65(B)

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$ $\frac{1}{2}$

$A + A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$ $\frac{1}{2}$

2. $y = x \log x$

$\Rightarrow \frac{dy}{dx} = 1 + \log x$ 1

3. $\int_0^{\pi} \cos^5 x \, dx = 0$ 1

4. AB: $\frac{x-3}{1} = \frac{y+2}{2} = \frac{z-5/2}{2}$

DR's of required line $< 1, 2, 2 >$ 1

SECTION B

5. $\frac{dx}{dt} = -3$ cm/min, $\frac{dy}{dt} = 2$ cm/min $\frac{1}{2}$

$A = x.y$

$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt}$ 1

$= 2 \text{ cm}^2/\text{min}$ $\frac{1}{2}$

i.e. Area is increasing at the rate of 2 cm^2/min.

6. $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$ $\frac{1}{2}$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right]$$ $\frac{1}{2}$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)$$ $\frac{1}{2}$

$$= \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{2}$$ $\frac{1}{2}$

7. $A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = O$ 1

Pre-multiplying (or Post multiplying) by A^{-1}, we get

$A^{-1} = \frac{1}{7}(A - 3I) = \begin{bmatrix} 2/7 & 3/7 \\ -1/7 & -5/7 \end{bmatrix}$ 1

8. Cartesian equation of required line is

$$\frac{x-3}{2} = \frac{y+7}{-1} = \frac{z+4}{3}$$ 1

Vector equation of required line

$\vec{r} = (3\hat{i} - 7\hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$ 1

9. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$f'(x) = 12x^2 - 12x - 72$ $\frac{1}{2}$

$f'(x) = 0 \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\Rightarrow \quad x = -2 \text{ or } x = 3$ $\frac{1}{2}$

Disjoint intervals are $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$ $\frac{1}{2}$

f(x) is strictly increasing on $(-\infty, -2), \cup(3, \infty)$

f(x) is strictly decreasing on $(-2, 3)$ $\frac{1}{2}$

10. $I = 3\int \frac{dx}{\sqrt{5 - 4x - x^2}}$

$= 3\int \frac{dx}{\sqrt{(3)^2 - (x+2)^2}}$ 1

$= 3\sin^{-1}\frac{x+2}{3} + C$ 1

11. Let amount invested in bond B_1 is Rs.x and in bond B_2 is Rs. y

L.P.P. is Maximum $Z = \frac{8}{100}x + \frac{9y}{100}$ $\frac{1}{2}$

subject to

$$\left.\begin{array}{l} x \geq 20000 \\ y \leq 35000 \\ x + y \leq 75000 \\ x \geq y \\ x, y \geq 0 \end{array}\right\}$$ $1\frac{1}{2}$

12. A and B are independent events

$\therefore \quad P(A \cap B) = P(A).P(B) = \frac{1}{8}$ $\frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$ 1

$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{5}{8} = \frac{3}{8}$ $\frac{1}{2}$

SECTION C

13. $x^2 = y$ (say) 1

$$\frac{y}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$$ 1

Solving we get, $A = -\frac{1}{3}, \; B = \frac{4}{3}$

$$\therefore \quad \int \frac{x^2}{(x^2+1)(x^2+4)} dx = -\frac{1}{3}\int \frac{dx}{(x^2+1)} + \frac{4}{3}\int \frac{dx}{x^2+4}$$

$$= -\frac{1}{3}\tan^{-1} x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$ 2

14. $R_1 \to R_1 - R_2 - R_3$ 1

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ 1

$= 2c[ab + b^2 - bc] - 2b[bc - c^2 - ac]$

$= 4abc$ 2

OR

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad A\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$ 1

$$\Rightarrow \quad A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$ 1

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$ 1

15. $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{4}, x > 0$

$$\Rightarrow \quad \tan^{-1}\frac{1}{x} - \tan^{-1}\frac{1}{x+2} = \frac{\pi}{4}$$ 1

