CBSE Class 12 Maths Question Paper Solution 2020 Set 65/3/1

QUESTION PAPER CODE 65/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $f \circ g(x)$ for $x < 0$ is

(A) $4x$ **(B)** $2x$ **(C)** 0 **(D)** $-4x$

Ans: (D) $-4x$ 1

2. The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

(A) $-\frac{\pi}{6}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{2\pi}{3}$ **(D)** $\frac{5\pi}{6}$

Ans: (D) $\frac{5\pi}{6}$ 1

3. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(A) 64 **(B)** 16 **(C)** 0 **(D)** -8

Ans: (A) 64 1

4. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

(A) 15 **(B)** 12 **(C)** 9 **(D)** 0

Ans: (A) 15 1

5. $\int \frac{e^x(1+x)}{\cos^2\left(xe^x\right)} dx$ is equal to

(A) $\tan\left(xe^x\right) + c$ **(B)** $\cot\left(xe^x\right) + c$

(C) $\cot\left(e^x\right) + c$ **(D)** $\tan\left[e^x(1+x)\right] + c$

Ans: (A) $\tan\left(xe^x\right) + c$ 1

6. The degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6

Ans: (A) 1 1

7. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

(A) 0 **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** $\sqrt{3}$

Ans: (B) $\frac{1}{\sqrt{3}}$ 1

8. The coordinates of the foot of the perpendicular drawn from the point (–2, 8, 7) on the XZ-plane is

(A) (–2, –8, 7) **(B)** (2, 8, –7) **(C)** (–2, 0, 7) **(D)** (0, 8, 0)

Ans: (C) (–2, 0, 7) 1

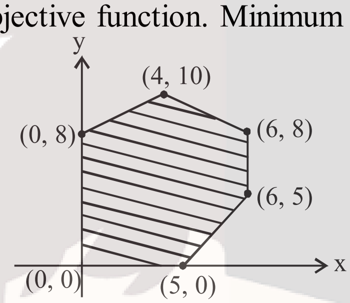
9. The vector equation of XY-plane is

(A) $\vec{r} \cdot \hat{k} = 0$ **(B)** $\vec{r} \cdot \hat{j} = 0$ **(C)** $\vec{r} \cdot \hat{i} = 0$ **(D)** $\vec{r} \cdot \vec{n} = 1$

Ans: (A) $\vec{r} \cdot \hat{k} = 0$ 1

10. The feasible region for an LPP is shown below:

Let z = 3x– 4y be the objective function. Minimum of z occurs at



(A) (0, 0) **(B)** (0, 8) **(C)** (5, 0) **(D)** (4, 10)

Ans: (B) (0, 8) 1

Fill in the blanks in questions numbers 11 to 15

11. If $y = \tan^{-1} x + \cot^{-1} x, x \in R$, then $\frac{dy}{dx}$ is equal to __________.

Ans: 0 1

OR

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi, n \in Z$,

then $\frac{dy}{dx}$ is equal to ____________.

Ans: $-\frac{y}{x}$ 1

12. The value of λ so that the function f defined by $f(x) = \begin{cases} \lambda x, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$

is continuous at $x = \pi$ is ______

Ans: $-\frac{1}{\pi}$ **1**

13. The equation of the tangent to the curve y = sec x at the point (0, 1) is __________.

Ans: $y = 1$ **1**

14. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is __________ square units.

Ans: 3 **1**

OR

The value of λ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ are orthogonal is ________.

Ans: $\frac{1}{2}$ **1**

15. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is ________

Ans: $\frac{2}{7}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. Construct a 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.

