MATHEMATICS (FINAL)

1. The system of equations $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

A. has no solution

B. has one and only one solution

C. has infinite number of solutions

D. None of the above

2. If a and b are real numbers, then $\sup\{a,b\} =$

A. $\dfrac{a+b-|a-b|}{2}$

B. $\dfrac{a-b-|a-b|}{2}$

C. $\dfrac{a-b+|a+b|}{2}$

D. $\dfrac{a+b+|a-b|}{2}$

3. If, for $x \in \mathbb{R}$, $\varphi(x)$ denotes the integer closest to x (if there are two such integers take the larger one), then $\int_{10}^{12} \varphi(x)\,dx$ equals

A. 22

B. 11

C. 20

D. 12

4. The eigen values of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ are

A. 1,1,1

B. 2,2,2

C. –2, –2, –2

D. –1, –1, –1

5. The value of $\sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n}$ is

A. log 2

B. e^2

C. 0

D. 1

6. The rank of the matrix $\begin{pmatrix} 0 & 0 & 0 & 0 & -2 \\ 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 13 \end{pmatrix}$ is

A. 5

B. 4

C. 3

D. 2

7. $\sum_{n=1}^{\infty} \frac{2^n}{(n-1)!}$ is

A. e

B. $2e$

C. $2e^2$

D. $2e^{-2}$

8. If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$, and $|A^2|=125$, then the value of α is

A. ± 5

B. ± 3

C. ± 2

D. ± 1

9. The algebraic equation $x^n + y^n = z^n$ has no integer solution when

A. $n>2$

B. $n=2$

C. $n=0$

D. $n<2$

10. Which of the function $f:R\to R$ is one-one and onto?

A. $f(x)=x^3+2$

B. $f(x)=\sin x$

C. $f(x)=\cos x$

D. $f(x)=x^4-x^2$

11. Let $P(x)$ be a non-constant polynomial such that $P(n)=P(-n)$ for all $n\in N$. Then $P'(0)$ is

A. -1

B. 1

C. 0

D. -2

12. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $A^2-5A+7I$ is equal to

A. $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

B. $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

C. $2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

D. None of the above

13. If A is any non-identity 3×3 matrix such that $A^2=A$, then A is

A. Singular

B. Non singular

C. Regular

D. None of the above

14. If every cross-section of a bounded surface in three dimensions is a circle, then the surface must be a

A. cylinder

B. sphere

C. cone

D. third-degree surface

15. For any complex number z, the minimum value of $|z|+|z-1|$ is

A. 1

B. 0

C. 1/2

D. 3/2

16. The number having a recurring decimal representation 1.414141… is

A. real but irrational

B. not real

C. rational

D. neither rational nor real

17. The derivative with respect to x of the product $(1+x)(1+x^2)(1+x^4)(1+x^8)+....+(1+x^{56})$ at $x=0$ is

A. 0

B. 1

C. 8

D. 56

18. If α, β and γ are roots of the equation $x^3+px^2+qx+r=0$, then $\alpha^2+\beta^2+\gamma^2$ is equal to

A. p^2-2q

B. p^2+2q

C. $2p+q^2$

D. $2p-q^2$

19. Given that $2+i\sqrt{3}$ is one of $x^3-5x^2+11x-7=0$ then the other roots are

A. $2-i\sqrt{3},-1$

B. $2-i\sqrt{3},1$

C. $2+i\sqrt{3},1$

D. None of the above

20. The quadratic function on a single variable attains its maximum value 5 at 3. The function is

A. $ax^2 - 6ax + 9a + 5,\ a < 0$

B. $ax^2 - 6ax + 9a + 5,\ a > 0$

C. $ax^2 + 6ax + 9a + 5,\ a < 0$

D. None of the above

21. Let n be a two digit number. $P(n)$ is the product of the digits of n and $S(n)$ is the sum of the digits of n. If $n = P(n) + S(n)$ then the unit digit of n is