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x(x+2)}}\right] = \frac{\pi}{4}$$ 1

$$\Rightarrow \quad \frac{2}{x^2 + 2x + 1} = 1 \Rightarrow x^2 + 2x - 1 = 0$$ 1

$$\Rightarrow \quad x = \sqrt{2} - 1$$ 1

OR

$$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$$ 1

$$= \tan^{-1}\left[\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{16}}\right]$$ 1

$$= \tan^{-1}\frac{56}{33} = \sin^{-1}\frac{56}{65}$$ 1+1

16. $y = (x\cos x)^x + (\sin x)^{\cos x}$

Let $u = (x\cos x)^x$

$\Rightarrow \quad \log u = x(\log x + \log\cos x)$ $\frac{1}{2}$

$\Rightarrow \quad \frac{1}{u}\cdot\frac{du}{dx} = x\left[\frac{1}{x} - \tan x\right] + \log(x\cos x)$

$\Rightarrow \quad \frac{du}{dx} = (x\cos x)^x\,[1 - x\tan x + \log(x\cos x)]$ ...(i) 1

$v = (\sin x)^{\cos x}$

$\log v = \cos x \log\sin x$ $\frac{1}{2}$

$\Rightarrow \quad \frac{1}{v}\cdot\frac{dv}{dx} = \cos x \,.\, \cot x + \log\sin x \,.\, (-\sin x)$

$\Rightarrow \quad \frac{dv}{dx} = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x \log\sin x\right]$ ...(ii) 1

$\therefore \quad \frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x\log\sin x\right]$ 1

OR

$y = a(\sin t - t\cos t)$

$\frac{dy}{dt} = a[\cos t + t\sin t - \cos t]$ 1

$= a\,t\sin t$

$x = a[\cos t + t\sin t]$

$\frac{dx}{dt} = a[-\sin t + t\cos t + \sin t]$ 1

$= a\,t\cos t$

$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx}$$

$$= \frac{\sec^2 t}{a\,t\cos t} = \frac{1}{a\,t}\sec^3 t$$ 1

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a}$$ $\frac{1}{2}$

17. Given differential equation can be written as

$$\frac{dx}{dy} = \frac{2x\,e^{x/y} - y}{2y\,e^{x/y}}$$ $\frac{1}{2}$

Put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$ 1

$$v + y\cdot\frac{dv}{dy} = v - \frac{1}{2e^v}$$

$$\int e^v\,dv = -\int\frac{dy}{2y}$$ 1

$$e^v = -\frac{1}{2}\log|y| + C$$

$$\Rightarrow \quad e^{x/y} = -\frac{1}{2}\log|y| + C$$ 1

when $x = 0$, $y = 1$, we get $C = 1$

$$\therefore \quad e^{x/y} = 1 - \frac{1}{2}\log|y|$$ $\frac{1}{2}$

18. $$I = \int_0^{\pi}\frac{(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)}dx$$ 1

$$\Rightarrow \quad 2I = \pi\int_0^{\pi}\frac{\sin x}{1 + \cos^2 x}dx$$ 1

$= 2\pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

$= -2\pi\int_1^0 \frac{dt}{1+t^2}$ 1

put $\cos x = t, -\sin x\, dx = dt$

$= 2\pi[\tan^{-1} t]_0^1$

$= \frac{\pi^2}{2}$

$\Rightarrow \quad I = \frac{\pi^2}{4}$ 1

19. $|\vec{a}\times\vec{b}| = 1$ 1

$\Rightarrow \quad |\vec{a}||\vec{b}|\sin\theta = 1$ 1

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$ 1

$\Rightarrow \quad \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$ 1

20. Unit vector perpendiculare to $\vec{a}$ and $\vec{b}$

$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ 1

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix} = 18\hat{i} - 18\hat{j} + 9\hat{k}$ 1

$|\vec{a}\times\vec{b}| = 27$

$\therefore \quad \hat{n} = \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$ 1

Required vector $= 2\hat{i} - 2\hat{j} + \hat{k}$ 1

21. Let A worked for x days and B worked for y days

Minimise z = 225x + 300y 1

subject to constraints

$$\left.\begin{aligned} 9x + 15y \geq 90 &\Rightarrow 3x + 5y \geq 30 \\ 6x + 6y \geq 48 &\Rightarrow x + y \geq 8 \end{aligned}\right\} \quad 2$$

