# LOGARITHMS

- A logarithm of a number with a base is equal to another number. A logarithm is just the opposite function of exponentiation. For example, if $10^2 = 100$ then $\log_{10} 100 = 2$.
- Hence, we can conclude that,

  **$\text{Log}_b\, x = n$ or $b^n = x$**

  Where b is the base of the logarithmic function.

- A logarithm is defined as the power to which number must be raised to get some other values. It is the most convenient way to express large numbers. A logarithm has various important properties that prove multiplication and division of logarithms can also be written in the form of logarithm of addition and subtraction.
- The logarithm of a positive real number a with respect to base b, a positive real number not equal to 1[nb 1], is the exponent by which b must be raised to yield a.

  **i.e $b^y = a$** and it is read as "the logarithm of a to base b."

- In other words, the logarithm gives the answer to the question "How many times a number is multiplied to get the other number?".

**Example: How many 3's are multiplied to get the answer 27?**

If we multiply 3 for 3 times, we get the answer 27.
Therefore, the logarithm is 3.
The logarithm form is written as follows:
$\text{Log}_3 (27) = 3$ ….(1)
Therefore, the base 3 logarithm of 27 is 3.
The above logarithm form can also be written as:
3x3x3 = 27
$3^3 = 27$ …..(2)
Thus, the equations (1) and (2) both represent the same meaning.

## Logarithm Types

In most cases, we always deal with two different types of logarithms, namely

- Common Logarithm
- Natural Logarithm

### Common Logarithm

The common logarithm is also called the base 10 logarithms. It is represented as log10 or simply log. For example, the common logarithm of 1000 is written as a log (1000). The common logarithm defines how many times we have to multiply the number 10, to get the required output.

For example, log (100) = 2

If we multiply the number 10 twice, we get the result 100.

## Natural Logarithm

The natural logarithm is called the base e logarithm. The natural logarithm is represented as ln or loge. Here, "e" represents the Euler's constant which is approximately equal to 2.71828. For example, the natural logarithm of 78 is written as ln 78. The natural logarithm defines how many we have to multiply "e" to get the required output.

For example, ln (78) = 4.357.

Thus, the base e logarithm of 78 is equal to 4.357.

## Logarithm Rules and Properties

There are certain rules based on which logarithmic operations can be performed. The names of these rules are:

- Product rule
- Division rule
- Power rule/Exponential Rule
- Change of base rule
- Base switch rule
- Derivative of log
- Integral of log

Let us have a look at each of these properties one by one

### Product Rule

In this rule, the multiplication of two logarithmic values is equal to the addition of their individual logarithms.

**$\text{Log}_b (mn) = \log_b m + \log_b n$**

For example: $\log_3 ( 2y ) = \log_3 (2) + \log_3 (y)$

### Division Rule

The division of two logarithmic values is equal to the difference of each logarithm.

**$\text{Log}_b (m/n) = \log_b m - \log_b n$**

For example, $\log_3 ( 2/ y ) = \log_3 (2) -\log_3 (y)$

### Exponential Rule

In the exponential rule, the logarithm of m with a rational exponent is equal to the exponent times its logarithm.

**$\text{Log}_b (m^n) = n \log_b m$**

For example: $\log_b(2^3) = 3 \log_b 2$

## Change of Base Rule

**$Log_b\, m = log_a\, m / log_a\, b$**

For example: $log_b\, 2 = log_a 2/log_a\, b$

## Base Switch Rule

**$log_b\,(a) = 1\,/\,log_a\,(b)$**

For example: $log_b\, 8 = 1/log_8\, b$

## Derivative of log

If $f\,(x) = log_b\,(x)$, then the derivative of $f(x)$ is given by;

$f'(x) = 1/(x\, ln(b))$

For example: Given, $f\,(x) = log_{10}\,(x)$

Then, $f'(x) = 1/(x\, ln(10))$

## Integral of Log

$\int log_b(x)dx = x(\, log_b(x) - 1/ln(b)\,) + C$

Example: $\int log_{10}(x)\, dx = x \cdot (\, log_{10}(x) - 1\,/\,ln(10)\,) + C$

## Other Properties

Some other properties of logarithmic functions are:

- $Log_b\, b = 1$
- $Log_b\, 1 = 0$
- $Log_b\, 0$ = undefined

## Logarithmic Formulas

$log_b(mn) = log_b(m) + log_b(n)$

$log_b(m/n) = log_b\,(m) - log_b\,(n)$

$Log_b\,(xy) = y\, log_b(x)$

$Log_b m\sqrt{n} = log_b\, n/m$

$m\, log_b(x) + n\, log_b(y) = log_b(x^m y^n)$

$log_b(m+n) = log_b\, m + log_b(1+nm)$

$log_b(m - n) = log_b\, m + log_b\,(1\text{-}n/m)$

## Solved Examples

**Question 1: Solve $\log_2 (64)$ =?**

**Solution:**

since $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, 6 is the exponent value and $\log_2 (64) = 6$.