A. 1

B. 5

C. 7

D. 9

22. The value of $\sqrt{20 + \sqrt{20 + \sqrt{20 + \ldots}}}$ is

A. – 4

B. 5

C. 4

D. – 5

23. If A, B, C are three sets with cardinality m, p, q respectively such that $B \cap C = \phi$, then the cardinality of $(A \times B) \cup (A \times C)$ equals

A. mpq

B. $m(p+q)$

C. $m(p+q-pq)$

D. $m + pq$

24. Which among the following is not a group under usual multiplication?

A. ¡ $-\{0\}$

B. ¤ $-\{0\}$

C. ¤ $^{+}$

D. ¥

25. If $\log_{27} x = \log_3 27,$ then x is

A. 27

B. 3

C. 3^{27}

D. 27^3

26. If ϕ is a homomorphism of a group G onto a group $\overline{G}$ with kernel K, then G/K is isomorphic to

A. G

B. $\overline{G}$

C. $\overline{G}/\overline{K}$

D. None of the above

27. If G is a group and for $a \in G$ no positive integer m exists such that $a^m = e$, then the order of a is

A. finite

B. m

C. $m + 1$

D. infinite

28. The derivative of e^t with respect to $\sqrt{t}$ is

A. $\dfrac{e^t}{2\sqrt{t}}$

B. $\dfrac{2\sqrt{t}}{e^t}$

C. $2\sqrt{t}e^t$

D. $2\sqrt{te^t}$

29. $\int_0^1\int_0^1 \dfrac{1}{\sqrt{(1-x^2)}\sqrt{(1-y^2)}}\,dx\,dy$, is equal to

A. $\dfrac{\pi^2}{2}$

B. $\dfrac{\pi^2}{3}$

C. $\dfrac{\pi^2}{4}$

D. None of the above

30. $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x dx$ is equal to

A. $\dfrac{\pi}{8}$

B. $\dfrac{\pi}{16}$

C. $\dfrac{\pi}{32}$

D. 1

31. $\int_0^{\infty} \frac{1}{\left(1+x^2\right)^4} dx$ is equal to

A. $\frac{-5\pi}{32}$

B. $\frac{5\pi}{16}$

C. $\frac{5\pi}{32}$

D. $\frac{-5\pi}{16}$

32. $\int_0^{\frac{\pi}{6}} \cos^2 \frac{\pi}{2}$ is equal to

A. $\frac{\pi}{12} - \frac{1}{4}$

B. $\frac{\pi}{12} + \frac{1}{4}$

C. $\frac{1}{4} - \frac{\pi}{2}$

D. $\frac{1}{4} + \frac{\pi}{2}$

33. The series $\sum \frac{1}{n(\log n)^p}$ is divergent if

A. $p > 1$

B. $p \leq 1$

C. $p < 1$

D. $p = 1$

34. A non-decreasing sequence which is bounded above is

A. divergent

B. convergent

C. oscillating

D. unbounded

35. Let $f :$ ¡ $\rightarrow$ ¡ be defined by

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x > 0 \end{cases}$$

If f is continuous for all x, then the value of a is

A. 0

B. 4

C. 8

D. 5

36. The cardinal number of the empty set is

A. 1

B. 0

C. ∞

D. -1

37. Every closed and bounded set in R^n is

A. empty

B. open

C. compact

D. convex

38. In which subspace the sequence $\left\{\frac{1}{n}\right\}$ is Cauchy but not convergent

A. $[0,1]$

B. $[0,1)$

C. $(0,1]$

D. $(0,1)$

39. The collection of open intervals $\left(\frac{1}{n}, \frac{2}{n}\right)$, $n = 1, 2, 3, ...$ is an open covering of the interval

