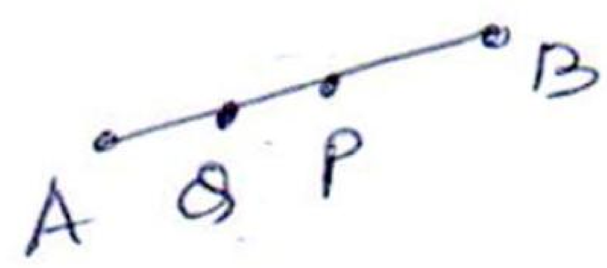
QUESTION PAPER CODE 65(B)

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. P.V. of $P = \frac{13\vec{a} - 6\vec{b}}{5} \left(\because \frac{AP}{PB} = \frac{3}{2}\right)$ ½ m



P.V. of $Q = \frac{23}{10}\vec{a} + \frac{9}{10}\vec{b} \left(\because \frac{AQ}{QP} = \frac{1}{1}\right)$ ½ m

2. $\vec{a} \cdot \vec{b} = 0$ as $\vec{a} \perp \vec{b}$ ½ m

$2\lambda - 3\lambda - 5 = 0$

$\Rightarrow \lambda = -5$ ½ m

3. D.R. of normal to plane 3 , 4 , 2 ½ m

Also point (3, 4, 2) lies on plane

$3x + 4y + 2z + d = 0$

$\Rightarrow d = -29$

So cartesian Equation of plane is

$3x + 4y + 2z - 29 = 0$ ½ m

4. $A = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{vmatrix}$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 3$ 1 m

5. Order = 2 ½ m

or Degree = 1

So A + B = 3 ½ m

6. $y = ax + x^2$

$y_1 = a + 2x$

$y_1 - 2x = a$ ½ m

So $y = (y_1 - 2x)x + x^2$

$\Rightarrow \quad xy_1 = y + x^2$ ½ m

SECTION - B

7. Total Expenditure incurred for villages x, y, z are

$$\begin{bmatrix} 200 & 400 & 200 \\ 350 & 600 & 300 \\ 225 & 375 & 150 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 15 \end{bmatrix} = \begin{bmatrix} 7000 \\ 11{,}000 \\ 6375 \end{bmatrix}$$ 2 m

So Expenditure on village x = ₹ 7000
So Expenditure on village y = ₹ 11,000
So Expenditure on village z = ₹ 6375 1 m

Value : Sensitization about hygehic habits ..... or Any other relevant value 1 m

8. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$A^2 - \lambda A + \mu I = 0$

$\Rightarrow \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 2\lambda & -\lambda \\ -\lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 1 m

$\Rightarrow \left.\begin{array}{l} 5 - 2\lambda + \mu = 0 \\ -4 + \lambda = 0 \end{array}\right\} \Rightarrow \begin{array}{l} \lambda = 4 \\ \mu = 3 \end{array}$ 1 m 1 m

OR

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1$$ 1 m

$c_{11} = 7 \quad c_{21} = -3 \quad c_{31} = -3$

$c_{12} = -1 \quad c_{22} = 1 \quad c_{32} = 0$

$c_{13} = -1 \quad c_{23} = 0 \quad c_{33} = 1$

$$\text{Adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$ 1½ m

$$A \cdot (\text{adj } A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{.................................... (i)}$$

Since $|A| = 1$

$$\text{So } |A| \, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{.................................... (ii)}$$ ½ m

from (i) & (ii)

