

2014 – MS

Booklet No. 600496

A

Test Paper Code : MS

QUESTION BOOKLET CODE A

Reg. No.

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Time : 3 Hours

Name :

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Maximum Marks : 100

### GENERAL INSTRUCTIONS

1. This Question-cum-Answer Booklet has 32 pages consisting of Part-I and Part-II.
2. An **ORS** (Optical Response Sheet) is inserted inside the Question-cum-Answer Booklet for filling in the answers of Part-I. Verify that the CODE and NUMBER Printed on the ORS matches with the CODE and NUMBER Printed on the **Question-cum-Answer Booklet**.
3. Based on the performance of Part-I, a certain number of candidates will be shortlisted. Part-II will be evaluated only for those shortlisted candidates.
4. The merit list of the qualified candidates will depend on the performance in both the parts.
5. Write your **Registration Number and Name** on the top right corner of this page as well as on the right hand side of the **ORS**. Also fill the appropriate bubbles for your registration number in the **ORS**.
6. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
7. **Non-Programmable Calculator is ALLOWED. But clip board, log tables, slide rule, cellular phone and other electronic gadgets are NOT ALLOWED.**
8. The Question-cum-Answer Booklet and the ORS must be returned in its entirety to the Invigilator before leaving the examination hall. **Do not remove any page from this Booklet.**
9. Refer to special instructions/useful data on the reverse of this page.

#### Instructions for Part-I

10. Part-I consists of 35 objective type questions. The first 10 questions carry **ONE** mark each and the rest 25 questions carry **TWO** marks each.
11. Each question has 4 choices for its answer: (A), (B), (C) and (D). Only **ONE** of the four choices is correct.
12. Fill the correct answer on the left hand side of the included **ORS** by darkening the appropriate bubble with a black ink ball point pen as per the instructions given therein.
13. There will be **negative marks for wrong answers**. For each 1 mark question the negative mark will be 1/3 and for each 2 mark question it will be 2/3.

#### Instructions for Part-II

14. Part-II has 8 subjective type questions. Answers to this part must be written in blue/black/blue-black ink only. The use of sketch pen, pencil or ink of any other color is not permitted.
15. Do not write more than one answer for the same question. In case you attempt a descriptive question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.

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## Special Instructions/ Useful Data

$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \dots, x_n) : x_i \in \mathbb{R}, i = 1, 2, \dots, n\}$
$E(X)$	Expectation of random variable $X$
$P(A)$	Probability of event $A$
$\bar{X}$	$\frac{1}{n} \sum_{i=1}^n X_i$
i.i.d.	Independent and identically distributed
$U[a, b]$	Continuous uniform distribution on $[a, b]$ , $-\infty < a < b < \infty$
$\text{Bin}(n, p)$	The binomial distribution with $n$ trials and success probability $p$
$N(\mu, \sigma^2)$	Normal distribution with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$
$\phi(x)$	Probability density function of $N(0, 1)$
$\Phi(x)$	Cumulative distribution function of $N(0, 1)$
Special values of $\Phi(x)$	$\Phi(0.66) = 0.7454$ , $\Phi(1) = 0.8413$ , $\Phi(1.5) = 0.9332$ , $\Phi(2) = 0.9772$



**IMPORTANT NOTE FOR CANDIDATES**

- Part-I consists of 35 objective type questions. The first ten questions carry one mark each and the rest of the objective questions carry two marks each. There will be negative marks for wrong answers. For each 1 mark question the negative mark will be  $1/3$  and for each 2 mark question it will be  $2/3$ .
- Write the answers to the objective questions by filling in the appropriate bubble on the left hand side of the included ORS.
- Part-II consists of 8 descriptive type questions each carrying five marks.