A. $(0,1)$

B. $(1,2)$

C. $(0,\infty)$

D. $(1,\infty)$

40. The set of all rational numbers is a

A. empty set

B. finite set

C. countable set

D. uncountable set

41. $\int_0^{2\pi} \frac{dx}{(2+\cos x)}$ is equal to

A. $\frac{2\pi}{\sqrt{3}}$

B. $\frac{\pi}{\sqrt{3}}$

C. $\frac{2\pi}{-3}$

D. None of the above

42. The whole length of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is

A. $6a$

B. $8a$

C. $4a$

D. None of the above

43. The condition for the point (x, y) to lie on the straight line joining the points $(0, b)$ and $(a, 0)$ is

A. $\frac{x}{a} + \frac{y}{b} = 1$

B. $\frac{x}{a} - \frac{y}{b} = 1$

C. $\frac{x}{a^2} + \frac{y}{b^2} = 1$

D. None of the above

44. The centroid of the triangle whose vertices are $(2, 4, -3)$, $(-3, 3, -5)$ and $(-5, 2, -1)$ is

A. $(-2, -3, -3)$

B. $(-3, 3, -2)$

C. $(3, -2, -3)$

D. $(-2, 3, -3)$

45. The triangle whose vertices are $(-1, 1, 0)$, $(3, 2, 1)$ and $(1, 3, 2)$ is

A. isosceles triangle

B. right-angle triangle

C. equilateral triangle

D. None of the above

46. The coordinates of the point at which the line joining the points $(4,3,1)$ and $(1,-2,6)$ meets the plane $3x-2y-z+3=0$ is

A. $(-2,-7,11)$

B. $(-2.7,11)$

C. $(-2,-7,-11)$

D. $(2,7,11)$

47. The center of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$ is

A. $(5,-4,3)$

B. $(3,4,-5)$

C. $(-5,-4,-3)$

D. $(3,-4,5)$

48. The equation of the right circular cone with its vertex at the origin, axis along z-axis and semi-vertical angle α is

A. $x^2+y^2=z^2\tan^2\alpha$

B. $x^2-y^2=z^2\tan^2\alpha$

C. $x^2+y^2=z\tan^2\alpha$

D. $x^2-y^2=z\tan^2\alpha$

49. $\nabla\times(\nabla\times A)$ is equal to

A. 0

B. $-\nabla^2 A+\nabla(\nabla.A)$

C. $\nabla^2 A+\nabla(\nabla.A)$

D. $(\nabla\times\nabla)\times A$

50. The curl of $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ at $(1,2,-3)$ is

A. $\vec{i} + 2\vec{j} - 3\vec{k}$

B. $\vec{i} - 2\vec{j} + 3\vec{k}$

C. $2\vec{i} - 4\vec{j} + 6\vec{k}$

D. $2\vec{i} + 4\vec{j} - 6\vec{k}$

51. Let $F = \dfrac{-yi + xj}{x^2 + y^2}$. Then $\nabla \times F$ is

A. 0

B. 1

C. – 1

D. None of the above

52. The probability mass function or probability density function for which the mean in units and the variance in square units are same is

A. Binomial

B. Poisson

C. Standard Normal

D. Geometric

53. The chance of getting at least 9, in a single throw with two dice is

A. $\dfrac{4}{36}$

B. $\dfrac{3}{36}$

C. $\dfrac{5}{18}$

D. $\dfrac{1}{18}$

54. A random variable X has a probability density function $f(x) = \frac{C}{1+x^2}$, $-\infty < x < \infty$. Then the value of C is