Ans: $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ $\frac{1}{2}$ mark for any two correct = **1**

17. Differentiate $\sin^2\left(\sqrt{x}\right)$ with respect to x.

Ans: $\frac{\sin\left(2\sqrt{x}\right)}{2\sqrt{x}}$ or $\frac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$ **1**

18. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

Ans: $f'(x) = -4 - 2x$ **1/2**

$\Rightarrow$ f(x) is increasing on $(-\infty, -2)$ **1/2**

19. Evaluate: $\int_{-2}^{2} |x|\,dx$.

Ans: $\int_{-2}^{2} |x|\,dx = -\int_{-2}^{0} x\,dx + \int_{0}^{2} x\,dx = 4$ **1/2+1/2**

OR

Find $\int \frac{dx}{9+4x^2}$

Ans: $\int \frac{dx}{9+4x^2} = \frac{1}{6}\tan^{-1}\frac{2x}{3} + c$ **1/2+1/2**

20. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

Ans: $1-\left(\frac{1}{2}\right)^4 = \frac{15}{16}$ **1/2+1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Solve for x: $\sin^{-1}4x + \sin^{-1}3x = -\frac{\pi}{2}$

Ans: $\sin^{-1}(4x) + \sin^{-1}(3x) = -\frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(4x) = -\frac{\pi}{2} - \sin^{-1}(3x)$

$\Rightarrow \quad 4x = -\sin\left(\frac{\pi}{2} + \sin^{-1}3x\right)$

$= -\cos(\sin^{-1}3x)$ **1**

$\Rightarrow \quad -4x = \sqrt{1-9x^2}$ **1/2**

$\Rightarrow \quad 16x^2 = 1 - 9x^2$

$\Rightarrow \quad 25x^2 = 1$

$\Rightarrow \quad x^2 = \frac{1}{25} \Rightarrow x = \pm\frac{1}{5}$

As $\sin^{-1}4x + \sin^{-1}3x < 0, x \neq \frac{1}{5}$ **1/2**

So, $x = -\frac{1}{5}$

OR

Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

Ans: $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$ **1**

$= \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$ **1**

22. Express $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

Ans: $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$ **1/2**

$P = \frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 8 & -1 \\ -1 & -2 \end{bmatrix}$ **1/2**

$Q = \frac{A-A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ **1/2**

Now, $A = P + Q$ **1/2**

$P+Q = \frac{1}{2}\begin{bmatrix} 8 & -6 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} = A$

23. If $y^2\cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

Ans: $y^2\cos\left(\frac{1}{x}\right) = a^2$

Then $2y\frac{dy}{dx}\cdot\cos\left(\frac{1}{x}\right) - y^2\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 0$ **1**

$\Rightarrow \quad 2y\cdot\cos\left(\frac{1}{x}\right)\frac{dy}{dx} = -\frac{y^2}{x^2}\sin\left(\frac{1}{x}\right)$

$\therefore \quad \frac{dy}{dx} = -\frac{y}{2x^2}\tan\left(\frac{1}{x}\right)$ **1**

24. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

Ans: $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow \quad |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow \quad 4\vec{a}\cdot\vec{b} = 0$ or $\vec{a}\cdot\vec{b} = 0$ or $\vec{a} \perp \vec{b}$ 1

Let $\vec{a} \perp \vec{b}$

Then $\vec{a}\cdot\vec{b} = 0$

Thus, $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ and $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

$\Rightarrow \quad |\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ 1

OR

Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, 3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+2\hat{k}$ form the sides of a right-angled triangle.

Ans: Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{b} = 3\hat{i}+7\hat{j}+\hat{k}$ and $\vec{c} = 5\hat{i}+6\hat{j}+2\hat{k}$

Since $\vec{c} = \vec{a}+\vec{b}$, three vectors form a triangle. 1

Also, $\vec{a}\cdot\vec{b} = 0$.

So, triangle is a right angled triangle. 1

25. Find the coordinates of the point where the line through (-1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

Ans: Let the line segment AB is cut by ZX-plane in the ratio 1 : λ. So, y-coordinate is zero. 1

i.e., $\dfrac{-2+\lambda}{1+\lambda} = 0$ i.e. $\lambda = 2$

$\therefore$ The point of intersection is (1, 0, –2) 1

26. If A and B are two events such that P(A) = 0.4, P(B) = 0.3 and P(A ∪ B) = 0.6, then find P(B′ ∩ A).

Ans: $P(A\cap B) = P(A)+P(B)-P(A\cup B) = 0.1$ 1

$P(B'\cap A) = P(A)-P(A\cap B) = 0.3$ 1

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Show that the function f: $(-\infty, 0) \to (-1, 0)$ defined by

$f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$ is one-one and onto.

