Pre-PhD course (Algebra):

Recall the basic commutative algebra: Noetherian, Artinian modules/rings, Hilbert basis theorem, Jordan-Holders theorem, Jacobson Radical, Nakayama Lemma, Localisation, Tensor products, primary decomposition, Integral extensions, Going-up, Going Down theorem, Noether's Normalisation lemma, Krull dimension.

Affine space, Algebraic sets, Zariski topology, irreducible sets, irreducible components, coordinate ring of an algebraic set, algebraic sets and radical ideals, irreducible closed sets and prime ideals, Hibert's Nullstellensatz. Regular maps, morphisms, isomorphisms, morphisms and k-algebra homomorphisms, rational morphisms, Birational equivalence, product of affine varieties and universal property.

Projective space, projective algebraic sets, Homogenisation/dehomogenistaion, affine open covering, Regular maps, morphisms, isomorphisms, morphisms and k-algebra homomorphisms,rational morphisms.

Tangent space, Jacobian criterion, smoth points, non-singular varieties, Tangent cones, plane curves, Rational curves, multiple points, Geometric multiplicity, nodes and cusp, Bezout's theorem.

References:

1. Atiya, Macdonald, Introduction to Commutative algebra, Addison-Wesley Publishing Company, Addsison-Wesley Series in Mathematics. 1969

2. D. Bump, Algebraic Geometry, World Scientific, 2001

3. Musili, Algebraic Geometry for beginners, Hindustan Book Agency, 2001

4. Hartshorne, Algebraic Geometry, Graduate text in Mathematics, Springer, 1977

Syllabus for

Advanced Real Analysis

For IXth Semester Integrated Mathematics (and Pre-PhD)

Text for Reference: ***Real and Complex Analysis***, Walter Rudin.

Chapter 1: Abstract Integration

Set-theoretic notations and terminology
The concept of measurability
Simple functions
Elementary properties of measures
Arithmetic in $[0, \infty]$
Integration of positive functions
Integration of complex functions
The role played by sets of measure zero

Chapter 2: Positive Borel Measures

Vector spaces
Topological preliminaries
The Riesz representation theorem
Regularity properties of Borel measures
Lebesgue measure
Continuity properties of measurable functions

Chapter 3: L^p-Spaces

Convex functions and inequalities
The L^p -spaces
Approximation by continuous functions

Chapter 6: Complex Measures

Total variation
Absolute continuity
Consequences of the Radon-Nikodym theorem
Bounded linear functionals on LP
The Riesz representation theorem

Chapter 8: Integration on Product Spaces and Fubini's Theorem

Abridged.

Syllabus for Classical Theory of Partial Differential Equations

Module-1 - Distribution Theory:

Test functions, distributions, order of the distribution, support of distributions, derivative of distributions, convolutions of distributions, fundamental solutions, the Schwartz space, Fourier Transforms, tempered distributions.

Module-2 – Classical PDE

Laplace equation in Higher dimensions: Fundamental solution, Green's functions, Poisson integral formula, mean value properties, weak and strong maximum principle, Hopf's lemma, gradient estimates, removable singularity, Perron's method.

Heat equation in Higher dimensions: Fourier transforms, fundamental solution, regularity of solutions, weak and strong maximum principle, Harnack's inequalities, radient estimates.

Wave equation in Higher dimensions.: Solution of wave equation in higher dimension using spherical means, Huygen's principle, Duhamel's principle, energy estimates.

References:

For Module-1

1. Kesavan, S., Topics in Functional Analysis and applications, New Age International, 2003.
2. Strichartz, R. S., A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003
3. Stakgold, I., Green's functions and boundary value problems, John Wiley & Sons, 1979.
4. Hormander, L., The Analysis of Linear Partial Differential Operators, Springer, Vol. 1, 1983.

For Module-2

1. Qing Han, A Basic Course in Partial Differential Equations, Graduate Studies in Mathematics, AMS, Vol. 120, 2013.