Sample Paper 5

ANSWERKEY																			
1	(c)	2	(d)	3	(c)	4	(b)	5	(c)	6	(d)	7	(d)	8	(c)	9	(b)	10	(b)
11	(c)	12	(a)	13	(b)	14	(a)	15	(c)	16	(b)	17	(a)	18	(c)	19	(c)	20	(c)
21	(b)	22	(c)	23	(a)	24	(b)	25	(a)	26	(c)	27	(c)	28	(a)	29	(b)	30	(d)
31	(a)	32	(c)	33	(d)	34	(c)	35	(a)	36	(d)	37	(a)	38	(b)	39	(a)	40	(d)
41	(a)	42	(d)	43	(c)	44	(d)	45	(b)	46	(b)	47	(a)	48	(c)	49	(a)	50	(d)

SOLUTIONS

1. (c) Let the three points be A(0, 0), B(3, $\sqrt{3}$) and C(3, λ).

$\therefore$ AB = BC = CA

[$\because$ ΔABC an equilateral Δ]

$\Rightarrow AB^2 = BC^2 = CA^2$ Now $AB^2 = BC^2$

$\Rightarrow (0-3)^2 + \left(0-\sqrt{3}\right)^2$

$= (3-3)^2 + \left(\sqrt{3}-\lambda\right)^2$

$\Rightarrow 9 + 3 = 0 + 3 - 2\lambda\sqrt{3} + \lambda^2$

$\Rightarrow \lambda^2 - 2\sqrt{3}\lambda - 9 = 0$

$\Rightarrow \lambda^2 - 3\sqrt{3}\lambda + \sqrt{3}\lambda - 9 = 0$

$\Rightarrow \lambda\left(\lambda - 3\sqrt{3}\right) + \sqrt{3}\left(\lambda - 3\sqrt{3}\right) = 0$

$\Rightarrow \left(\lambda - 3\sqrt{3}\right)\left(\lambda + \sqrt{3}\right) = 0$

$\Rightarrow \lambda = 3\sqrt{3}$ or $\lambda = -\sqrt{3}$.

2. (d) $\alpha + \beta = \dfrac{2}{k^2 - 14}$

$\Rightarrow \dfrac{2}{k^2 - 14} = 1$ (Given)

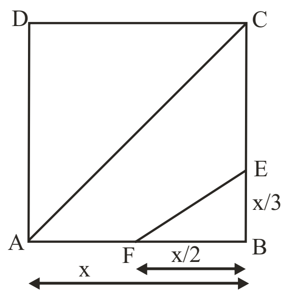
$\Rightarrow k = \pm 4$

3. (c) We have ABCD is square,

AF = BF, BE = $\frac{1}{3}$ BC,

Area ΔFBE = 108 sq cm.

Let AB = x $\Rightarrow$ BF = $\frac{x}{2}$ and BE = $\frac{x}{3}$



Area of ΔFBE = $\frac{1}{2}$ BF × BE

$= \frac{1}{2} \times \frac{x}{2} \times \frac{x}{3} = \frac{x^2}{12}$

= 108 sq. cm. (Given)

$\Rightarrow x^2 = 12 \times 108 \Rightarrow x^2 = 12 \times 12 \times 3 \times 3$

$\Rightarrow x = 12 \times 3 = 36$ cm

In rt. ΔABC, AC = $\sqrt{AB^2 + BC^2}$

(By Pythagoras Theorem)

$= \sqrt{36^2 + 36^2} = 36\sqrt{2}$ cm

4. (b) Let x = 0.31783178...... ...(i)

Multiply by 10000

10000 x = 3178.31783178........ ...(ii)

Subtracting (i) from (ii)

10000 x = 3178.3178........

x = 0.3178........

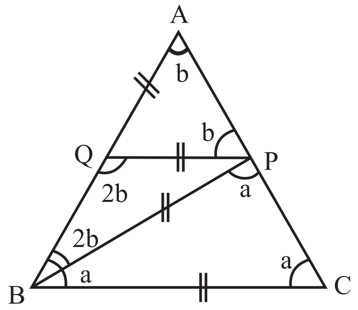
– –

999 x = 3178

$x = \dfrac{3178}{999}$

5. **(c)** Either a rational number or an irrational number.