**Part –I: Objective Questions**

Q. 1 – Q. 10 carry one mark each.

Q.1 The equation

$$3x^2 - 12x + 11 + \frac{1}{5}(x^3 - 6x^2 + 11x - 6) = 0$$

has

- (A) exactly one root in the interval (1, 2)  
 (B) exactly two distinct roots in the interval (1, 2)  
 (C) exactly three distinct roots in the interval (1, 2)  
 (D) NO roots in the interval (1, 2)

Q.2 A circle of random radius  $R$  (in cm) is constructed, where the random variable  $R$  has  $U[0, 1]$  distribution. The probability that the area of the circle is less than  $1 \text{ cm}^2$ , is

- (A)  $\frac{1}{4\sqrt{\pi}}$       (B)  $\frac{1}{3\sqrt{\pi}}$       (C)  $\frac{1}{2\sqrt{\pi}}$       (D)  $\frac{1}{\sqrt{\pi}}$

Q.3 Let the random variable  $X$  have moment generating function

$$M_X(t) = e^{2t(1+t)}, \quad t \in \mathbb{R}.$$

Then  $P(X \leq 2)$  is

- (A)  $\frac{1}{4}$       (B)  $\frac{1}{2}$       (C)  $\frac{1}{3}$       (D)  $\frac{2}{3}$

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Q.4 A system consisting of  $n$  components functions if, and only if, at least one of  $n$  components functions. Suppose that all the  $n$  components of the system function independently, each with probability  $\frac{3}{4}$ . If the probability of functioning of the system is  $\frac{63}{64}$ , then the value of  $n$  is

- (A) 2 (B) 4  
(C) 3 (D) 5

Q.5 The matrix

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

- (A) is an elementary matrix  
(B) can be written as a product of elementary matrices  
(C) does NOT have linearly independent eigenvectors  
(D) is a nilpotent matrix

Q.6 Let the mappings  $T_1, T_2, T_3, T_4$  from  $\mathbb{R}^3$  to  $\mathbb{R}^3$  be defined by

$$T_1(x, y, z) = (x^2 + y^2, x + z, x + y + z);$$

$$T_2(x, y, z) = (y + z, z + x, x + y);$$

$$T_3(x, y, z) = (x + y, xy, x - z);$$

$$T_4(x, y, z) = (x, 2y, 3z).$$

Then which of these are linear transformations of  $\mathbb{R}^3$  over  $\mathbb{R}$ ?

- (A)  $T_1$  and  $T_2$  (B)  $T_2$  and  $T_3$   
(C)  $T_2$  and  $T_4$  (D)  $T_3$  and  $T_4$

Q.7 Let

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Then the matrix of the linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by  $T(X) = TX$ ; with

respect to the basis  $B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$  over  $\mathbb{R}$  is

(A)  $\begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & -1 \\ 1 & 2 & 3 \end{pmatrix}$       (B)  $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$       (C)  $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$       (D)  $\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}$

Q.8 Let  $X_1, X_2, X_3$  and  $X_4$  be independent random variables. Then which of the following pairs of random variables are independent?

- (A)  $(X_1 + X_2, X_2 + X_3)$       (B)  $(X_1, X_1 + X_3)$   
 (C)  $(X_1 + X_2, X_3)$       (D)  $(X_2, X_1 + X_2 + X_3)$

Q.9 The series

$$\sum_{n=1}^{\infty} \frac{\log\left(1 + \frac{1}{n}\right)}{n^\alpha}$$

- (A) converges if  $\alpha > 0$       (B) diverges for all  $\alpha \in \mathbb{R}$   
 (C) converges if  $\alpha = 0$       (D) converges if  $\alpha < 0$

Q.10 Let  $X$  be a random variable of continuous type with probability density function

$$f(x|\theta) = \begin{cases} \frac{\theta}{x} \left(\frac{3}{x}\right)^\theta, & \text{if } x > 3 \\ 0, & \text{otherwise} \end{cases}; \theta > 0.$$

Based on single observation  $X$ , the most powerful test of size  $\alpha = 0.1$ , for testing  $H_0: \theta = 1$  against  $H_1: \theta = 2$ , rejects  $H_0$  if  $X < k$ . Then the value of  $k$  is

