and R all

Answer: (b)

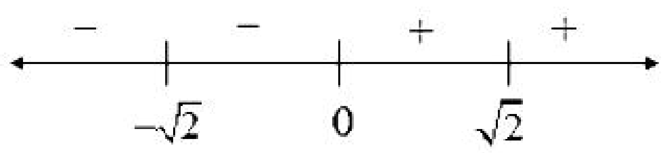
Solution:

Given $f(x) = 3^{(x^2-2)^3} + 4$

$\therefore f'(x) = 3^{(x^2-2)^3} \cdot \ln 3 \cdot 3(x^2 - 2)^2 \, 2x$

Now, $f'(x) = 0$, we get $x = 0$, $x = \pm\sqrt{2}$

$\therefore\ f(x)$ will have inflation point at $x=\sqrt{2}$



So, $f(x)$ is increasing for $x>\sqrt{2}$

And will make minimum at $x=0$

Question: Let $f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||x^2-5x+4|$. The number of points where the function is not differentiable is:

Options:

(a) 3

(b) 4

(c) 5

(d) 6

Answer: (a)

Solution:

$f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||x^2-5x+4|$

$f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||(x-1)(x-4)|$

$f(x)=|(x-1)|\sin|x-1|\cdot\cos|x-2|+|x-3||(x-1)||(x-4)|$

We know, $|(x-a)|g(|x-a|)$ is differentiable when $x-a=0$

$\therefore\ f(x)$ is non-differentiable at $x=1,3,4$

Question: Let A and B are two 3×3 non-zero real matrices and $AB=0$, then which of the following options is correct?

Options:

(a) $AX=B$ has unique solution

(b) $AX=B$ has infinite solutions

(c) B is invertible

(d) $(adj(A))B$ is invertible

Answer: (b)

Solution:

$\because\ AB=0\Rightarrow|A|=0=|B|$

So, B is not invertible as $|B|=0$

$(adj(A))B$ is not invertible as $|adj(A)B|=|adj(A)||B|=0$

$AX=B$ has either no solution nor infinitely many solutions.

Question: If $|x-1| \le y \le \sqrt{5-x^2}$, then the area of region bounded by the curves is:

Options:

(a) $\frac{5\pi}{4}-\frac{1}{2}$

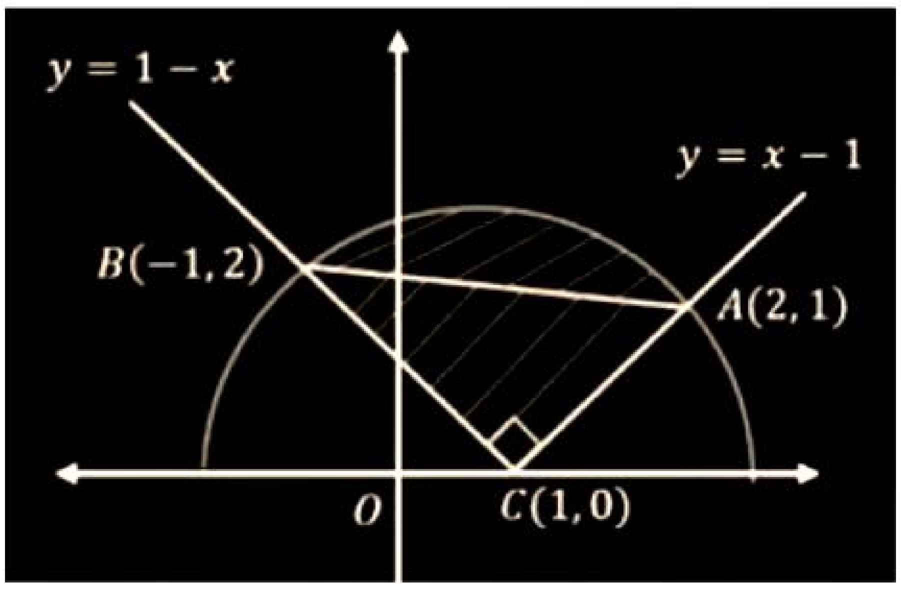
(b) $\frac{5\pi}{4}-\frac{3}{2}$

(c) $\frac{3\pi}{4}-\frac{1}{2}$

(d) $\cos^{-1}\frac{1}{3}-\frac{1}{2}$

Answer: (a)

Solution:



Clearly chord AB subtends a right angle at centre.

Required area = area of $\Delta ABC +$ area of segment of circle on chord AB

$= AC \cdot BC +$ [area of quarter circle – area of ΔAOB]

$=\frac{1}{2}\sqrt{2}\cdot 2\sqrt{2}+\left(\frac{5\pi}{4}-\frac{1}{2}\sqrt{5}\cdot\sqrt{5}\right)$

$=\frac{5\pi}{4}-\frac{1}{2}$

Question: A matrix of 3×3 order, should be filled either by 0 or 1 and sum of all elements should be prime number. Then the number of such matrix is equal to _____.

Answer: 282.00

Solution:

Let $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Now, sum of all elements to be prime numbers.

So, for 2, number of ways $={}^9C_2=36$

For 3, number of ways $={}^9C_3=84$

For 5, number of ways $= {}^9C_5 = 126$

For 7, number of ways $= {}^9C_7 = 36$

That's all prime number we can get

So total number of such matrix = 282.

Question: Let $a_1, a_2, a_3, ..., a_n$ are in A.P. and $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4,$ then $4a_2$ is equal to ____.

Answer: 16.00

Solution:

Given, $\sum_{r=1}^{\infty} \frac{a^r}{2^r} = 4$

$$\Rightarrow 4 = \frac{a_1}{2} + \frac{a^2}{2^2} + \frac{a^3}{2^3} + ...$$

$$\frac{4}{2} = \qquad \frac{a_1}{2^2} + \frac{a_2}{2^2} + ...$$

$$2 = \frac{a_1}{2} + \left(\frac{d}{2^2} + \frac{d}{2^3} + ...\right)$$

$$2 = \frac{a_1}{2} + \frac{\frac{d}{4}}{1 - \frac{1}{2}}$$

$$a_1 + d = 4$$

$$\Rightarrow 4a_2 = 4(a_1 + d) = 4 \times 4 = 16$$

Question: If $\frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + ... + \frac{1}{100\cdot101\cdot102} = \frac{k}{101},$ then $34k$ is equal to ____.

Answer: 286.00

Solution:

$$\frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + ... + \frac{1}{100\cdot101\cdot102} = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{4-2}{2\cdot3\cdot4} + \frac{5-3}{3\cdot4\cdot5} + ... + \frac{102-100}{100\cdot101\cdot102}\right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2\cdot3} - \frac{1}{3\cdot4} + \frac{1}{3\cdot4} - \frac{1}{4\cdot5} + ... + \frac{1}{100\cdot101} - \frac{1}{101\cdot102}\right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2\cdot3} - \frac{1}{101\cdot102}\right) = \frac{k}{101}$$

$$\Rightarrow k = \frac{1}{2}\left(\frac{101}{2\cdot3} - \frac{1}{102}\right) = \frac{1}{2}\left(\frac{10296}{2\cdot3\cdot102}\right) = \frac{858}{102}$$

$$\therefore 34k = 286$$