$A \cdot (\text{adj } A) = |A| I$ ½ m

9. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1$$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$ 3 m

$$= a \begin{vmatrix} a & 2a+b \\ 3a & 7a+3b \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & 2a+b \\ 3 & 7a+3b \end{vmatrix}$$

$$= a^2 \, (7a + 3b - 6a - 3b)$$

$$= a^3$$ 1 m

10. $$I = \int_0^{\pi/4} \log(1 + \tan x)\, dx$$

$$= \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$$ 1 m

$$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$$ 1 m

$$= \int_0^{\pi/4} (\log 2 - \log(1 + \tan x))\, dx$$

$$I = \int_0^{\pi/4} \log 2 \, dx - I$$ 1 m

$$2I = \frac{\pi}{4} \log 2$$

$$\text{or } \; I = \frac{\pi}{8} \log 2$$ 1 m

11. $$\int \frac{\sqrt{1-\sin x}}{1+\cos x} \cdot e^{-x/2} dx \; ; \; 0 \le x \le \frac{\pi}{2}$$

$$= \int \frac{-\sin \frac{x}{2} + \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} \cdot e^{-x/2} dx$$ 1 m

$$= \frac{1}{2} \int \left(-\sec \frac{x}{2} \tan \frac{x}{2} + \sec \frac{x}{2} \right) e^{-x/2} dx$$ 1 m

Put $-\frac{x}{2} = t$

$\Rightarrow \frac{-1}{2} dx = dt$

$$= -\int (\sec t + \sec t \tan t) \, e^t dt$$

$$= -e^t \sec t + c$$ 1 m

$$= -e^{-x/2} \sec \left(\frac{-x}{2} \right) + c$$

$$= -e^{-x/2} \sec \left(\frac{x}{2} \right) + c$$ 1 m

OR

$$\int \frac{x^2+1}{x^2-5x+6} \, dx$$

$$= \int \left(1 + \frac{5x-5}{x^2-5x+6}\right) dx = \int \left(1 + \frac{5x-5}{(x-2)(x-3)}\right) dx$$ 1 m

$$= \int dx + \int \frac{-5}{x-2}\, dx + \int \frac{10}{x-3}\, dx$$ 1½ m

$$= x - 5\log|x-2| + 10\log|x-3| + c$$ 1½ m

12. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Event A : No. on card is 'more than 5' 1 m

$A = \{6, 7, 8, 9, 10\}$

Event B : Even no. on card

$B = \{2, 4, 6, 8, 10\}$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$ 1 m

$$= \frac{3/10}{5/10} = 3/5$$ 2 m

13. Given $|\vec{a}| = |\vec{b}|$

$\cos\theta = \cos 60^\circ = \frac{1}{2}$, θ angle between $\vec{a}$ & $\vec{b}$ 1 m

$$\vec{a} \cdot \vec{b} = \frac{1}{2}$$

Use $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$$\Rightarrow \frac{1}{2} = \frac{\frac{1}{2}}{|\vec{a}||\vec{b}|} = \frac{\frac{1}{2}}{|\vec{a}|^2}$$

$\Rightarrow \; |\vec{a}|^2 = 1$

$\Rightarrow \; |\vec{a}| = |\vec{b}| = 1$ 2 m

14. $\vec{r_1} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

$\vec{r_2} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

S.D. between $\vec{r_1}$ & $\vec{r_2}$ $= \left| \dfrac{\vec{b} - \vec{a} \cdot \vec{c} \times \vec{d}}{|\vec{c} \times \vec{d}|} \right|$ 1 m

$\left(\vec{b} - \vec{a}\right) \cdot \left(\vec{c} \times \vec{d}\right) = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$ 1 m

$= 10$

$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{c} \times \vec{d}\right| = \sqrt{59}$ 1 m

Hence S.D. $= \left| \dfrac{10}{\sqrt{59}} \right| = \dfrac{10}{\sqrt{59}}$ units 1 m

15. $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$

$y = \dfrac{\pi}{2} - 2\tan^{-1}\left(\sqrt{\cos x}\right) \quad \because \left(\cot^{-1}x + \tan^{-1}x = \dfrac{\pi}{2}\right)$ 1 m

or $y - \dfrac{\pi}{2} = -2\tan^{-1}\left(\sqrt{\cos x}\right)$

or $\frac{\pi}{2} - y = \cos^{-1}\left(\frac{1-\cos x}{1+\cos x}\right) \quad \left(\because\ 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$ 1 m

