QUESTION PAPER CODE 65/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. π 1

2. Order of AB is 3×4 1

3. $\frac{dy}{dx} = \cos x$ $\frac{1}{2}$

Slope of tangent at (0, 0) is 1

Equation of tangent is $y = x$ $\frac{1}{2}$

4. Putting $(1 + \log x)$ or $\log x = t$ $\frac{1}{2}$

$\log|1 + \log x| + C$ $\frac{1}{2}$

SECTION B

5. Let A be $10\hat{i} + 3\hat{j}$, B be $12\hat{i} - 5\hat{j}$, C be $\lambda\hat{i} + 11\hat{j}$

$\overrightarrow{AB} = 2\hat{i} - 8\hat{j}$ $\frac{1}{2}$

$\overrightarrow{AC} = (\lambda - 10)\hat{i} + 8\hat{j}$ $\frac{1}{2}$

As $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear

$\frac{2}{\lambda - 10} = \frac{-8}{8}$ $\frac{1}{2}$

So $\lambda = 8$ $\frac{1}{2}$

6. Let number of large vans = x

and number of small vans = y

Minimize cost $z = 400x + 200y$ $\frac{1}{2}$

Subject to constraints

$$\left.\begin{array}{l} 200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ x \leq y \\ 400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \end{array}\right\}$$ $1\frac{1}{2}$

$x \geq 0, y \geq 0$

7. $R_2 \to R_2 + R_1$ implies

$$\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 17 & -7 \end{pmatrix}$$ 1+1

1 mark for pre matrix on LHS and 1 mar for matrix on RHS

8. $\frac{dr}{dt} = -3$ cm/min, $\frac{dh}{dt} = 2$ cm/min $\frac{1}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right]$$ 1

$$\left(\frac{dV}{dt}\right)_{\text{at } r=9,\, h=6} = -54\pi \text{ cm}^3/\text{min}$$ $\frac{1}{2}$

$\Rightarrow$ Volume is decreasing at the rate 54π cm^3/min.

9. Differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$ 1

Eliminating 4a, we get

$$y^2 = 2y\frac{dy}{dx} \cdot x$$

or $$2xy\frac{dy}{dx} - y^2 = 0$$ 1

10. $\lim_{x\to\pi/4} f(x) = f(\pi/4)$

$$\lim_{x\to\pi/4} \frac{\sqrt{2}\sin(x - \pi/4)}{4(x - \pi/4)} = k$$ $1\frac{1}{2}$

$\therefore \quad k = \frac{\sqrt{2}}{4}$ $\frac{1}{2}$

11. Given integral $= \int \frac{1}{\sqrt{(x-2)^2 - 2^2}}\,dx$ 1

$$= \frac{(x-2)}{2}\sqrt{x^2-4x} - 2\log\left|x-2+\sqrt{x^2+4x}\right| + C$$ 1

12. Integrating factor is $e^{\int \frac{2}{x}dx} = x^2$ $\frac{1}{2}$

Solution is $y.x^2 = \int x \cdot x^2 dx + C$ $\frac{1}{2}$

$$\left.\begin{aligned} y\cdot x^2 &= \frac{x^4}{4} + C \\ \text{or} \quad y &= \frac{x^2}{4} + \frac{C}{x^2} \end{aligned}\right\}$$ 1

SECTION C

13. $\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta)$

$= a\,\theta\cos\theta$ $1\frac{1}{2}$

$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$= a\,\theta\sin\theta$ 1

$\therefore \quad \frac{dy}{dx} = \tan\theta$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{d\theta}{dx} = \frac{\sec^3\theta}{a\theta}$ 1

14. Differentiating $y = \cos(x+y)$ wrt x we get

$\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$ 1

Slope of given line is $\frac{-1}{2}$ $\frac{1}{2}$

As tangent is parallel to line $x + 2y = 0$

$$\therefore \quad \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$$\Rightarrow \quad \sin(x+y) = 1$$

$$\Rightarrow \quad x + y = n\pi + (-1)^n \frac{\pi}{2}, n \in Z \qquad ...(1)$$ 1