- (A) 1      (B)  $\frac{10}{3}$       (C)  $\frac{11}{3}$       (D) 4

**Q. 11 – Q. 35 carry two marks each.**

Q.11 Let  $X$  be a random variable of continuous type with probability density function  $f(x)$ . Then, based on single observation  $X$ , the most powerful test of size  $\alpha = 0.1$  for testing  $H_0 : f(x) = 2x, 0 < x < 1$ , against  $H_1 : f(x) = 4x^3, 0 < x < 1$ , has power

- (A)  $\frac{9}{10}$                       (B)  $\frac{1}{10}$                       (C)  $\frac{81}{100}$                       (D)  $\frac{19}{100}$

Q.12 Let  $X$  and  $Y$  be two random variables of discrete type with respective probability mass functions as

$$p_X(0) = 2p, p_X(1) = 2p, p_X(2) = 1 - 4p, 0 < p < \frac{1}{4},$$

and

$$p_Y(0) = \frac{p}{2}, p_Y(1) = 4p^2, p_Y(2) = 1 - \frac{p}{2} - 4p^2, 0 < p < \frac{1}{4}.$$

Then, among statistics  $X$  and  $Y$ ,

- (A) both  $X$  and  $Y$  are complete  
 (B)  $X$  is complete but  $Y$  is NOT complete  
 (C) both  $X$  and  $Y$  are NOT complete  
 (D)  $X$  is NOT complete but  $Y$  is complete
- Q.13 Let  $X$  and  $Y$  denote the lifetimes (in years) of two independent components connected in a series with respective probability density functions

$$f_X(x) = \frac{1}{2} e^{-\frac{x}{2}}, x > 0, \quad \text{and} \quad f_Y(y) = \frac{y}{4} e^{-\frac{y}{2}}, y > 0.$$

Then the probability that the system will survive for at least 2 years, is

- (A)  $e^{-2}$                       (B)  $2e^{-2}$                       (C)  $3e^{-2}$                       (D)  $4e^{-2}$

Q.14 The distribution function of a random variable  $X$  is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}, & 0 \leq x < \frac{1}{4} \\ \frac{1}{2}, & \frac{1}{4} \leq x < \frac{1}{2} \\ \frac{3}{4}, & \frac{1}{2} \leq x < \frac{3}{4} \\ \frac{x+3}{5}, & \frac{3}{4} \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

Then  $P\left(\frac{1}{4} \leq X \leq 1\right)$  is

- (A)  $\frac{1}{20}$                       (B)  $\frac{11}{20}$                       (C)  $\frac{7}{20}$                       (D)  $\frac{13}{20}$

Q.15 Four persons P, Q, R and S take turns (in the sequence P, Q, R, S, P, Q, R, S, P, ... ) in rolling a fair die. The first person to get a six wins. Then the probability that S wins is

- (A)  $\frac{125}{671}$                       (B)  $\frac{125}{571}$                       (C)  $\frac{125}{471}$                       (D)  $\frac{125}{371}$

Q.16 The function

$$f(x, y) = 3(x^2 + y^2) - 2(x^3 - y^3) + 6xy, (x, y) \in \mathbb{R}^2,$$

has

- (A) a point of maxima                      (B) a point of minima  
(C) a saddle point                      (D) NO saddle point

Q.17 The area of the region bounded by  $y = 8$  and  $y = |x^2 - 1|$ , is

- (A)  $\frac{50}{3}$                       (B)  $\frac{100}{3}$                       (C)  $\frac{110}{3}$                       (D)  $\frac{52}{3}$

- Q.18 Let  $X_1, X_2, \dots$  be a sequence of i.i.d.  $U[0, 1]$  random variables. If  $Y_n = \sum_{i=1}^n X_i$ ,  $n=1, 2, \dots$ , then

$$\lim_{n \rightarrow \infty} P\left(Y_n \leq \frac{n}{2} + \sqrt{\frac{n}{12}}\right) =$$