A. π

B. 1

C. $\frac{1}{\pi}$

D. $\frac{2}{\pi}$

55. If $f(z)$ and $f(\bar{z})$ are analytic in a region D, then

A. $f(z)$ is constant in D

B. $f(z)$ is continuous in D

C. $f(z)$ is not differentiable everywhere in D

D. $f(z) = |z|$

56. The value of $\int_C \frac{z^2+1}{z^2-1}\, dz$, where C is a circle of unit radius with center at $z = 1$ is

A. 0

B. $2\pi i$

C. $-2\pi i$

D. 1

57. If $(x+iy)^{1/3} = a+ib$, then the value of $\frac{x}{a}+\frac{y}{b}$ is

A. $4\left(a^2-b^2\right)$

B. $4ab$

C. $4\left(a^2+b^2\right)$

D. $5ab$

58. The value of $(1+i)\left(1+i^2\right)\left(1+i^3\right)\left(1+i^4\right)......\left(1+i^{50}\right)$ is

A. 50

B. 1

C. 0

D. i

59. If $\omega=\frac{z}{z-i}$ and $|\omega|=1$, then z lies on

A. a circle

B. an ellipse

C. a parabola

D. a straight line

60. The residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1$ is

A. –8

B. 1/2

C. –6

D. 0

61. If $u(x, y) = xy$ is a harmonic function, a harmonic conjugate of u is

A. $\frac{x^2}{2}$

B. $x^2 + y^2$

C. $\frac{x^2}{2} + \frac{y^2}{2}$

D. $\frac{y^2}{2} - \frac{x^2}{2}$

62. The solution of a homogeneous initial value problem with constant coefficient is $y = 3xe^{2x} + 6\cos 4x$. Then the least possible order of the differential equation is

A. 4

B. 5

C. 6

D. 3

63. The differential equation of the curve $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is

A. $\frac{dy}{dx} = x$

B. $\frac{dy}{dx} = -x$

C. $\frac{dy}{dx} = y$

D. $\frac{dx}{dy} = y$

64. The differential equation of all circles which pass through the origin and whose centers are on the x-axis is

A. $y^2 + x^2 - 2xy\frac{dy}{dx} = 0$

B. $x^2 - y^2 + 2x\frac{dy}{dx} = 0$

C. $y^2 - x^2 - 2xy\frac{dy}{dx} = 0$

D. $y^2 + 2x\frac{dy}{dx} = 0$

65. The correlation coefficient lies between

A. 0 and 1

B. –1 and 0

C. –1 and 1

D. None of the above

66. The average monthly production of a factory for the first 8 months is 2500 units, the next 4 months is 1200 units. The average monthly production of the year will be

A. 2066.55

B. 5031.10

C. 4021.10

D. 3012.11

67. The equation of the curve that passes through the point $(1,1)$ and has at every point the slope $\frac{-y}{x}$ is

A. $xy = 1$

B. $xy = -1$

C. $xy = 2$

D. $xy = -2$

68. The solution of the differential equation $\frac{d^2y}{dx^2}+-3\frac{dy}{dx}+2y=0,$ subject to initial conditions $y(0)=0, y'(0)=1$ is

A. e^x+e^{2x}

B. e^x-e^{2x}

C. $-e^x+e^{2x}$

D. None of the above

69. The general solution of the differential equation $\frac{d^2y}{dx^2}+y=x$ is

A. $\sin x+\cos x+x$

B. $\sin x+\cos x+1$

C. $\sin x-\cos x+x$

D. $\sin x-\cos x-1$

70. The partial differential equation $u_t=ku_{xx}$ is

A. one dimension wave equation

B. elliptic

C. one dimension heat equation

D. not parabolic

71. The partial differential equation $(u_{xx})^2+u_{yy}+a(x,y)u_x+b(x,y)u-4e^x=0$ is

A. linear

B. quasilinear

C. nonlinear

D. homogeneous

72. The solution of the IVP $\frac{dy}{dx} = x^2 y - 3x^2, y(0) = 1$ is $y =$

A. $3 + ce^{x^3/3}, c$ is a constant

B. $3 - 2e^{x^3/3}$

C. $3 + 3e^{x^3/3}$

D. $3 - 2e^{x^3}$

73. If $q(x) \leq 0$, then any nontrivial solution of $y'' + q(x)y = 0$

A. can have more than one zero

B. can have at most one zero

C. cannot have any zero

D. has exactly one zero

74. A solution of the partial differential equation $u_t + cu_x = 0$ is $u(x,t) =$

A. $\sin(x - t)$

B. $\cos(x - ct)$

C. $\cos(cx - t)$

D. $\cos xt$

75. The general solution of the partial differential equation $\frac{\partial x \partial x}{\partial x \partial y} = xy$ is