or $\cos\left(\frac{\pi}{2} - y\right) = \frac{2\sin^2 x/2}{2\cos^2 x/2}$ 1 m

or $\sin y = \tan^2\left(\frac{x}{2}\right)$ 1 m

Hence proved

OR

$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$

$\tan^{-1}\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right) = \tan^{-1}(-7)$ 1 m

or $\tan^{-1}\left(\frac{2x^2+1-x}{1-x}\right) = \tan^{-1}(-7)$ 1 m

or $2x^2 + 1 - x = -7(1-x)$ ½ m

or $2x^2 - 8x + 8 = 0$

or $(x-2)^2 = 0$

$\Rightarrow \quad x = 2$ 1 m

since $x = 2$ does not satisfy the given equation.

Hence no solution ½ m

16. $y = \left(3\cot^{-1}x\right)^2$

$y_1 = 2\left(3\cot^{-1}x\right)\left(\frac{-3}{1+x^2}\right)$

$= -18\frac{\cot^{-1}x}{1+x^2}$ 2 m

or $y_1\left(1+x^2\right) = -18\cot^{-1}x$

or $y_2\left(1+x^2\right) + 2xy_1 = \frac{18}{1+x^2}$ 1 m

or $y_2\left(1+x^2\right)^2 + 2x\left(1+x^2\right)y_1 = 18$ 1 m

OR

$f(x) = |x-3|, \quad x \in R$

$f(x) = x-3, \quad x \geq 3$

$= -(x-3), \quad x < 3$

To show continuity

$\underset{x\to 3^+}{\text{lt}} f(x) = \underset{x\to 3^-}{\text{lt}} f(x) = f(3)$ 1 m

$\underset{x\to 3^+}{\text{lt}} f(x) = \underset{x\to 3}{\text{lt}} x-3 = 0$

$\underset{x\to 3^-}{\text{lt}} f(x) = \underset{x\to 3}{\text{lt}} -(x-3) = 0$

$f(3) = 3-3 = 0$

So f(x) is continuous at $x = 3$ 1 m

For derivability at $x = 3$ need to show that

R.H.D = LHD

In this case

$$\text{R.H.D}(3) = \lim_{h\to 0} \frac{h}{h} = 1$$

$$\text{L.H.D}(3) = \lim_{h\to 0} \frac{h}{-h} = -1$$ 1 m

So func is not differentiable at $x = 3$ 1 m

17. $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

or $y = e^{x \log\left(x+\frac{1}{x}\right)} + e^{\left(1+\frac{1}{x}\right)\log x}$ 1 m

$$\frac{dy}{dx} = e^{x\log\left(x+\frac{1}{x}\right)}\left[\log\left(1+\frac{1}{x}\right) + \frac{x\left(1-\frac{1}{x^2}\right)}{1+\frac{1}{x}}\right]$$

$$+ e^{\left(1+\frac{1}{x}\right)\log x}\left[\left(\frac{-1}{x^2}\right)\log + \left(1+\frac{1}{x}\right)\left(\frac{1}{x}\right)\right]$$

$$= \left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right]$$ 1½+1½ m

$$+ (x)^{\left(1+\frac{1}{x}\right)}\left[\frac{x^2+1-\log x}{x^2}\right]$$

18. $y = (x-2)^2$

$$\frac{dy}{dx} = 2(x-2)$$ 1 m

Let (x_1, y_1) be the point of contact

$$\left.\frac{dy}{dx}\right|_{(x_1, y_1)} = 2(x_1 - 2)$$

Slope of chord = $m = \frac{4-0}{4-2} = 2$

$2(x_1 - 2) = 2$

$\Rightarrow \quad x_1 = 3$

since (x_1, y_1) lies on curve $y = (x-2)^2$

So $y_1 = (3-2)^2 = 1$

So point of contact is (3, 1) 2 m

Also, equation of tangent is

$y - 1 = 2(x - 3)$

or $y - 2x + 5 = 0$ 1 m

19. $I = \int (6x+5)\sqrt{6+x-x^2}\,dx$

$6x + 5 = A(1-2x) + B$

$\Rightarrow \quad A = -3, \quad B = 8$ 1 m

So, $I = -3\int (1-2x)\sqrt{6+x-x^2}\,dx + 8\int\sqrt{6+x-x^2}\,dx$

$= -2(6+x-x^2)^{3/2} + 8\int\sqrt{\left(\frac{5}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}\,dx$ 1 m