- (A) 0.9413                      (B) 0.7413                      (C) 0.8413                      (D) 0.6413

- Q.19 Let  $\underline{X} = (X_1, X_2)$  have a bivariate normal distribution with

$$E(X_1) = E(X_2) = 0, E(X_1^2) = E(X_2^2) = 1 \text{ and } E(X_1 X_2) = \frac{1}{2}.$$

$$\text{Then } P(X_1 + 2X_2 > \sqrt{7}) =$$

- (A) 0.1587                      (B) 0.5000                      (C) 0.7612                      (D) 0.8413

- Q.20 There are two urns  $U_1$  and  $U_2$ .  $U_1$  contains four white and four black balls, and  $U_2$  is empty. Four balls are drawn at random from  $U_1$  and transferred to  $U_2$ . Then a ball is drawn at random from  $U_2$ . The probability that the ball drawn from  $U_2$  is white is

- (A)  $\frac{1}{3}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{3}{4}$

- Q.21 The integral

$$\int_0^1 \int_{x^2}^{2x} f(x, y) dy dx$$

is equal to

- (A)  $\int_0^1 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy + \int_1^2 \int_{y/2}^1 f(x, y) dx dy$
- (B)  $\int_0^2 \int_y^{y/2} f(x, y) dx dy$
- (C)  $\int_0^1 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy + \int_1^2 \int_y^{2y} f(x, y) dx dy$
- (D)  $\int_0^2 \int_y^{2\sqrt{y}} f(x, y) dx dy$





Q.22 In which case the system of equations

$$\begin{aligned}x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - 5x_2 + 2x_3 &= 2 \\ x_1 + 2x_2 + \lambda x_3 &= \mu\end{aligned}$$

has infinite number of solutions?

- (A)  $\lambda = 1, \mu = -19$  (B)  $\lambda = -1, \mu = 19$   
(C)  $\lambda = 2, \mu = 18$  (D)  $\lambda = 1, \mu = 19$

Q.23 Which of the following differential equations is satisfied by functions  $y_1(x) = e^{(-1+\sqrt{3})x}$  and  $y_2(x) = e^{-2x}$ ?

- (A)  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$   
(B)  $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$   
(C)  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$   
(D)  $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 4y = 0$

Q.24 Let

$$t_n = \frac{1}{n} \left( 1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \right), \quad n = 1, 2, \dots$$

Then

- (A) the series  $\sum_{n=1}^{\infty} t_n$  converges and the sequence  $\{t_n\}$  converges to 0  
(B) the series  $\sum_{n=1}^{\infty} t_n$  converges but the sequence  $\{t_n\}$  does NOT converge to 0  
(C) the series  $\sum_{n=1}^{\infty} t_n$  diverges but the sequence  $\{t_n\}$  converges to 0  
(D) the series  $\sum_{n=1}^{\infty} t_n$  diverges and the sequence  $\{t_n\}$  does NOT converge to 0

Q.25 Let  $\{a_n\}$  be a sequence of positive real numbers such that  $\lim_{n \rightarrow \infty} a_n^{\frac{1}{n}} = \frac{1}{4}$ . Then the function

$$f(x) = \begin{cases} x \sin \frac{1}{x^2}, & x \neq 0 \\ \lim_{n \rightarrow \infty} \frac{\log(1+a_n)}{\sin\left(a_n + \frac{\pi}{2}\right)}, & x = 0 \end{cases}$$

is

- (A) continuous at  $x = 0$  but NOT differentiable at  $x = 0$
- (B) continuous everywhere except at  $x = 0$
- (C) differentiable at  $x = 0$
- (D) nowhere differentiable

Q.26 The value of the limit

$$\lim_{x \rightarrow \frac{1}{2}} \frac{\int_{1/2}^x \cos^2 \pi t \, dt}{\frac{e^{2x}}{2} - e\left(x^2 + \frac{1}{4}\right)}$$

is

- (A) 0
- (B)  $\frac{\pi}{e}$
- (C)  $\frac{\pi^2}{2e}$
- (D)  $-\frac{\pi^2}{2e}$