$x, y \geq 0$

Value: Any relevant value 1

22. Let P = probability of doublet

$P = \frac{1}{6}, q = \frac{5}{6}$ $\frac{1}{2}$

x:	0	1	2	3	$\frac{1}{2}$
P(x):	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$	2
xP(x):	0	$\frac{75}{216}$	$\frac{30}{216}$	$\frac{3}{216}$	$\frac{1}{2}$

Mean = $\Sigma xP(x) = \frac{108}{216} = \frac{1}{2}$ $\frac{1}{2}$

23. Let H_1 be the event that bolt is manufactured by machine A

H_2 be the event that bolt is manufactured by machine B

H_3 be the event that bolt is manufactured by machine C

and E be the event that bolt selected is defective

$P(H_1) = \frac{25}{100},\ P(H_2) = \frac{35}{100},\ P(H_3) = \frac{40}{100}$ 1

$P(E/H_1) = \frac{5}{100},\ P(E/H_2) = \frac{4}{100},\ P(E/H_3) = \frac{2}{100}$ 1

Reqd prob. is

$$P(H_2/E) = \frac{P(H_2)\cdot P(E/H_2)}{P(H_1)\cdot P(E/H_1) + P(H_2)P(E/H_2) + P(H_3)\cdot P(E/H_3)} \quad 1$$

$= \frac{28}{69}$ 1

SECTION D

24. $AB = 8I$ 1

$$\Rightarrow \quad A^{-1} = \frac{1}{8}B = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$ $1\frac{1}{2}$

Given equation in matrix form is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad AX = C$$

$$\Rightarrow \quad X = A^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad x = 3,\ y = -2,\ z = -1$$ $1\frac{1}{2}$

OR

$$A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix},\ A^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$ $1\frac{1}{2}+1\frac{1}{2}$

$A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 11 \\ 0 & 0 & 11 \end{bmatrix}$$ $1\frac{1}{2}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$ $1\frac{1}{2}$

25. $f: A \to A$

Let $x_1, x_2 \in A$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{x_1 - 2}{x_1 - 1} = \dfrac{x_2 - 2}{x_2 - 1}$

$\Rightarrow \quad x_1 = x_2$ 2

$\Rightarrow \quad$ f is one-one

Now $y = \dfrac{x-2}{x-1} \quad \Rightarrow \quad x - 2 = xy - y$

$\Rightarrow \quad x(y-1) = y - 2$

$\Rightarrow \quad x = \dfrac{y-2}{y-1}$ 1

For each $y \in A = R - \{1\}$, there exists $x \in A$

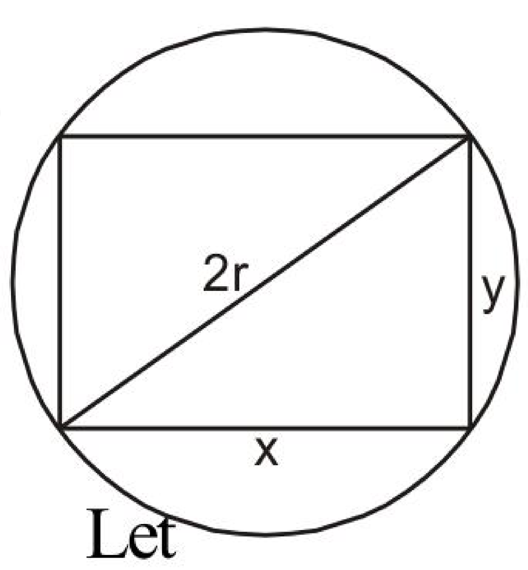
Thus f is onto. Hence f is bijective $\frac{1}{2}$

and $f^{-1}(x) = \dfrac{x-2}{x-1}$ $\frac{1}{2}$

(i) $f^{-1}(x) = \dfrac{5}{6} \quad \Rightarrow \quad \dfrac{x-2}{x-1} = \dfrac{5}{6} \quad \Rightarrow \quad x = 7$ 1

(ii) $f^{-1}(2) = 0$ 1

26.