A. $z = a\frac{x^2}{2} - \frac{y^2}{2a} + b$

B. $z = a\frac{x^2}{2} + \frac{y^2}{2a} - b$

C. $z = a\frac{x^2}{2} + \frac{y^2}{2a} + b$

D. $z = a\frac{x^2}{2} - \frac{y^2}{2a} - b$

76. The function $f:(-1,1) \to \mathbb{R}$ defined by $f(x) = \frac{x}{1-|x|}$ is

A. one-one but not onto

B. not onto

C. one-one and onto

D. neither one-one nor onto

77. For each $n \in \mathbb{N}$, let $a_n = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k}$. Then the sequence (a_n) is

A. not a Cauchy sequence

B. a convergent sequence

C. not a bounded sequence

D. convergent to 0

78. If a function f is continuous in $[a,b]$ and differentiable (a,b), then there exists at least one number $\theta \in (a,b)$ such that $f(a+h) = f(a) + hf'(a+\theta h)$. It is the statement of the

A. Roll's Theorem

B. Lagrange's Mean Value Theorem

C. Cauchy's Mean Value Theorem

D. Taylor's Mean Value Theorem

79. The set of all polynomials with rational coefficients

A. is not countable

B. is finite

C. does not contain Q

D. countable

80. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$.

Then at the point 0

A. f is continuous

B. f is not continuous

C. f is differentiable

D. None of the above

81. $\lim_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ is equal to

A. 3

B. 2

C. 1

D. 0

82. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases} x-1 \text{ if } x\geq 1 \\ 1-x \text{ if } x<1 \end{cases}$

Then at the point 1

A. f is continuous

B. f is not continuous

C. f is differentiable

D. None of the above

83. The smallest number with 18 divisors is

A. 360

B. 175

C. 185

D. 180

84. The value of the $\int_{|z|=2} \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2} dz$

A. –1

B. 1

C. 0

D. π

85. $\left(\frac{1+i}{\sqrt{2}}\right)^4$ is equal to

A. 1

B. 0

C. $\sqrt{2}$

D. –1

86. The set $\left\{z \in \pounds : |z - 3i| = 3\right\}$ geometrically represents a

A. Parabola

B. Circle

C. Ellipse

D. Hyperbola

87. The bilinear transformation which maps the points $z_1 = 0,\ z_2 = -i$ and $z_3 = -1$ into $w_1 = i,\ w_2 = 1$ and $w_3 = 0$ respectively is

A. $\frac{-i(z+1)}{z-1}$

B. $\frac{-i(z-1)}{z+1}$

C. $\frac{-1(z+i)}{z-i}$

D. $\frac{(z+i)}{z-i}$

88. The first two terms of the Laurent's series of $f(z) = \frac{z}{(z-1)(2-z)}$, valid for $|z| > 2,$ is

A. $\frac{1}{z} - \frac{3}{z^2}$

B. $-\frac{1}{z} - \frac{3}{z^2}$

C. $\frac{1}{z} + \frac{3}{z^2}$

D. None of the above

89. Z_6 is

A. a ring

B. an integral domain

C. a field

D. None of the above

90. The set of all square matrices of order 2 over the set of real number is

A. Ring

B. Integral Domain

C. Field

D. None of the above

91. Let S be the set of functions $f : \mathbb{R} \to \mathbb{R}$ which are solutions to the differential equation $f''' + f' - 2f = 0$. Then S is

A. not a vector space

B. a vector space of dimension greater than 3

C. a vector space of dimension 3

D. a vector space of dimension less than 3

92. The dimension of the subspace $W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0\}$

A. 1

B. 2

C. 3

D. 4

93. Let W be the subspace spanned by
$S = \{(1,0,0,0),(0,1,0,0),(1,1,0,0),(1,1,1,0),(2,0,3,0)\}$.
Then the dimension of W is