6. **(d)**



In ΔABC

$AB = AC$

$\Rightarrow \angle C = \angle B \Rightarrow \angle B = \angle C = a$

By angle sum properly in ΔABC,

$b + a + a = 180$

$\Rightarrow b + 2a = 180°$ ...(i)

In ΔQPB

$\Rightarrow \angle QPB = 180 - 4b$

Since 'APC' is a straight line

$\Rightarrow 180 - 4b + a + b = 180$

$\Rightarrow a = 3b$ ...(ii)

From equations (i) & (ii)

$b + 2(3b) = 180 \Rightarrow b = \frac{180}{7}$

$\angle AQP = 180° - 2\left(\frac{180}{7}\right) = \frac{5}{7}\pi$

7. **(d)** $2\pi r = 4\pi \Rightarrow r = 2$

Area $= \pi(2)^2 = 4\pi$

When, $2\pi r = 8\pi$

$\Rightarrow r = 4$

Area $= 16\pi$

8. **(c)** $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$= \left\{1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right\}\times\left\{1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right\}$

$= \frac{\{(\cos\theta+\sin\theta)+1\}\times\{(\cos\theta+\sin\theta)-1\}}{\cos\theta\times\sin\theta}$

$= \frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta\times\sin\theta}$ $\{\because (a+b)(a-b)=a^2-b^2\}$

$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta\times\sin\theta} = 2.$

9. **(b)**

10. **(b)** Let the number of boys and girls in classroom is x and y.

According to question

$\frac{x-x/5}{y} = \frac{2}{3} \Rightarrow \frac{4x}{5y} = \frac{2}{3} \Rightarrow \frac{x}{y} = \frac{5}{6}$ ...(i)

Also, $\frac{x-x/5}{y-44} = \frac{5}{2} \Rightarrow \frac{4x}{5(y-44)} = \frac{5}{2}$

$\Rightarrow 8x = 25y - 1100$ ...(ii)

From Eqs. (i) and (ii), we get, $x = 50$, $y = 60$

Let n number of boy leaves the class so number of boys and number of girls become equal.

$\therefore 50 - 10 - n = 60 - 44$

$n = 40 - 16 = 24$

11. **(c)** Perimeter $= \frac{1}{4}\times 2\pi r + 2r$

$= \left(\frac{1}{2}\times\frac{22}{7}\times 7 + 2\times 7\right)$ cm = 25 cm

12. **(a)** $\Delta ABC \sim \Delta ANM$

$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ANM} = \frac{AC^2}{AM^2}$ ...(i)

$\Delta ABC \sim \Delta MPC$

$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta MPC} = \frac{AC^2}{MC^2}$ ...(ii)

From Eqs. (i) and (ii,) we get

$\frac{\text{Area of } \Delta ANM}{\text{Area of } \Delta MPC} = \frac{AM^2}{MC^2}$

$\frac{\text{Area of } \Delta ANM + \text{Area of } \Delta MPC}{\text{Area of } \Delta MPC} = \frac{AM^2+MC^2}{MC^2}$

Now, Area of ΔANM + Area of ΔMPC

= Area of ΔABC – Area of $BNMP$

Using Area of BNMP $= \frac{5}{18}$ of area of ΔABC

$\therefore \frac{13}{18}\frac{(\text{Area of } \Delta ABC)}{(\text{Area of } \Delta MPC)} = \frac{AM^2+MC^2}{MC^2}$ ...(iii)

From Eq. (iii), $\frac{13}{18}\left(\frac{AC^2}{MC^2}\right) = \frac{AM^2+MC^2}{MC^2}$

$\Rightarrow 13\,(AM+MC)2 = 18\,(AM2+MC2)$

$\Rightarrow \frac{AM}{MC} = 5, \frac{1}{5}$. Hence, option (a) is correct.

13. **(b)**

14. **(a)** Condition for infinite many solutions.

$$\frac{p}{12} = \frac{3}{p} = \frac{p-3}{p}\left\{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right\}$$

$p^2 = 36$; $p =$ {From I and II}

$p^2 - 3p = 3p$ {From II and III}

$p = 6$

$\therefore \quad p = 6$

15. **(c)** Radius of outer concentric circle = (35 + 7) m = 42 m.