Let

Let x, y respectively be the sides of rectangle $\therefore \; y = \sqrt{4r^2 - x^2}$ ...(1)

$A = xy$ $\frac{1}{2}$

$Z = A^2 = 4x^2r^2 - x^4$ $1\frac{1}{2}$

$\dfrac{dZ}{dx} = 8r^2x - 4x^3$ 1

$\dfrac{dZ}{dx} = 0 \quad \Rightarrow \quad 4x(2r^2 - x^2) = 0$

$\Rightarrow \quad x = \sqrt{2}r$ 1

$$\frac{d^2Z}{dr^2} = 8r^2 - 12x^2$$

$$\left.\frac{d^2Z}{dx^2}\right|_{x=\sqrt{2}r} = -16r^2 < 0$$ 1

$\Rightarrow$ Area is maximum when $x = \sqrt{2}r$

$\therefore$ $y = \sqrt{2}r$ (From (i))

i.e. $x = y$

Hence, Area is maximum when rectangle is a square 1

27. Equation of plane passing through (–1, 3, 2)

$a(x+1) + b(y-3) + c(z-2) = 0$ ...(i) 1

Required plane is perpendicular to $x + 2y + 3z = 5$

and $3x + 3y + z = 0$

$$\therefore \quad \left.\begin{array}{l} a + 2b + 3c = 0 \\ 3a + 3b + c = 0 \end{array}\right\}$$ 1+1

$$\Rightarrow \quad \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3}$$ 1

$\therefore$ Equation (i) $\Rightarrow$

$7x - 8y + 3z + 25 = 0$ 1

Vector Equation of plane is

$$\vec{r} \cdot (7\hat{i} - 8\hat{j} + 3\hat{k}) = -25$$ 1

OR

Equation of plane passing through (1, 1, 4), (3, – 1, 2) and (4, 1, – 2) is

$$\begin{vmatrix} x-1 & y-1 & z-4 \\ 2 & -2 & -2 \\ 3 & 0 & -6 \end{vmatrix} = 0$$ 2

$\Rightarrow$ $2x + y + z = 7$ ...(i)

Equation of line passing through (3, –4, –5) and (2, –3, 1)

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k$$ 1

$\Rightarrow$ $x = 3 - k,\ y = k - 4,\ z = 6k - 5$ ...(ii) 1

$(3 - k, k - 4, 6k - 5)$ lies on (i)

$6 - 2k + k - 4 + 6k - 5 - 7 = 0$

$\Rightarrow$ $k = 2$ 1

Eqn (ii) $\Rightarrow$ point of intersection is (1, –2, 7) 1

28. $\frac{dy}{dx} + y \cdot \cot x = 4x \operatorname{cosec} x$ ...(i)

Here $P = \cot x$, I.F. $= e^{\int \cot x . dx} = e^{\log \sin x} = \sin x$ $1\frac{1}{2}$

Hence the solution is

$y \sin x = \int 4x \cdot dx$ 2

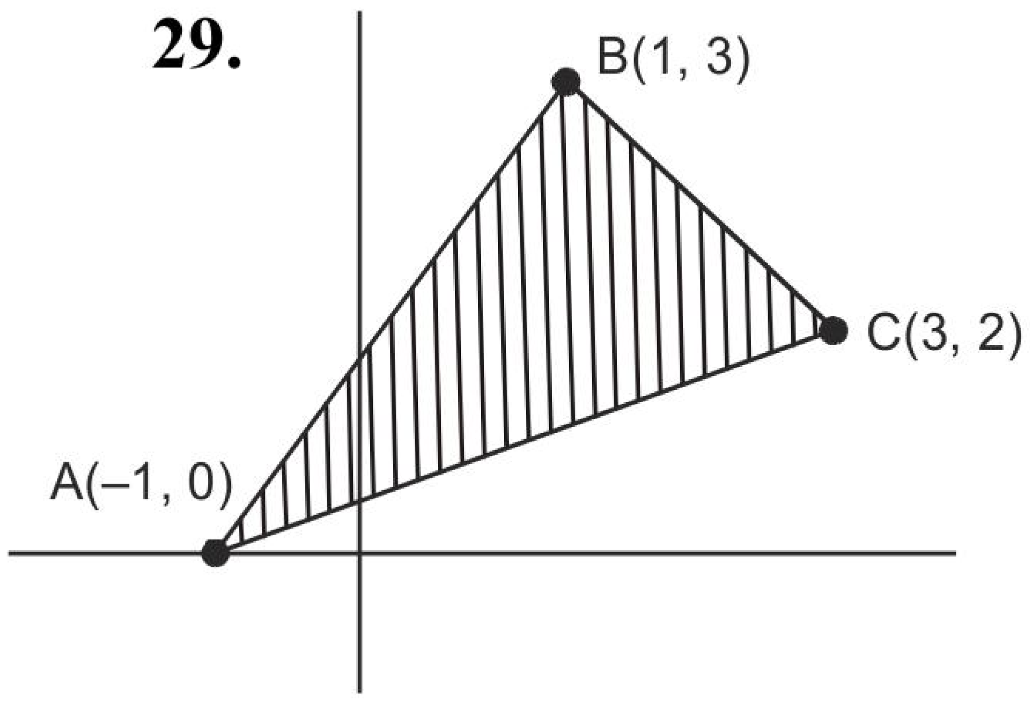
$\Rightarrow$ $y \sin x = 2x^2 + C$ $1\frac{1}{2}$

