Series $HKP25/C

SET~1

Code No. 65/1/1

Roll No.							

Candidates must write the Code on the title page of the answer-book.

NOTE :

(i) Please check that this question paper contains **9** printed pages.

(ii) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) Please check that this question paper contains **38** questions.

(iv) Please write down the serial number of the question in the answer-book before attempting it.

(v) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

*(a) This question paper contains **two** parts — A and B. Each part is **compulsory**. Part A carries **24** marks and part B carries **56** marks.*

(b) Part A has objective type questions and part B has descriptive type questions.

Part A : (Objective Type Questions)

*(i) It consists of **two** sections – I and II.*

*(ii) Section – I comprises of **16** very short answer type questions with choices in 5 questions. Each question is of **1** mark.*

*(iii) Section – II contains **2** case studies. Each comprises of 5 MCQs. You are to attempt any **4** out of 5 MCQs. Each MCQ is of **1** mark.*

Part B : (Descriptive Type Questions)

*(i) It consists of **three** sections – III, IV and V.*

*(ii) Section – III comprises of **10** questions of **2** marks each.*

*(iii) Section – IV comprises of **7** questions of **3** marks each.*

*(iv) Section – V comprises of **3** questions of **5** marks each.*

*(v) Internal choice is provided in 3 questions of Section – III, 2 questions of Section – IV and 3 questions of Section – V. You are to attempt only **one alternative** in all such questions.*

PART A
(Section – I)

*Question numbers **1** to **16** carry **1** mark each.* *16×1=16*

1. If A is a square matrix of order 3 such that A(adj A) = $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find $|A|$. *1*

2. (a) Find the order of the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}.$$ *1*

OR

(b) If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A. *1*

3. Write the smallest reflexive relation on set A = {a, b, c}. *1*

4. (a) Find :

$$\int e^x \left(\log \sqrt{x} + \frac{1}{2x} \right) dx$$ *1*

OR

(b) Find :

$$\int e^{2 \log x} \, dx$$ *1*

5. (a) Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$. *1*

OR

(b) Write the projection of the vector $\vec{r} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ on (i) x-axis, and (ii) y-axis. *1*

6. If $\vec{a} = \alpha \hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \beta\hat{k}$, find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear. *1*

7. If f = {(1, 2), (2, 4), (3, 1), (4, k)} is a one-one function from set A to A, where A = {1, 2, 3, 4}, then find the value of k. *1*

8. (a) Check whether the relation R defined on the set {1, 2, 3, 4} as R = {(a, b) : b = a + 1} is transitive. Justify your answer. *1*

OR

(b) If the relation R on the set A = {x : 0 ≤ x ≤ 12} given by R = {(a, b) : a = b} is an equivalence relation, then find the set of all elements related to 1. *1*

9. If $A = [1 \quad 0 \quad 4]$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, find AB. *1*

10. (a) Write the order and degree of the differential equation :

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$ *1*

OR

(b) Find the general solution of the differential equation $\frac{dy}{dx} = a$, where a is an arbitrary constant. *1*

11. Show that the function $f(x) = \frac{3}{x} + 7$ is strictly decreasing for $x \in R - \{0\}$. *1*

12. Find the magnitude of vector $\vec{a}$ given by $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$. *1*

13. Write the equation of the plane that cuts the coordinate axes at (2, 0, 0), (0, 4, 0) and (0, 0, 7). *1*

14. Find the distance between the two parallel planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 12$. *1*

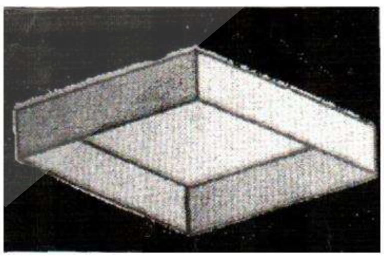
15. If A and B are two independent events and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, find $P(\bar{A} \mid \bar{B})$. *1*

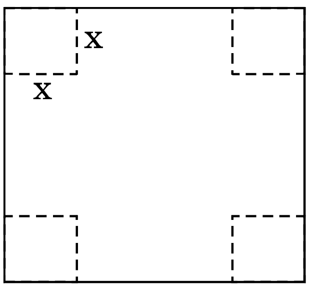
16. A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6. *1*

(Section – II)

*Both the case study based questions (17 & 18) are compulsory. Attempt any **4** subparts out of **5** from each of question numbers **17** and **18**. Each subpart carries **1** mark.*

17. A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps.





Based on the above information, answer ***any four*** of the following ***five*** questions, if x is the length of each square cut from corners : *4×1=4*

(i) The volume of the open box is :

(A) $4x(x^2 - 18x + 81)$

(B) $2x(2x^2 + 36x + 162)$

(C) $2x(2x^2 + 36x - 162)$

(D) $4x(x^2 + 18x + 81)$

(ii) The condition for the volume (V) to be maximum is :

(A) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$

(B) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} > 0$

(C) $\frac{dV}{dx} > 0$ and $\frac{d^2V}{dx^2} = 0$

(D) $\frac{dV}{dx} < 0$ and $\frac{d^2V}{dx^2} = 0$

(iii) What should be the side of square to be cut off so that the volume is maximum ?