Ans: Let $x_1, x_2 \in (-\infty, 0)$ such that $f(x_1) = f(x_2)$

i.e., $\frac{x_1}{1+|x_1|} = \frac{x_2}{1+|x_2|}$

$\Rightarrow \quad \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$ **1**

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one. **1**

Let $y \in (-1, 0)$ such that $y = \frac{x}{1+|x|}$

$\Rightarrow \quad y = \frac{x}{1-x}$

$\Rightarrow \quad x = \frac{y}{1+y}$ **1**

For each $y \in (-1, 0)$, there exists $x \in (-\infty, 0)$,

such that $f(x) = f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1+\left|\frac{y}{1+y}\right|}$

$= \frac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$

Hence f is onto. **1**

OR

Show that the relation R in the set A = {1, 2, 3, 4, 5, 6} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation.

Ans: **Reflexive:** $|a-a| = 0$, which is divisible by 2 for all $a \in A$.

$\therefore$ $(a, a) \in R \Rightarrow R$ is reflexive. **1**

Symmetric: Let $(a, b) \in R$ i.e., $|a-b| = 2\lambda$, $\lambda \in \omega$

then $|b-a| = |-(a-b)| = |a-b| = 2\lambda$

$\Rightarrow$ (b, a) $\in$ R $\Rightarrow$ R is symmetric. **1**

Transitive : Let (a, b), (b, c) $\in$ R i.e., $|a-b| = 2\lambda$, $|b-c| = 2\mu$

$$a - c = (a-b) + (b-c) = \pm 2\lambda \pm 2\mu = \pm 2(\lambda + \mu)$$

$|a-c| = 2|\lambda + \mu|$, which is divisible by 2

$\Rightarrow$ (a, c) $\in$ R $\Rightarrow$ R is transitive. **1**

Hence R is an equivalence relation. **1**

28. If $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

Ans: Let $u = x^3(\cos x)^x$ and $v = \sin^{-1}\sqrt{x}$ so that y = u + v

$\log u = 3\log x + x\log(\cos x)$ **1/2**

$\Rightarrow \quad \frac{1}{u}\frac{du}{dx} = \frac{3}{x} - x\tan x + \log\cos x$ **1**

$\Rightarrow \quad \frac{du}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right]$ .... (i) **1/2**

and $v = \sin^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$ ... (ii) **1**

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$\Rightarrow \quad \frac{dy}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right] + \frac{1}{2\sqrt{x - x^2}}$ **1**

29. Evaluate: $\int_{-1}^{5}(|x| + |x+1| + |x-5|)\,dx$

Ans: If $x \in [-1, 0] \Rightarrow f(x) = -x + x + 1 - x + 5 = 6 - x$ **1**

If $x \in [0, 5] \Rightarrow f(x) = x + x + 1 - x + 5 = x + 6$ **1**

$\therefore \int_{-1}^{5}\left(|x| + |x+1| + |x-5|\right)dx = \int_{-1}^{0}(6-x)dx + \int_{0}^{5}(x+6)dx$ **1**

$$= \left[\frac{(6-x)^2}{-2}\right]_{-1}^{0} + \left[\frac{(x+6)^2}{2}\right]_{0}^{5}$$