A. 4

B. 5

C. 2

D. 3

94. The number of subsets (including the empty subset and the whole set) for a set of n elements is

A. n

B. n^2

C. n^n

D. 2^n

95. Let G be the complete graph on n vertices. Then the number of edges in G is

A. n

B. n^2

C. $2n$

D. $\frac{n(n-1)}{2}$

96. For $n \geq 4$, let G be a graph with n vertices and n edges. Then

A. G is a star

B. G should contain a cycle

C. G is acyclic

D. G is a complete graph

97. If Z is the optimal solution of a LPP and Z' is the optimal solution of its dual, then

A. $Z < Z'$

B. $Z > Z'$

C. $Z \neq Z'$

D. $Z = Z'$

98. The differential equation obtained by eliminating f from $z = f\left(x^2 + y^2\right)$ when $p = \frac{\partial f}{\partial x}$ and $q = \frac{\partial f}{\partial y}$ is

A. $py = qx$

B. $pq = xy$

C. $px = qy$

D. $x = y$

99. The differential equation of the family of curves $y = e^{2x}\left(A\cos x + B\sin x\right)$ where A and B are constants is

A. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

B. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

C. $\frac{d^2y}{dx^2} - 4y\frac{dy}{dx} + 5y = 0$

D. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4x = 0$

100. Let $\left(y - c\right)^2 = cx$ be the primitive of the differential equation $4x\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$. Then number of integral curve(s) which will pass through $\left(1, 2\right)$ is

A. one

B. two

C. three

D. four

101. If y_1 and y_2 are two independent solutions of the differential equation $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$, where $P(x)$ and $Q(x)$ are continuous functions of x, then which of the following is true?

A. $y_1y_1^{'}-y_2y_2^{'}=Ce^{-\int Pdx}$

B. $y_1y_2^{'}-y_2y_1^{'}=Ce^{-\int Pdx}$

C. $y_1y_1^{'}+y_2y_2^{'}=Ce^{\int Pdx}$

D. $y_1y_2^{'}+y_2y_1^{'}=Ce^{\int Pdx}$

102. Let the chances of solving a problem given to three students are $\frac{1}{2},\frac{1}{3},\frac{2}{5}$. The probability that the problem will be solved is

A. $\frac{1}{15}$

B. $\frac{1}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

103. To measure flatness or peakedness of a distribution we use

A. skewness

B. kurtosis

C. correlation

D. standard deviation

104. A method for solving linear programming problem without using artificial variables is

A. Two-phase method

B. Big-M method

C. Dual simplex method

D. Revised simplex method

105. In an assignment problem of order n, in the reduced cost matrix, the minimum number of lines needed to cover all zeros will be

A. n

B. $n-1$

C. $n+1$

D. $(n-1)(n+1)$

106. The angle between two forces of equal magnitude P when their resultant also has the same magnitude P is

A. 120^{o}

B. 60^{o}

C. 30^{o}

D. 45^{o}

107. The horizontal range of the projectile is maximum when the particle is projected at an angle of

A. 90^{o} to the horizontal

B. 30^{o} to the horizontal

C. 60^{o} to the vertical

D. 45^{o} to the vertical

108. The period of oscillation of a simple pendulum is

A. $2\pi\sqrt{\frac{g}{l}}$

B. $\frac{\sqrt{2\pi l}}{g}$

C. $\sqrt{\frac{2\pi l}{g}}$

D. $2\pi\sqrt{\frac{l}{g}}$

109. The series $x + x^3/3! + x^5/5! + \ldots$ represents the function

A. $\cos x$

B. $\cosh x$

C. $\sin x$

D. $\sinh x$

110. The value of $\frac{1}{2}\left(\log(1+x) - \log(1-x)\right)$ equals

A. $\tan x$

B. $\tanh x$

C. $\tan^{-1} x$

D. $\tanh^{-1} x$

111. The value of $p(4)$, where p is the number of possible partitions of 4 is

A. 10

B. 5

C. 4

D. 1

112. Let $f(x) \in R[x] = \{a_0 + a_1 x + ... + a_n x^n : a_i \in R \text{ a ring}\}$, where n is a non-negative integer. If $f(a) = f'(a) = 0$, then $(x-a)^2$ divides

A. $f(x)$

B. $f'(x)$

C. $f(x) - a$

D. $f'(x) - a$

113. Let G be a graph with $n > 2$ vertices. If all the n vertices in G form a cycle, then the degree of any vertex in G is

A. 1

B. 2

C. $n-1$

D. n

114. Consider the graph G with 6 vertices given by $v_1, v_2, v_3, v_4, v_5, v_6$ and edges a, b, c, d, e, f, g, h. Then which among the following is a possibility for a path?