Putting (1) in $y = \cos(x+y)$

we get $y = 0$

$$\Rightarrow \quad x = n\pi + (-1)^n \pi/2, n \in Z$$

$$x = \frac{-3\pi}{2} \in [-2\pi, 0]$$ $\frac{1}{2}$

$\therefore$ Required equation of tangent is

$$y = \frac{-1}{2}\left(x + \frac{3\pi}{2}\right)$$

or $\quad 2y + x + \frac{3\pi}{2} = 0$ 1

15. Let $\frac{x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ 1

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$ $1\frac{1}{2}$

Thus integral becomes

$$\frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$ $1\frac{1}{2}$

16. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad ....(i)$$ $\frac{1}{2}$

Clearly it is homogenous

Let $\frac{y}{x} = v,\ \frac{dy}{dx} = v + \frac{dv}{dx}$ 1

(1) becomes

$$v + x\frac{dv}{dx} = v + \sec v$$

$$\Rightarrow \quad \cos v\, dv = \frac{dx}{x}$$ 1

integrating both sides we get

$$\sin v = \log |x| + C$$ 1

$$\sin\frac{y}{x} = \log |x| + C$$ $\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

As A, B, C & D are coplanar

$\therefore \quad \overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = 0$

i.e. $\begin{vmatrix} 1 & x-3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$ $1\frac{1}{2}$

which gives

$$x = 6$$ 1

18. Given equation of lines can be written as

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{1} \quad ...(1)$$ 1

$$\frac{x-1}{-3p/7} = \frac{y-5}{-1} = \frac{z-11}{-7} \quad ...(2)$$ 1

(1) & (2) are perpendicular

So $-3\left(\frac{-3p}{7}\right)+\frac{2p}{7}(-1)+1(-7)=0$ 1

which gives p = 7 1

OR

Required equation of plane is $x+y+z-1+\lambda(2x+3y+4z-5)=0$ for some λ. 1

i.e. $(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z=1+5\lambda$

according to question

$$2\left(\frac{1+5\lambda}{1+3\lambda}\right)=3\left(\frac{1+5\lambda}{1+4\lambda}\right)$$ 1

Solving we get $\lambda=-1$ 1

Thus the equation of required plane is

$-x-2y-3z=-4$

or $x+2y+3z=4$ 1

19. Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix}=0$$ 1

$C_1 \rightarrow C_1+C_2+C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix}=0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix}=0$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix}1 & b/y-1 & c/z-1\\0 & 1 & 0\\0 & 0 & 1\end{vmatrix} = 0$$ 1

$$\therefore \quad \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1 = 0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$ $\frac{1}{2}$

OR

We know that

$IA = A$ 1

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}A = \begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix}1 & 0\\-2 & 1\end{pmatrix}A = \begin{pmatrix}1 & 2\\0 & -5\end{pmatrix}$$ 1

$R_2 \to \frac{R_2}{-5}$

$$\begin{pmatrix}1 & 0\\2/5 & -1/5\end{pmatrix}A = \begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}$$ $\frac{1}{2}$