$= \frac{13}{2} + \frac{85}{2} = 49$ **1**

30. Find the general solution of the differential equation $x^2y\,dx - \left(x^3 + y^3\right)dy = 0$

Ans: $\frac{dx}{dy} = \frac{x^3 + y^3}{x^2y}$

Put $x = vy \Rightarrow \frac{dx}{dy} = v + y\cdot\frac{dv}{dy}$ **1**

$\therefore \quad v + y\frac{dv}{dy} = \frac{y^3(v^3+1)}{y^3v^2}$ **1**

$\Rightarrow \quad y\frac{dv}{dy} = \frac{1}{v^2}$

$\Rightarrow \quad v^2dv = \frac{dy}{y}$ **1**

Integrating both sides, we get

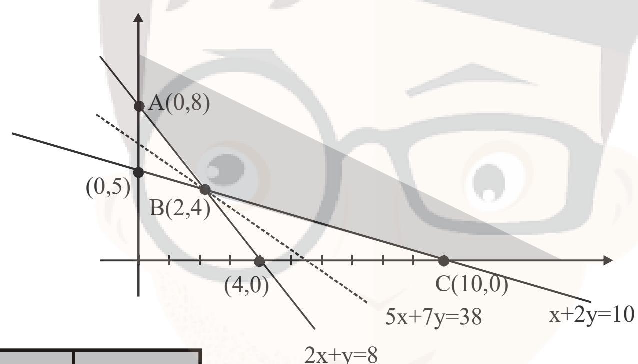
$\frac{v^3}{3} = \log y + c \Rightarrow \frac{x^3}{3y^3} = \log y + c$ **1**

$\Rightarrow x^3 = 3y^3\log y + 3cy^3$

31. Solve the following LPP graphically:
Minimise z = 5x + 7y
subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

Ans: **1+1**



Corner Points	Z
A (0, 8)	56
B (2, 4)	38
C (10, 0)	50

← Smallest value (for B (2, 4)) **$1\frac{1}{2}$**

To verify whether the smallest value of z = 38 is the minimum value we draw open half plane.

5x + 7y < 38. Since there is no common point with the possible feasible region except (2, 4).

Hence minimum value of z = 38 at x = 2 and y = 4. **1/2**

32. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

Ans: Let E_1 be the event that unbiased coin is tossed.
E_2 be the event that biased coin is tossed.
A be the event that coin tossed shows tail 1

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}, P(A|E_1)=\frac{1}{2}, P(A|E_2)=\frac{2}{5}$$ 1

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$ 1

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{2}{5}}=\frac{5}{9}$$ 1

OR

The probability distribution of a random variable X, where k is a constant is given below:

$$P(X=x)=\begin{cases} 0\cdot 1, & \text{if} \quad x=0 \\ k x^2, & \text{if} \quad x=1 \\ k x, & \text{if} \quad x=2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine

(a) the value of k

(b) $P(x \le 2)$

(c) Mean of the variable X.

Ans:

x_i	P_i
0	0.1
1	k
2	2k
3	3k

(i) $\sum P_i = 1$

$\Rightarrow 0\cdot 1 + 6k = 1$

$\Rightarrow \quad k=\frac{3}{20}$ 1

(ii) $P(x \le 2) = 0.1 + 3k$

$=\frac{1}{10}+\frac{9}{20}=\frac{11}{20}$ $1\frac{1}{2}$

(iii) Mean $= \sum P_i x_i = 14k = \frac{21}{10}$ $1\frac{1}{2}$

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$
$$2x - y + 3z = 12$$
$$3x + 2y - z = 5$$

Ans: Writing given equations in matrix form

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$$

Which is of the form AX = B **1**

Here |A| = –2 ≠ 0 **1**

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$ **2**

$$\therefore \quad X = A^{-1}B = \frac{1}{-2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$ **1**

$\Rightarrow \quad x = 2, y = 1, z = 3$ **1**

OR

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

Ans: Using elementary row transformation,

$$A = IA \Rightarrow \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ **1**

Operating $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

[4 marks for correct operations]

$R_2 \to R_2 + R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 5 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -5 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -2 & 0 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_3$, $R_2 \to R_2 - 5R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_3 \to - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ 3 & 3 & -1 \end{bmatrix} \cdot A$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ 3 & 3 & -1 \end{bmatrix}$$ **1**

34. Find the points on the curve $9y^2 = x^2$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

Ans: Equation of given curve, $9y^2 = x^3$ ... (i)

$$\Rightarrow \quad 18y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$$ **1/2**

Slope of normal $= \dfrac{-6y}{x^2}$ **1/2**

$$-\frac{6y}{x^2} = \pm 1 \quad \text{(given)}$$

$\Rightarrow \quad y = \pm\frac{x^2}{6}$ ... (ii) **1**

From (i) & (ii), we get

$9\cdot\frac{x^4}{36} = x^3 \Rightarrow x^3(x-4) = 0 \Rightarrow x = 0, 4$ (x = 0 is rejected)