Area of path = $\pi\,(42^2 - 35^2)\text{ m}^2 = \frac{22}{7}(42^2 - 35^2)\text{ m}^2$

16. **(b)** $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$
$= 9 \times 1 = 9.$

17. **(a)** Total three digit number are : $3 \times 3 \times 2 = 18$

Now, numbers divisible by 5 are :

$2 \times 3 \times 1 + 2 \times 2 \times 1 = 10$

So, probability that the slip bears a number divisible by 5 = $\frac{10}{18} = \frac{5}{9}$

18. **(c)** Let $x = 0.\overline{235}$ ...(i)

$1000x = 235.\overline{235}$ ...(ii)

Subtract (i) from (ii), $999x = 235 \Rightarrow x = \frac{235}{999}$

19. **(c)** Joining B to O and C to O

Let the radius of the outer cirlce be r

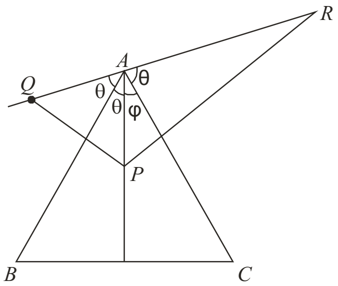
$\therefore$ perimeter = $2\pi r$

But $OQ = BC = r$ [diagonals of the square BQCO]

$\therefore$ Perimeter of $ABCD$ = 4r.

Hence, ratio = $\frac{2\pi r}{4r} = \frac{\pi}{2}$

20. **(c)** Here, ABC is a triangle & P be interior point of a ΔABC, Q and R be the reflections of P in AB and AC, respectively.



As QAR are collinear

$\therefore \quad \angle QAR = 180°$

Q is reflection of P on AB

$\therefore \quad \angle QAB = \angle BAP$

R is reflection of P on AC

$\therefore \quad \angle RAC = \angle CAP$

$\angle QAR = 180°$

$\therefore \quad 2\angle BAP + 2\angle CAP = 180°$

$\angle BAP + \angle CAP = 90° \quad \Rightarrow \quad \angle BAC = 90°$

21. **(b)** $\alpha + \beta = -a$ and $\alpha\beta = -b$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-a}{-b} = \frac{a}{b}$$

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-\beta}$$

$\therefore$ The required polynomial is

$x^2 - \frac{a}{b}x - \frac{1}{b}.$

22. **(c)** H.C.F. of 20 and 15 = 5

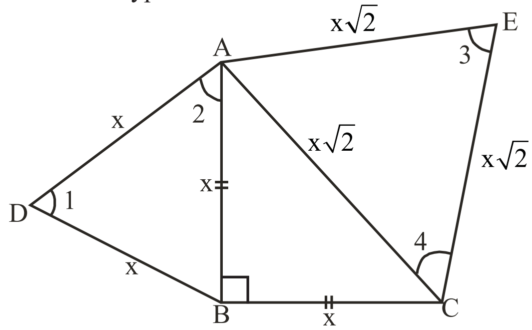
So the 5 students are in each group so

$$n = \frac{20 + 15}{5} = \frac{35}{5} = 7$$

Hence, x = 4, y = 3 and n = 7

23. **(a)** Let AB = BC = x units.

Then Hyp. AC = side $\sqrt{2}$ = $x\sqrt{2}$ units



In ΔABD and ΔCAE

$\angle 1 = \angle 3$ [In equilateral Δ each angle = 60°]

$\angle 2 = \angle 4$

$\therefore \quad \Delta ABD \sim \Delta CAE$ (By AA rule)

$$\therefore \quad \frac{\text{area}(\Delta ABD)}{\text{area}(\Delta CAE)} = \frac{AB^2}{CA^2}$$ (By the theorem)

$$= \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

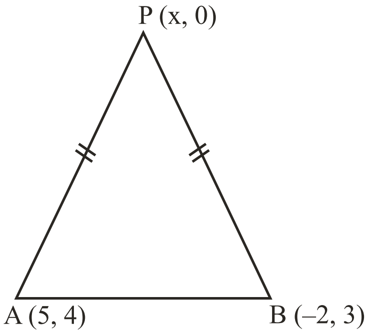
$\therefore \quad \dfrac{\text{area}(\Delta ABD)}{\text{area}(\Delta CAE)} = \dfrac{1}{2}$

$\Rightarrow$ area (ΔABD) = $\dfrac{1}{2}$ area (ΔCAE)

24. **(b)** Degree of quotient = degree of dividend – degree of divisor

Degree of quotient = 7 – 4 = 3.

25. **(a)** Since, the required point (say P) is on the x-axis, its ordinate will be zero. Let the abscissa of the point be x. Therefore, coordinates of the point P are (x, 0).