$R_1 \to R_1 - 2R_2$

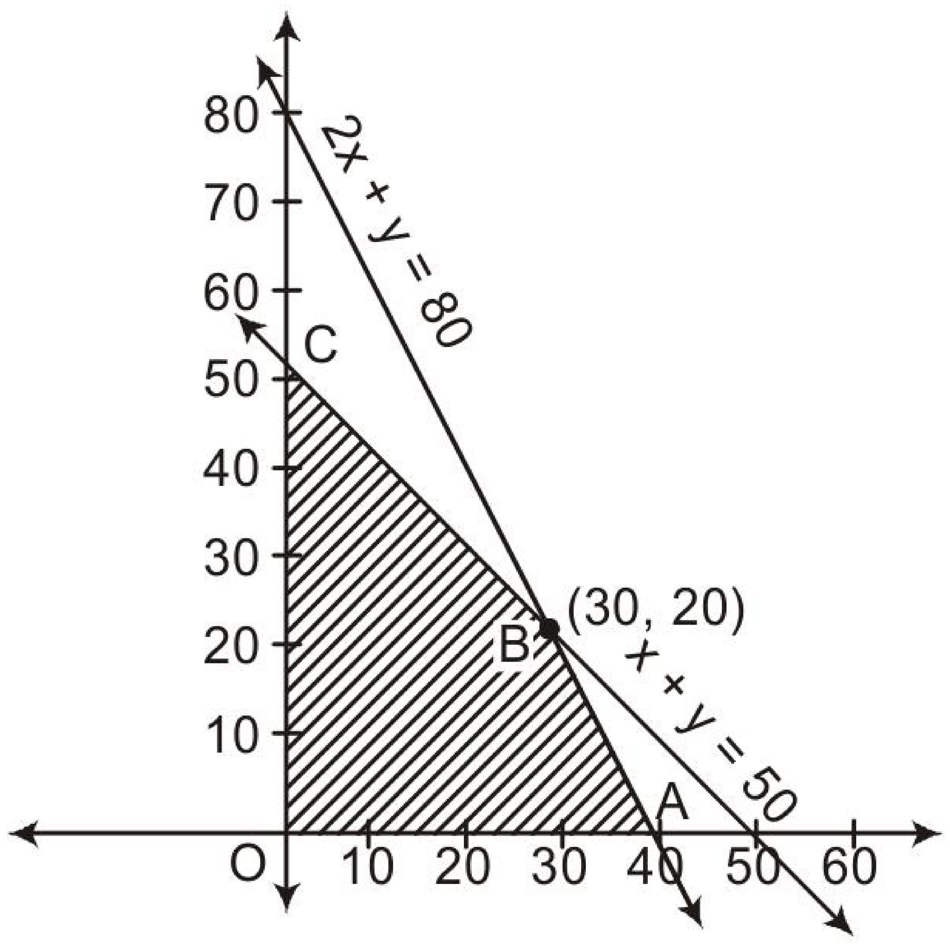
$$\begin{pmatrix}1/5 & 2/5\\2/5 & -1/5\end{pmatrix}A = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$ 1

$$\therefore \quad A^{-1} = \begin{pmatrix}1/5 & 2/5\\2/5 & -1/5\end{pmatrix}$$ $\frac{1}{2}$

Full marks for finding correct A^{-1} using column transformations with $AI = A$

20. **Correct lines** 1

Correct shading 1



Corner points	Value of Z	
O(0 , 0)	0	
A(40, 0)	4200	
B(30, 20)	4950 maximum	$\frac{1}{2}$
C(0, 50)	4500	

Maximum Z = 4950

at x = 30, y = 20 $\frac{1}{2}$

21. Let $x + 3 = A(2x - 4) + B$

which gives $A = \frac{1}{2}, \quad B = 5$ 1

Given integral becomes

$$I = \frac{1}{2}\int \frac{2x-4}{\sqrt{5-4x+x^2}}\,dx + 5\int \frac{dx}{\sqrt{5-4x+x^2}}$$ 1

$$= \sqrt{5-4x+x^2} + 5\int \frac{dx}{\sqrt{(x-2)^2+1^2}}$$ $\frac{1}{2}+\frac{1}{2}$

$$= \sqrt{5-4x+x^2} + 5\log\left|x-2+\sqrt{5-4x+x^2}\right| + C$$ 1

22. LHS becomes

$$\cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$ 2

$$= \cot^{-1}\left[\frac{2\cos x/2}{2\sin x/2}\right]$$ 1

$$= \cot^{-1}\cot\frac{x}{2}$$

$$= \frac{x}{2}$$ 1

23. E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$P(E_1) = \frac{1}{6} \quad P(E_2) = \frac{3}{6} \quad P(E_3) = \frac{2}{6}$ 1

$P(S/E_1) = 0.002\ P(S/E_2) = 0.02,\ P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\,P(S/E_3)}{P(E_1)\,P(S/E_1) + P(E_2)\,P(S/E_2) + P(E_3)\,P(S/E_3)}$$

$$= \frac{\frac{2}{6} \times 0.2}{\frac{1}{6} \times .002 + \frac{3}{6} \times .02 + \frac{2}{6} \times 0.2}$$ 1

$$= \frac{200}{231}$$ 1

Value: Hardwork and Regularity 1

SECTION D

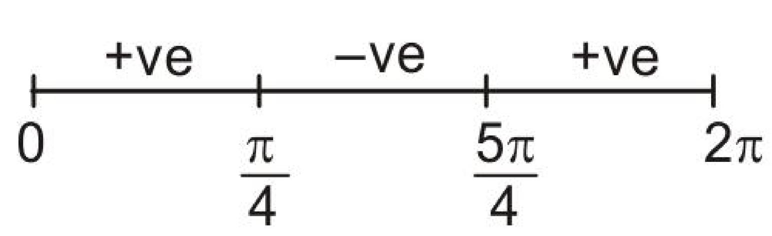
24. $f(x) = \sin x + \cos x \qquad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$ 1