Q.27 The number of distinct eigenvalues of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Q.28 Let  $X_1, X_2, \dots$  be i.i.d. random variables with common probability density function

$$f(x) = \begin{cases} \frac{1}{2} e^{-\left(\frac{x-1}{2}\right)}, & \text{if } x > 1 \\ 0, & \text{otherwise} \end{cases}$$

Define  $Y_n = \frac{1}{n} \sum_{i=1}^n (X_i - 1)^2, n = 1, 2, \dots$ . If  $\lim_{n \rightarrow \infty} P(|Y_n - k| > \varepsilon) = 0, \forall \varepsilon > 0$ , then  $k =$

- (A) 7                      (B) 9                      (C) 8                      (D) 10

Q.29 Let  $X_1, \dots, X_n$  ( $n > 2$ ) be a random sample from a population with probability density function

$$f(x|\theta) = \frac{\theta}{2} e^{-\theta|x|}, -\infty < x < \infty, \theta > 0.$$

Then a uniformly minimum variance unbiased estimator of  $\theta$  is

- (A)  $\frac{1}{n} \sum_{i=1}^n |X_i|$                       (B)  $\frac{1}{n-1} \sum_{i=1}^n |X_i|$   
 (C)  $\frac{n}{\sum_{i=1}^n |X_i|}$                       (D)  $\frac{n-1}{\sum_{i=1}^n |X_i|}$

Q.30 Let  $\underline{X} = (X_1, X_2)$  have joint probability density function

$$f(x_1, x_2) = \begin{cases} \frac{e^{-\frac{x_2^2}{2}}}{x_2 \sqrt{2\pi}}, & \text{if } 0 < |x_1| \leq x_2 < \infty. \\ 0, & \text{otherwise} \end{cases}$$

Then the variance of random variable  $X_1$  is

- (A)  $\frac{1}{3}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{3}{4}$

Q.31 Let  $S_n$  denote the number of heads obtained in  $n$  independent tosses of a fair coin. Using Chebyshev's inequality, the smallest values of  $n$  such that

$$P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| \leq 0.1\right) \geq \frac{3}{4},$$

is

- (A) 400                      (B) 200                      (C) 300                      (D) 100

- Q.32 Let  $X$  and  $Y$  be independent  $\text{Bin}\left(3, \frac{1}{3}\right)$  random variables. Then the probability that the matrix

$$P = \begin{pmatrix} \frac{X}{\sqrt{2}} & \frac{Y}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

is orthogonal, is

- (A)  $\frac{5}{9}$                       (B)  $\frac{65}{81}$                       (C)  $\frac{4}{9}$                       (D)  $\frac{16}{81}$

- Q.33 Let  $X_1, \dots, X_n$  be a random sample from a population with the probability density function

$$f(x, \alpha) = 3\alpha x^2 e^{-\alpha x^3}, \quad x > 0, \quad \alpha > 0.$$

Then the maximum likelihood estimator of  $\alpha$  is

- (A)  $\frac{n}{\sum_{i=1}^n X_i^3}$                       (B)  $\frac{n}{\sum_{i=1}^n X_i^2}$                       (C)  $\frac{\sum_{i=1}^n X_i^2}{n}$                       (D)  $\frac{\sum_{i=1}^n X_i^3}{n}$

- Q.34 Let  $X_1, \dots, X_n$  be a random sample from a  $U[0, \theta]$  distribution,  $\theta > 0$ . Let  $T_1 = \frac{2}{n} \sum_{i=1}^n X_i$

and  $T_2 = \max(X_1, \dots, X_n)$ . Then which one of the following is NOT a correct statement?