$= -2(6+x-x^2)^{3/2} + \frac{8}{4}\left((2x-1)\sqrt{6+x-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{2x-1}{5}\right)\right)$ 1 m

$= -2(6+x-x^2)^{3/2} + 2\left((2x-1)\sqrt{6+x-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{2x-1}{5}\right)\right) + c$ 1 m

SECTION - C

20. $f(x) = 3x + 2, \quad f: R \to R$

$g(x) = \frac{x}{x^2+1}, \quad g: R \to R$

(i) $fog(x) = f(g(x)), \quad fog: R \to R$

$$= f\left(\frac{x}{x^2+1}\right)$$

$$= 3\left(\frac{x}{x^2+1}\right) + 2$$

$$= \frac{2x^2+3x+2}{x^2+1}$$ 2 m

(ii) $fof(x) = f(f(x)), \quad fof: R \to R$

$$= f(3x+2)$$

$$= 3(3x+2)+2$$

$$= 9x+8$$ 2 m

(iii) $gog(x) = g(g(x)), \quad gog: R \to R$

$$= g\left(\frac{x}{x^2+1}\right)$$

$$= \frac{\frac{x}{x^2+1}}{\left(\frac{x}{x^2+1}\right)^2+1} = \frac{x(x^2+1)}{3x^2+1+x^4}$$

$$= \frac{x(x^2+1)}{x^4+3x^2+1}$$ 2 m

OR

$f: A \times B \to B \times A$ s.t.

$f(a, b) = (b, a)$

To show f is one – one

Let $(a, b,)$ & (c, d) be any arbitrany element in $A \times B$ s.t.

$a \neq c, \quad a, c \in A$

$b \neq d, \quad b, d \in B$

then $f(a, b) = (b, a)$

$f(c, d) = (d, c)$

$(b, a) \neq (d, c) \quad (\because b \neq d, a \neq c)$

$\Rightarrow f(a, b) \neq f(c, d)$

$\Rightarrow$ f is one – one .... (i) 2 m

f is onto

$\forall \; a \in A, \; b \in B,$

$(b, a) \in B \times A$

$\Rightarrow (a, b) \in A \times B$

So f is onto .... (ii) 1 m

Hence, from (i) & (ii)

f is bijective function 1 m

21. Area $= = \int_0^{ae} y\,dx$

$= 2\int_0^{ae} \frac{b}{a}\sqrt{a^2 - x^2}\,dx$ 1 m

$= \frac{2b}{a}\int_0^{ae} \sqrt{a^2 - x^2}\,dx$ 1 m

$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^{ae}$ 2 m

$$= \frac{2b}{a}\left[\frac{ae}{2}\sqrt{a^2(1-e^2)} + \frac{a^2}{2}\sin^{-1}e\right] - 0$$ 1 m

$$= b\left[eb + a\sin^{-1}e\right]$$

or $b^2e + ab\sin^{-1}e$ 1 m

22. $$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

or $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$ 1 m

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$x\frac{dv}{dx} = \sec v$$

$\cos v\, dv = \frac{dx}{x}$ 1 m

$$\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$$

or $\sin v = \log|x| + c$ 1 m

when $y = \frac{\pi}{4}, \ x = 1$

$\frac{1}{\sqrt{2}} = \log 1 + c$ 1 m

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

Particular solution is $\sin\left(\frac{y}{x}\right) = \log|x| + \frac{1}{\sqrt{2}}$ 1 m

OR

$$\frac{dy}{dx} - y = \cos x$$

$\text{I.F.} = e^{\int -dx} = e^{-x}$ $\left(\text{Here } P = -1, Q = \cos x \text{ and equation is in form } \frac{dy}{dx} + Py = Q(x)\right)$ 1 m