$f'(x) = 0 \Rightarrow \cos x = \sin x$ 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 1

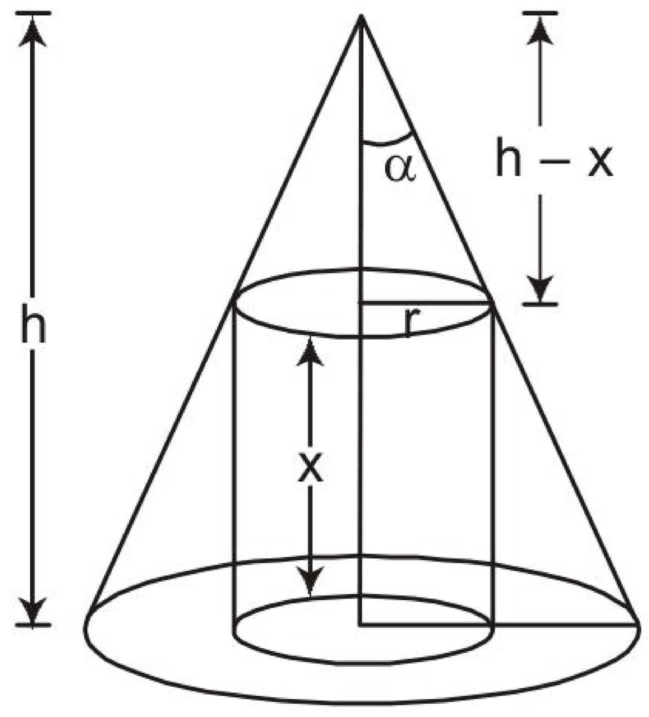
Sign of $f^1(x)$



2

So f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

OR



For Figure **1**

$$\frac{r}{h-x} = \tan\alpha$$ 1

$$r = (h-x)\tan\alpha$$

Volume of cylinder

$$V = \pi r^2 x$$

$$V = \pi(h-x)^2 x \tan^2\alpha$$ $\frac{1}{2}$

$$\frac{dV}{dx} = \pi\tan^2 (h-x)(h-3x)$$

$$\frac{dV}{dx} = 0 \Rightarrow h = x \text{ or } h = 3x$$

i.e. $x = \frac{h}{3}$ $1\frac{1}{2}$

$$\frac{d^2V}{dx^2} = \pi\tan^2\alpha\,(6x-4h)$$

$\therefore \quad \frac{d^2V}{dx^2} < 0 \quad \text{at} \quad x = \frac{h}{3}$ 1

$\therefore \quad$ V is maximum at $x = \frac{h}{3}$

and maximum volume is $V = \frac{4}{27}\pi h^3 \tan^2 \propto$ 1

25. $n = 8, P = \frac{1}{2}, \quad q = \frac{1}{2}$ 1

(i) $P(X=5) = 8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{7}{32}$ $1\frac{1}{2}$

(ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$ 2

$$= \frac{37}{256}$$ $\frac{1}{2}$

(iii) $P(X \leq 6) = 1 - [P(X = 7) + P(X = 8)]$

$$= 1 - \frac{9}{256} = \frac{247}{256}$$ 1

OR

Let X denote number of red cards drawn

X(xi)	P(X)	p_i	$p_i x_i$	$p_i x_i^2$
0	${}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	0	0
1	${}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	${}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	${}^3C_3\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$

Correct table 4

$$\text{mean} = \Sigma p_i x_i = \frac{12}{8} = \frac{3}{2}$$ 1

$$\text{Variance} = \Sigma p_i x_i^2 - (\text{mean})^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$ 1

26. For one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{4x_1}{3x_1+4} = \dfrac{4x_2}{3x_2+4}$

$\Rightarrow \quad 12\cancel{x_1}x_2 + 16x_1 = 12\cancel{x_1}x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one 3