**Question 2: What is the value of $\log_{10}(100)$?**

**Solution:** In this case, $10^2$ yields you 100. So, 2 is the exponent value, and the value of $\log_{10}(100) = 2$

**Question 3: Use of the property of logarithms, solve for the value of x for $\log_3 x = \log_3 4 + \log_3 7$**

**Solution:** By the addition rule, $\log_3 4 + \log_3 7 = \log_3 (4 * 7)$

$\log_3 (28)$. Thus, x= 28.

**Question 4: Solve for x in $\log_2 x = 5$**

**Solution:** This logarithmic function can be written In the exponential form as $2^5 = x$

Therefore, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, X= 32.

## Logarithmic Function Definition

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For x, a > 0, and a≠1,

**$y = \log_a x$, if $x = a^y$**

Then the logarithmic function is written as:

**$f(x) = \log_a x$**

The most common bases used in logarithmic functions are base e and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by $\log_{10}$ or simply log.

**$f(x) = \log_{10}$**

The log function to the base e is called the natural logarithmic function and it is denoted by $\log_e$.

**$f(x) = \log_e x$**

To find the logarithm of a number, we can use the logarithm table instead of using a mere calculation. Before finding the logarithm of a number, we should know about the characteristic part and mantissa part of a given number:

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Difference | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |

## How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

**Step 1**: Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.

**Step 2:** Identify the characteristic part and mantissa part of the given number. For example, if you want to find the value of $\log_{10}(15.27)$, first separate the characteristic part and the mantissa part.

Characteristic Part = 15

Mantissa part = 27

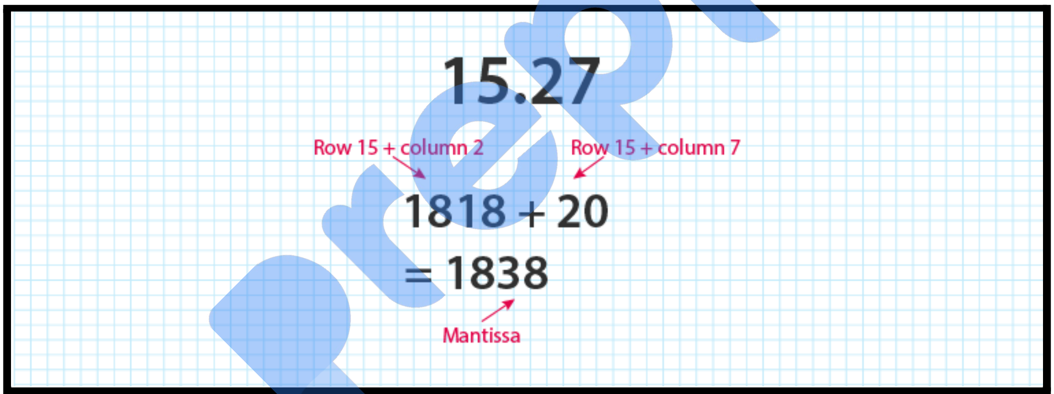
**Step 3**: Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.

**Step 4:** Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.

15.27

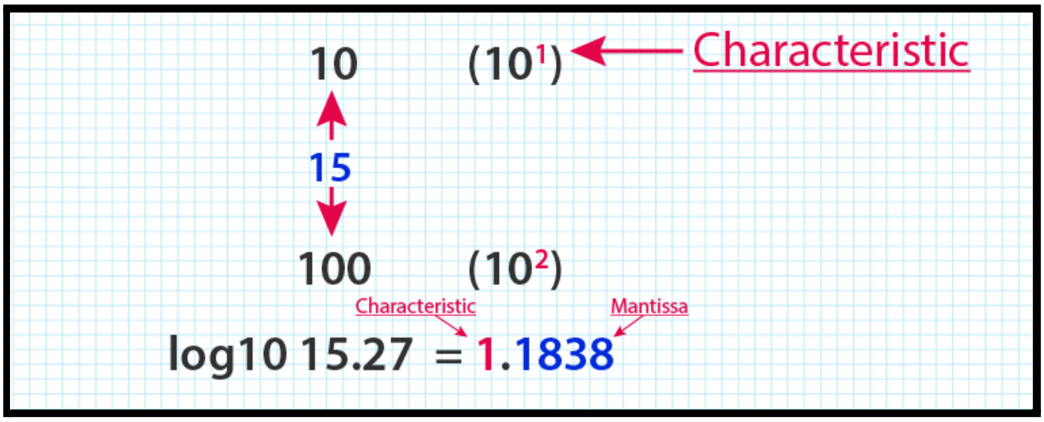
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Difference 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 |
| 14 | 1431 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 7 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 9 | 12 | 6 | 7 |

**Step 5**: Add both the values obtained in step 3 and step 4. That is 1818+20= 1838. Therefore, the value 1838 is the mantissa part.