When $x = \frac{\pi}{2}, y = 0, C = -\frac{\pi^2}{2}$

$\therefore$ Requried solution is

$y \sin x = 2x^2 - \frac{\pi^2}{2}$ 1

29.



Equation of AB: $y = \frac{3}{2}(x+1)$

Equation of BC: $y = \frac{-1}{2}x + \frac{7}{2}$

Equation of AC: $y = \frac{1}{2}(x+1)$ $1\frac{1}{2}$

Required area

$$= \frac{3}{2}\int_{-1}^{1}(x+1)dx - \frac{1}{2}\int_{1}^{3}(x-7)\,dx - \frac{1}{2}\int_{-1}^{3}(x+1)dx$$ $1\frac{1}{2}$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} - \frac{1}{2}\left[\frac{x^2}{2}-7x\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$ $1\frac{1}{2}$

$= 3 + 5 - 4$

$= 4$ sq.units $1\frac{1}{2}$

OR

$$\int_1^3 (3x^2 + e^{2x})dx$$

$h = \frac{2}{n}$, as $n \to \infty, h \to 0$

$$\int_1^3 f(x)\,dx = \lim_{\substack{h\to 0\\ n\to\infty}} h[f(x) + f(1+h) + f(1+2h) + \ldots + f(1+n)]$$

$$\int_1^3 (3x^2 + e^{2x})dx = \lim_{\substack{h\to 0\\ n\to\infty}} 3h^3\Sigma(n-1)^2 + 6h^2.\Sigma(n-1) + 3nh + he^2[1 + e^{2h} + 2^{4h} + \ldots + f(1-n)]$$ 2

$$= \lim_{\substack{h\to 0\\ n\to\infty}} \frac{24}{n^3}\cdot\frac{n(n-1)(2n-1)}{6} + \frac{24}{n^2}\cdot\frac{n(n-1)}{2} + 6 + he^2 1\cdot\frac{e^{2nh}-1}{e^{2h}-1}$$ 2

$$= \lim_{\substack{h\to 0\\ n\to\infty}} \frac{24\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} + \frac{24\left(1-\frac{1}{n}\right)}{2} + 6 + e^2$$ 1

$$= 26 + \frac{1}{2}(e^6 - e^2)$$ 1

Alternately $f(x) = (3x^2 + e^{2x})$

$$f(x)dx = \lim_{h\to 0} \{[3(1)^2 + e^2] + [3(1+h)^2 + e^{2(1+h)}] + 3[3(1+2h)^2 + e^{2(1+2h)}] + \ldots +$$

$$[3(1+(n-1)h)^2] + e^{2(1+(n-1)h)}]\} \quad 2$$

$$= \lim_{h\to 0} h\left[\frac{3n + 6hn(n-1)}{2} + \frac{3h^2 n(n-1)(2n-1)}{6} + e^2\{1 + e^{2h} + e^{4h} + \ldots + e^{2(n-1)h}\}\right] \quad 2$$

$$= \lim_{h\to 0} h\left[3nh + 3nh(nh - h) + \frac{nh(nh-h)(2nh-h)}{2} + \frac{e^2 h\, e^{2nh} - 1}{e^{2h} - 1}\right] \quad 1$$

$$= 6 + 12 + 8 + \frac{e^2(e^4 - 1)}{2} \quad 1$$