$x = 4, y^2 = \frac{64}{9} \Rightarrow y = \pm\frac{8}{3}$ **1**

Point of contacts are $\left(4, \frac{8}{3}\right), \left(4, \frac{-8}{3}\right)$ **$1\frac{1}{2}$**

Equation of normal at $\left(4, \frac{8}{3}\right)$ is $y - \frac{8}{3} = -(x-4)$

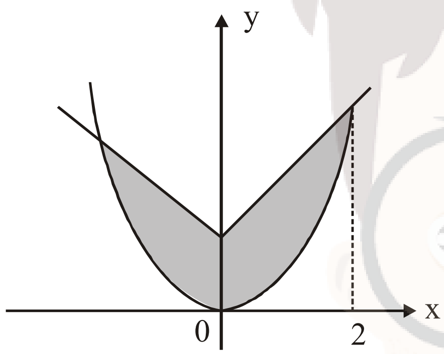
$\Rightarrow 3x + 3y - 20 = 0$ **1**

and equation of normal at $\left(4, -\frac{8}{3}\right)$ is $y + \frac{8}{3} = -(x-4)$

$\Rightarrow 3x + 3y = 20$ **1/2**

35. Find the area of the following region using integration: $\{(x, y) : y \le |x| + 2, y \ge x^2\}$.

Ans: **[Correct figure and shade (2)]**



$y = x^2$

$y = |x| + 2 = x + 2, \text{ if } x \ge 0$

$= -x + 2, \text{ if } x < 0$

Solving, $y = x^2$ and $y = x + 2$

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, -1$ (x = –1 rejected) **1**

Required area $= 2\left[\int_0^2 (x+2)dx - \int_0^2 x^2 dx\right]$ **1**

$= 2\left[\frac{(x+2)^2}{2} - \frac{x^3}{3}\right]_0^2$ **1**

$= 2\left[6 - \frac{8}{3}\right] = \frac{20}{3}$ sq. units **1**

OR

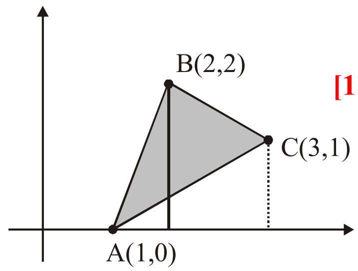
Using integration, find the area of a triangle whose vertices are (1,0), (2, 2) and (3,1).

Ans:

Equations of AB; $y = 2x - 2$

BC; $y = 4 - x$

AC; $y = \frac{1}{2}x - \frac{1}{2}$



[1½ marks for correct equations]

[1 mark for figure and shade]

Required area $= 2\int_1^2 (x-1)dx + \int_2^3 (4-x)dx - \frac{1}{2}\int_1^3 (x-1)dx$ **$1\frac{1}{2}$**

$= 2\left[\frac{(x-1)^2}{2}\right]_1^2 - \left[\frac{(4-x)^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{(x-1)^2}{2}\right]_1^3$ **1**

$= 1 + \frac{3}{2} - 1 = \frac{3}{2}$ sq. units **1**

36. Show that the lines

$\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect.

Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

Ans: $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1} = \lambda$ (say)

and $\frac{x-2}{1} = \frac{y-3}{3} = \frac{z-4}{2} = \mu$ (say)

Arbitrary points on the lines are

$(\lambda + 2, 3\lambda + 2, \lambda + 3)$ and $(\mu + 2, 4\mu + 3, 2\mu + 4)$

$\Rightarrow$ $\lambda + 2 = \mu + 2$, and $\lambda + 3 = 2\mu + 4$

$\Rightarrow$ $\lambda = \mu$, solving we get $\lambda = -1$, $\mu = -1$ **1**

$\lambda = -1$, $\mu = -1$ satisfying y-coordinates $3\lambda + 2 = 4\mu + 3$ **1**

$\therefore$ Point of intersection is $(1, -1, 2)$ **1**

Equation of plane passing through two given lines are **1**

$$\begin{vmatrix} x-2 & y-2 & z-3 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 0$$ **1**

$\Rightarrow \quad 2x - y + z - 5 = 0$ **1**