**Step 6**: Find the characteristic part. Since the number lies between 10 and 100, ($10^1$ and $10^2$), the characteristic part should be 1.

**Step 7**: Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.



**Example: Find the value of $\log_{10}$ 2.872**

**Solution:**

Step 1: Characteristic Part= 2 and mantissa part= 872

Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.

Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3

Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.

Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So, the characteristic part is 0

Step 6: Finally combine the characteristic part and the mantissa part. So, it becomes 0.4582.

Therefore, the value of log 2.872 is 0.4582.

## Practice Questions

**1. Express $5^3 = 125$ in logarithm form.**

**Solution:**

$5^3 = 125$

As we know,

$a^b = c \Rightarrow \log_a c=b$

Therefore;

$\text{Log}_5 125 = 3$

**2. Express $\log_{10} 1 = 0$ in exponential form.**

**Solution:**

Given, $\log_{10} 1 = 0$

By the rule, we know;

$\log_a c=b \Rightarrow a^b = c$

Hence,

$10^0 = 1$

**3. Find the log of 32 to the base 4.**

**Solution:** $\log_4 32 = x$

$4^x = 32$

$(2^2)^x = 2x2x2x2x2$

$2^{2x} = 2^5$

$2x=5$

$x=5/2$

Therefore,

$\log_4 32 = 5/2$

**4. Find x if $\log_5(x-7)=1$.**

**Solution:** Given,

$\log_5(x-7)=1$

Using logarithm rules, we can write;

$5^1 = x-7$

$5 = x-7$

$x=5+7$

$x=12$

**5. If $\log_a m=n$, express $a^{n-1}$ in terms of a and m.**

**Solution:**

$\log_a m=n$

$a^n=m$

$a^n/a=m/a$

$a^{n-1}=m/a$

**6. Solve for x if $\log(x-1)+\log(x+1)=\log_2 1$**

**Solution:** $\log(x-1)+\log(x+1)=\log_2 1$

$\log(x-1)+\log(x+1)=0$

$\log[(x-1)(x+1)]=0$

Since, $\log 1 = 0$

$(x-1)(x+1) = 1$

$x^2-1=1$

$x^2=2$

$x = \pm\sqrt{2}$

Since, log of negative number is not defined.

Therefore, $x = \sqrt{2}$

**7. Express log(75/16) - 2log(5/9) + log(32/243) in terms of log 2 and log 3.**

**Solution:** log(75/16) - 2log(5/9) + log(32/243)

Since, $n\log_a m = \log_a m^n$

⇒ $\log(75/16) - \log(5/9)^2 + \log(32/243)$

⇒ log(75/16) - log(25/81) + log(32/243)

Since, $\log_a m - \log_a n = \log_a(m/n)$

⇒ log[(75/16) ÷ (25/81)] + log(32/243)

⇒ log[(75/16) × (81/25)] + log(32/243)

⇒ log(243/16) + log(32/243)

Since, $\log_a m + \log_a n = \log_a mn$

⇒ log(32/16)

⇒ log2

**8. Express 2logx + 3logy = log a in logarithm free form.**

**Solution:** 2logx + 3logy = log a

$\log x^2 + \log y^3 = \log a$

$\log x^2 y^3 = \log a$

$x^2 y^3 = \log a$

**9. Prove that: 2log(15/18) - log(25/162) + log(4/9) = log2**

**Solution:** 2log(15/18) - log(25/162) + log(4/9) = log2

Taking L.H.S.:

⇒ 2log(15/18) - log(25/162) + log(4/9)

⇒ $\log(15/18)^2 - \log(25/162) + \log(4/9)$

⇒ log(225/324) - log(25/162) + log(4/9)

⇒ log[(225/324)(4/9)] - log(25/162)

⇒ log[(225/324)(4/9)]/(25/162)

$\Rightarrow \log(72/36)$

$\Rightarrow \log 2$ (R.H.S)

**10. Express $\log_{10}(2+1)$ in the form of $\log_{10}x$.**

**Solution:** $\log_{10}(2+1)$

$= \log_{10}2 + \log_{10}1$

$= \log_{10}(2\times 10)$

$= \log_{10}20$