Let A and B denote the points (5, 4) and (–2, 3) respectively.

Given that AP = BP, we have

$AP^2 = BP^2$

i.e. $(x-5)^2 + (0-4)^2$

$= (x+2)^2 + (0-3)^2$

$\Rightarrow x = 2$

26. **(c)**

27. **(c)** $\dfrac{2\tan 30°}{1-\tan^2 30°} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \sqrt{3} = \tan 60°.$

28. **(a)** $x = -1$ is the root of the quadratic polynomial $p(x)$

So, quadratic polynomial $p(x) = k(x+1)^2$

$p(-2) = k(-2+1)^2 = 2 \Rightarrow k = 2 \ \therefore\ p(x) = 2(x+1)^2$

Also, $p(2) = 2(2+1)^2 = 2 \times 3 \times 3 = 18$

29. **(b)**

30. **(d)** Given $2x + y = 10$

on adding y both sides, we get, $2x + y + y = 10 + y$

$\Rightarrow \quad 2(x+y) = 10 + y \quad \Rightarrow \quad x + y = 5 + \dfrac{y}{2}$

Now, $(x+y)_{max}$ when y is maximum & maximum value of y will be 10. ($\because$ y = 10 – 2x)

So $(x+y)_{max} = 5 + 5 = 10$ & $(x+y)_{min}$ when y = 0

$\therefore$ minimum value of $x + y = 5$

So, sum of $(x+y)_{max}$ & $(x+y)$min = 15

31. **(a)** Here, when A = 0°

LHS = sin 2 A = sin 0° = 0

and RHS = 2 sin A = 2 sin 0° = 2 × 0 = 0

In the other options, we will find that

LHS ≠ RHS

32. **(c)** $\dfrac{1}{7} = 0.\overline{142857}$

The second positive integer whose reciprocal have six different repeating decimals is

$\dfrac{1}{13} = 0.\overline{076923}$

And the third positive integer whose reciprocal have six different repeating decimals is

$\dfrac{1}{21} = 0.\overline{047619}$

Therefore, the values of x are 7, 13, 21

Hence, the required sum is = 7 + 13 + 21 = 41

33. **(d)**

34. **(c)** Let distance = d,

Time taken upstream = $\dfrac{d}{15-5} = \dfrac{d}{10}$

Time taken downstream = $\dfrac{d}{15+5} = \dfrac{d}{20}$

Hence, average speed

$= \dfrac{2d}{\frac{d}{10}+\frac{d}{20}} = \dfrac{2d \times 20}{3d} = \dfrac{40}{3}$ km/hr

Ratio = $\dfrac{40}{3} : 15 = 40 : 45 = 8 : 9$

35. **(a)** $P(E) + P(\overline{\ \ }) = 1$

36. **(d)** $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} = \dfrac{1-(1)^2}{1+(1)^2} = 0.$

37. **(a)** Let the required number be 33a and 33b.

Then 33a + 33b = 528 $\Rightarrow$ a + b = 16.

Now, co-primes with sum 16 are (1, 15), (3, 13), (5, 11) and (7, 9).

$\therefore$ Required numbers are (33 × 1, 33 × 15), (33 × 3, 33 × 13), (33 × 5, 33 × 11), (33 × 7, 33 × 9).

The number of such pairs is 4.

38. **(b)** Upstream speed = 4 km/hr and time = x hrs.

Downstream speed = 8 km/hr and

time taken = $x/2$ hrs.

Hence average speed = $\dfrac{4x+8\times x/2}{x+x/2}=\dfrac{16}{3}$ km/hr.

39. **(a)** $\dfrac{2\tan 30°}{1+\tan^2 30°}=\dfrac{2\left(\dfrac{1}{\sqrt{3}}\right)}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2}$

$$=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{2}{\sqrt{3}}\times\frac{3}{4}=\frac{\sqrt{3}}{2}=\sin 60°.$$

40. **(d)** For given numbers,

$(55)^{725}$, unit digit = 5; $(73)^{5810}$, unit digit = 9

$(22)^{853}$, unit digit = 2

Unit digit in the expression $55^{725}+735^{810}+22^{853}$ is 6

41.	**(a)**	**42.**	**(d)**	**43.**	**(c)**
44.	**(d)**	**45.**	**(b)**	**46.**	**(b)**
47.	**(a)**	**48.**	**(c)**	**49.**	**(a)**
50.	**(d)**				