So general solution is

$y \cdot e^{-x} = \int e^{-x} \cos x \, dx + c$ .... (i) 1 m

consider

$$I = \int e^{-x} \cos x \, dx = -\cos x \, e^{-x} + \int \left(-\sin x \cdot e^{-x}\right) dx$$

$= -\cos x \cdot e^{-x} - \left[-\sin x \cdot e^{-x} + \int \cos x \, e^{-x} dx\right]$ 2 m

$$2I = \left(-\cos x + \sin x\right) e^{-x} + c$$

$I = \left(\frac{\sin x - \cos x}{2}\right) e^{-x} + c$ .... (ii) 1 m

From (i) & (ii), general solution of given D.E. is

$y \cdot e^{-x} = \left(\frac{\sin x - \cos x}{2}\right) e^{-x} + c$ 1 m

or $2y = \sin x - \cos x + ce^{x}$

23. Given planes are

$$2x + 2y - 3z - 7 = 0$$

and $2x + 5y + 3z - 9 = 0$

Equation of plane passing through intersection of two given planes is

$(2x + 2y - 3z - 7) + k\,(2x + 5y + 3z - 9) = 0$ 1½ m

or $(2+ 2k)\,x + (2 + 5k)\,y + (-3 + 3k) = -7 - 9\,k = 0$ 1 m

This plane passes through point (2, 1, 3)

So $(2 + 2k)\,(2) + (2 + 5k)\,(1) + (-3 + 3k)\,(3) - 7 - 9k = 0$

$-10 + 9k = 0$ 2 m

or $k = \frac{10}{9}$

So equation of plane is

$$\left(2 + 2\left(\frac{10}{9}\right)\right)x + \left(2 + \frac{5(10)}{9}\right)y + \left(-3 + \frac{3(10)}{9}\right)z - 7 - \frac{9(10)}{9} = 0$$

$38x + 68y + 3k - 153 = 0$ 1 m

Hence vec. equ. of plane passing through the intersection of plane is

$\vec{r} \cdot \left(38\hat{i} + 68\hat{j} + 3\hat{k}\right) = 153$ ½ m

24. E_1 : Ball from bag I

E_2 : Ball from bag II 1 m

E_3 : Drawing black ball

$P(E_1) = P(E_2) = \frac{1}{2}$

$P(B/E_1) = \frac{4}{7}, \quad P(B/E_2) = \frac{6}{11}$ 2 m

Prob. of ball drawn found to be black, drawn from bag II

$$P(E_2/B) = \frac{P(E_2) \cdot P(B/E_2)}{P(E_1) \cdot P(B/E_1) + P(E_2) \cdot P(B/E_2)}$$ 1 m

$$= \frac{\frac{1}{2}\left(\frac{6}{11}\right)}{\frac{1}{2}\left(\frac{4}{7}\right) + \frac{1}{2}\left(\frac{6}{11}\right)} = \frac{21}{43}$$ +1 m

25.

	Returns	Investment
Bond A	10%	x
Bond B	15%	y

L.P.P. is

objective func. $z = \frac{10}{100} x + \frac{15}{100} y = 0.1x + 0.15y$ 2 m

Subject to

$x + y \leq 50{,}000$ 1 m

$x \geq 15{,}000$ 1 m

$y \leq 20{,}000$ 1 m

$x, y \geq 0$ +1 m

26. Let the two numbers be x and y

$x + y = 16$

$f(x) = x^3 + y^3$

$= x^3 + (16 - x)^3$ 1½ m

$f'(x) = 3x^2 + 3(16 - x)^2(-1)$

$= 96x - 768$ 1½ m

$f'(x) = 0 \Rightarrow x = 8$ 1 m

So x = 8 may be point of maximum or minimum

consider $f''(x) = 96 > 0$ 1 m

$\Rightarrow$ x = 8 is point of minima

when x = 8, y = 8

So 8 and 8 are numbers such that their sum is 16 and sum of their cubes is minimum. 1 m