(A) 6 cm

(B) 9 cm

(C) 3 cm

(D) 4 cm

(iv) Maximum volume of the open box is :

(A) 423 cm^3

(B) 432 cm^3

(C) 400 cm^3

(D) 216 cm^3

(v) The total area of the removed squares is :

(A) 324 cm^2

(B) 144 cm^2

(C) 36 cm^2

(D) 64 cm^2

18. In answering a multiple choice test for class XII, a student either knows or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$.
Let E_1, E_2, E_3 be the events that the student guesses, copies or knows the answer respectively and A is the event that the student answers correctly.

Based on the above information, answer ***any four*** of the following ***five*** questions : *4×1=4*

(i) What is the probability that the student knows the answer ?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

(ii) What is the probability that he answers correctly given that he knew the answer ?

(A) 1

(B) 0

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

(iii) What is the probability that he answers correctly given that he had made a guess ?

(A) $\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{1}{8}$

(iv) What is the probability that he knew the answer to the question, given that he answered it correctly ?

(A) $\frac{24}{29}$

(B) $\frac{4}{29}$

(C) $\frac{1}{29}$

(D) $\frac{3}{29}$

(v) $\sum_{k=1}^{3} P(E_k \mid A)$ is :

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{11}{8}$

PART B
(Section – III)

*Question numbers **19** to **28** carry **2** marks each.* *10×2=20*

19. A random variable X has the probability distribution :

X :	0	1	2	3	4
P(X) :	0	K	4K	3K	2K

Find the value of K and $P(X \leq 2)$. *2*

20. Simplify $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right), 0 < x < \dfrac{1}{\sqrt{2}}$. *2*

21. If the matrix $A = \begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the values of x. *2*

22. (a) Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1 & \text{if} \quad x \leq 3 \\ bx+3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at x = 3. *2*

OR

(b) Check the differentiability of $f(x) = |x-3|$ at $x = 3$. *2*

23. Find :

$$\int \frac{x^2+2}{x^2+1}\, dx$$ *2*

24. (a) Evaluate :

$$\int_{-1}^{1} \frac{|x|}{x}\, dx$$ *2*

OR

(b) Evaluate :

$$\int_{0}^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$$ *2*

25. Find the integrating factor of $x\dfrac{dy}{dx} + (1 + x\cot x)\, y = x$. *2*

26. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a} + 2\vec{b} + 3\vec{c}|$. *2*

27. If the sides AB and BC of a parallelogram ABCD are represented as vectors $\vec{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{BC} = \hat{i} + 2\hat{j} + 3\hat{k}$, then find the unit vector along diagonal AC. *2*

28. (a) Using integration, find the area bounded by the curve $y^2 = 4x$, y-axis and $y = 3$. *2*

OR

(b) Using integration, find the area of the region bounded by the line $2y = -x + 8$, x-axis, $x = 2$ and $x = 4$. *2*

(Section – IV)

*Question numbers **29** to **35** carry **3** marks each.* *7×3=21*

29. Show that the function $f : R - \{-1\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective. *3*

30. (a) If $x = a \cos\theta + b \sin\theta$, $y = a \sin\theta - b \cos\theta$,

then show that $\frac{dy}{dx} = -\frac{x}{y}$

and hence show that

$$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$ *3*

OR

(b) If $e^{y-x} = y^x$, prove that

$$\frac{dy}{dx} = \frac{y(1 + \log y)}{x \log y}$$ *3*

31. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$. *3*

32. (a) Find the equation of the normal to the curve $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$. *3*

OR

(b) Find the equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis. *3*

33. Find :

$$\int \frac{x^2}{(x-1)(x+1)^2}\, dx$$ *3*

34. If the solution of the differential equation $\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ is $\frac{ax}{y} = b \log |x| + C$, find the value of a and b. *3*

35. Using integration, find the area bounded by the circle $x^2 + y^2 = 9$. *3*

(Section – V)

*Question numbers **36** to **38** carry **5** marks each.* *3×5=15*

36. (a) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}.

Hence, solve the following system of equations :

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$ *5*

OR

(b) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. *5*

37. (a) Find the shortest distance between the following lines :

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ *5*

OR

(b) Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$. *5*

38. (a) Solve the following linear programming problem graphically :

Maximise $z = 3x + 9y$

subject to constraints

$x + 3y \le 60$

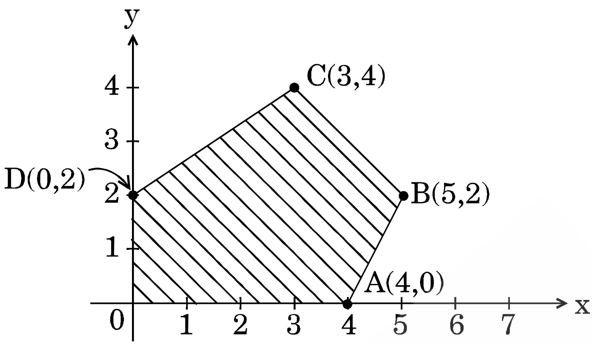
$x + y \ge 10$

$x \le y$

$x, y \ge 0$ *5*

OR

(b) The corner points of the feasible region determined by the system of linear inequations are as shown below :



Answer each of the following :

(i) Let $z = 13x - 15y$ be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.

(ii) Let $z = kx + y$ be the objective function. Find k, if the value of z at A is same as the value of z at B. 5