Clearly f: $R - \left\{-\dfrac{4}{3}\right\} \to$ Range f is onto 1

Let f(x) = y

i.e. $\dfrac{4x}{3x+4} = y$

$\Rightarrow \quad x = \dfrac{4y}{4-3y}$ 1

So f^{-1}: Range f $\to R - \left\{-\dfrac{4}{3}\right\}$ is

$f^{-1}(y) = \dfrac{4y}{4-3y}$ 1

OR

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b)

(a, b) * (c, d) = (c, d) * (a, b)

$\therefore$ * is commutative 2

((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)

(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)

As ((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))

$\therefore$ * is associative 2

Let (e_1, e_2) be identity

$(a, b) * (e_1, e_2) = (a, b)$

$(a + e_1, b + e_2) = (a, b)$

$e_1 = 0, e_2 = 0$

$(0, 0) \in R \times R$ is the identity element. 2

27. Clearly order of A is 2×3 1

Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ 1

So $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

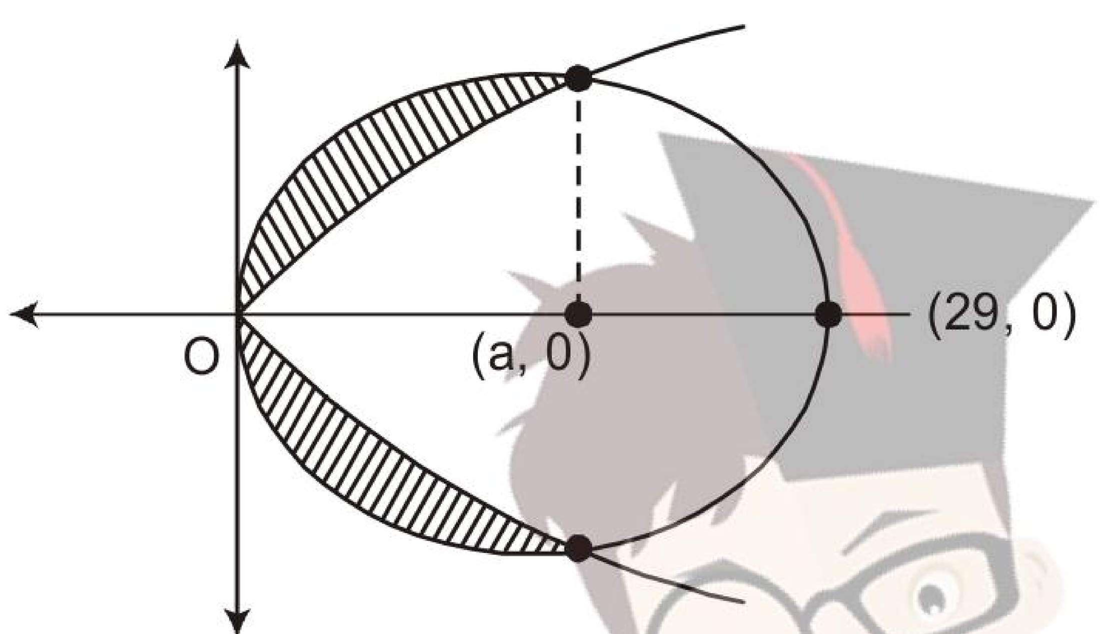
gives

$$\left.\begin{array}{l} 2a - d = -1,\ 2b - e = -8,\ 2c - f = -10 \\ a = 1,\ b = -2,\ c = -5 \end{array}\right\}$$ 2

$\Rightarrow$ $d = 3,\ e = 4,\ f = 0$ 1

Thus $A = \begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}$ 1

28. **Correct figure** 1



x-coordinate of point of intersection is, $x = a$ 1

Required area $= 2\left[\int_0^a \left(\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}\right) dx\right]$ 2

$$= 2\left[\frac{x-a}{2}\sqrt{a^2 - (x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - 2\sqrt{a}\,\frac{x^{\frac{3}{2}}}{3}\right]_0^a$$ 1

$$= \left(\frac{\pi}{2} - \frac{4}{3}\right)a^2$$ 1

29. Required equation of plane is given by

$$\begin{vmatrix} x+1 & y-3 & z-2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$$ 3

$\Rightarrow$ $(x+1)(-7) - (y-3)(-8) + (z-2)(-3) = 0$ 2

$\Rightarrow$ $-7x + 8y - 3z = 25$ 1

or $7x - 8y + 3z + 25 = 0$