- (A)  $T_1$  is method of moments estimator and  $T_2$  is the maximum likelihood estimator of  $\theta$   
 (B) Both  $T_1$  and  $T_2$  are consistent estimators of  $\theta$   
 (C) Both  $T_1$  and  $T_2$  are unbiased estimators of  $\theta$   
 (D)  $T_1$  is NOT a sufficient statistic, but  $T_2$  is a sufficient statistic

- Q.35 Consider the differential equation

$$x \frac{dy}{dx} = y + \sqrt{\frac{y}{x}}, \quad x > \frac{1}{3}, \quad y > 0.$$

If  $y(1) = 1$ , then  $y(4)$  is

- (A)  $\frac{331}{16}$                       (B)  $\frac{121}{16}$                       (C)  $\frac{9}{16}$                       (D)  $\frac{225}{16}$

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## Part -II: Descriptive Questions

Q. 36 – Q. 43 carry five marks each.

- Q.36 The solid sphere  $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$  is cut into two parts by the plane  $z = \frac{1}{2}$ .  
Find the volume of the smaller part.

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- Q.37 Suppose that 5 distinct balls are distributed at random into 3 distinct boxes in such a way that each of the 5 balls can get into any one of the 3 boxes. Find the probability that exactly one box is empty. Also find the probability that all boxes are occupied.



MS-15/32



Q.38 Let  $X_1, X_2$  be i.i.d. random variables with common probability density function

$$f(x) = \begin{cases} e^{-(x-\theta)}, & \text{if } x > \theta \\ 0, & \text{otherwise} \end{cases}$$

Let  $Y = \min\{X_1, X_2\}$ . Find a confidence interval, for  $\theta$ , of the type

$$[Y - b, Y], \quad 0 \leq b < \infty,$$

having confidence coefficient 0.95.



MS-17/32

- Q.39 Two cars  $A$  and  $B$  are travelling independently in the same direction. The speed of car  $A$  is normally distributed with the mean 100 kilometers per hour and standard deviation  $\sqrt{5}$  kilometers per hour, and the speed of car  $B$  is normally distributed with the mean 100 kilometers per hour and standard deviation 2 kilometers per hour. Initially (at time  $t = 0$ ) the car  $A$  is 3 kilometers ahead of car  $B$ . Find the probability that after 3 hours these two cars will be within a distance of 3 kilometers.

MS-19/32



Q.40 Let the conditional probability density function of  $X$ , given  $Y = y$  ( $y > 0$ ), be given by

$$f(x|y) = \begin{cases} e^{y-x}, & x > y \\ 0, & \text{otherwise} \end{cases}$$

and let  $Y$  have the probability density function

$$g(y) = \lambda e^{-\lambda y}, \quad y > 0, \quad \lambda > 0, \quad \lambda \neq 1.$$

Find the marginal density of  $X$ . Also find the correlation coefficient between  $X$  and  $Y$ .



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Q.41 Let  $X_1, X_2, \dots, X_5$  be a random sample from a population with probability density function

$$f(x|\mu) = \begin{cases} e^{\mu-x}, & \text{if } x \geq \mu, \\ 0, & \text{otherwise} \end{cases}; \mu > 0.$$

Find the likelihood ratio test of size  $\alpha$  for testing  $H_0: \mu \leq 1$  against  $H_1: \mu > 1$ .



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Q.42 Solve the differential equation

$$\frac{dy}{dx} - \left(\frac{1}{x} + 3x^2\right)y = xy^2, \quad y > 0, 0 < x \leq 1$$

with the condition  $y(1) = \frac{3}{2}$ .



MS-25/32



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Q.43 Expand the function  $f(x, y) = \sin(x + y)$ ,  $(x, y) \in \mathbb{R}^2$ , into its Taylor's series about the point  $\left(0, \frac{\pi}{2}\right)$  having terms up to second degree.

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**Space for rough work**

**MS-28/32**



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**MS-32/32**

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Question Number	Marks	
36		
37		
38		
39		
40		
41		
42		
43		
<b>Total Marks in the Subjective Part</b>		

<b>Total Marks (in words)</b>	:	
<b>Signature of Examiner(s)</b>	:	
<b>Signature of Head Examiner(s)</b>	:	
<b>Signature of Scrutinizer</b>	:	
<b>Signature of Chief Scrutinizer</b>	:	
<b>Signature of Coordinating Head Examiner</b>	:	

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