A. $v_1\, a\, v_2\, b\, v_3\, c\, v_3\, d\, v_4\, e\, v_2\, f\, v_5$

B. $v_2\, b\, v_3\, d\, v_4\, e\, v_2\, a\, v_1$

C. $v_1\, a\, v_2\, b\, v_3\, d\, v_4$

D. $v_2\, b\, v_3\, d\, v_4\, h\, v_5\, f\, v_2$

115. The maximum possible number of edges in a simple graph with n vertices and 2 components is

A. $(n-1)(n-2)/2$

B. $n(n-1)/2$

C. $n(n+1)/2$

D. $n^2/2$

116. A Hamiltonian circuit is possessed by every graph with three or more vertices if it is

A. connected

B. a tree

C. simple

D. complete

117. The rank of incidence matrix $A(G)$ of a disconnected graph G with n vertices and k components is

A. k

B. $k-1$

C. $n-k$

D. $n-k+1$

118. Let T be a tree with four vertices v_1, v_2, v_3, v_4. Then the possible number of paths between the vertices v_1 and v_4 is

A. atleast two

B. atleast one

C. exactly one

D. exactly two

119. For a graph G, both the incidence matrix $A(G)$ and adjacency matrix $X(G)$ contain the entire information about G if,

A. G has no self-loops

B. G has no parallel edges

C. G has self-loops but no parallel edges

D. G is simple

120. Which of the integrals does not have a definite value?

A. $\int_a^\infty \sin x \, dx,\ a > 0$

B. $\int_1^\infty \frac{1}{x^2} dx$

C. $\int_{-\infty}^0 e^x \, dx$

D. $\int_0^1 \frac{1}{\sqrt{x}} dx$

121. If p is a prime number, then $(p-1)!+1$ is

A. an odd number

B. an even number

C. a prime number

D. divisible by p

122. The curvature of a circle or radius r is

A. $\frac{1}{r^2}$

B. $-\frac{1}{r^2}$

C. r^2

D. $\frac{1}{r}$

123. The area enclosed by the curve $|x|+|y|=1$ is

A. 1

B. $\sqrt{2}$

C. 2

D. 4

124. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then the value of $C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n$ is

A. $(n+2)2^{n-1}$

B. $(n+2)2^n$

C. $(n+1)2^{n-1}$

D. $(n+1)(n+2)2^n$

125. The value of $\varsigma(4)$ is

A. $\pi^2/12$

B. $\pi^4/20$

C. $\pi^4/90$

D. π

126. The area under one arc of the cycloid $x = a(\theta - \sin\theta),\ y = a(1-\cos\theta)$ is

A. $\dfrac{\pi a^2}{8}$

B. $\dfrac{3\pi a^2}{16}$

C. $3\pi a^2$

D. $\dfrac{3\pi a^2}{32}$

127. The area between the parabola $y^2 = 4ax$ and the line $y = x$ is

A. $\frac{3a^2}{8}$

B. $\frac{8a^2}{3}$

C. $\frac{a^2}{8}$

D. $\frac{5a^2}{8}$

128. If $u = \tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

A. $\frac{2xy}{x^2 + y^2}$

B. 1

C. 0

D. $\frac{x^2}{x^2 + y^2}$

129. The velocity of a particle moving in a straight line is given by $v^2 = se^8$, where s is the displacement in time t. The acceleration of the particle is given by

A. $(1+s)v/2$

B. $v^2/2(1+s)$

C. $v/2(s-1)$

D. $v/2(1+1/s)$

130. The directional derivative of $f(x,y) = 2x^2 + 3y^2 + z^2$ at point $(2,1,3)$ in the direction $\vec{i} - 2\vec{k}$ is

A. $4/\sqrt{5}$

B. $-4/\sqrt{5}$

C. $\sqrt{5}/4$

D. $-\sqrt{5}/4$

131. The value of $\iiint xyz\ dx\ dy\ dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ is

A. $\frac{a^3}{48}$

B. $\frac{a^6}{48}$

C. $\frac{a^6}{8}$

D. $\frac{a^5}{48}$

132. If Σ is the boundary surface of a three dimensional region V having ς as the unit vector along the exterior normal to the surface Σ and y is a vector function, then $\iiint_V \nabla . y dV = \iint_\Sigma y . \varsigma d\Sigma$ is called

A. Stokes theorem

B. Gauss theorem

C. Green's theorem

D. Cauchy's theorem

133. The vector equation of a sphere one of whose diameters has the extremities with the position vectors a and b is

A. $(r-a)\times(r-b)=0$

B. $(r+a)\times(r+b)=0$

C. $(r+a).(r+b)=0$

D. $(r-a).(r-b)=0$

134. Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S with $\hat{n}$ the unit normal on S, the value of the integral $\iint 5\vec{r}.\hat{n}dS$ is

A. $3V$

B. $5V$

C. $10V$

D. $15V$

135. The dimension of the vector space V of all polynomials in $\mathbb{R}[x]$ of degree ≤ 20 is given by

A. 21

B. 20

C. 10

D. 8

136. If $A=(a_{ij})$ is an $n\times n$ matrix defined over a field F, then trace of A is

A. 0

B. I

C. $\sum_{i=1}^{n} a_{ij}$

D. $\sum_{ij=1}^{n} a_{ij}$

137. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, then $F(x)F(y)=$

A. $F(xy)$

B. $F(x)+F(y)$

C. $F(x+y)$

D. $F(x-y)$

138. If $A=\begin{bmatrix}1 & 0 & 0\\ i & \frac{-1+i\sqrt{3}}{2} & 0\\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2}\end{bmatrix}$, then the trace of A^{102} is

A. 0

B. 1

C. 2

D. 3

139. If the matrix $A=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ x & -2 & 2\end{bmatrix}$ is singular, then the value of x is

A. 6

B. 5

C. 3

D. 2

140. If V and W are vector spaces of dimensions m and n respectively over a field F, then the dimension of the vector space of all homomorphisms of V into W is

A. $m+n$

B. $m-n$

C. mn

D. m/n

141. The eigen values of a real symmetric matrix are always

A. positive

B. imaginary

C. real

D. complex conjugate pairs

142. If A is an $n \times n$ matrix with diagonal entries a and other entries b, then one eigen value of A is $a-b$. Another eigen value of A is

A. $b-a$

B. $nb+a-b$

C. $nb-a+b$

D. 0

143. What is the degree of the first order forward difference of a polynomial of degree n?

A. n

B. $n-1$

C. $n-2$

D. $n+1$

144. The rate of convergence of bisection method is

A. linear

B. faster than linear but slower than quadratic

C. quadratic

D. cubic

145. The set ¥ of natural numbers where $x * y = \max\{x, y\}$ is a

A. ring

B. complete lattice

C. semi group

D. field

146. In a three dimensional Euclidean space, the direction cosines of a line which is equally inclined to the axes are

A. 1, 1, 1

B. $1/3, 1/3, 1/3$

C. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$

D. $1/2, 1/2, 1/2$

147. In two dimensions, the distance between the origin and the centroid of the triangle joining the points $(-1,0)$, $(4,0)$ and $(0, 3)$ is

A. 0

B. 1

C. $\sqrt{2}$

D. $\sqrt{3}$

148. Let P be the point $(1,0)$ and Q a point on the locus $y^2 = 4x$. Then the locus of mid point of PQ is

A. $y^2 + 2x + 1 = 0$

B. $y^2 - 2x + 1 = 0$

C. $x^2 - 2y + 1 = 0$

D. $x^2 + 2y + 1 = 0$

149. The Laplace transform of $\frac{\sin at}{at}$ is

A. $\tan\left(\frac{a}{s}\right)$

B. $\tan^{-1}\left(\frac{a}{s}\right)$

C. $\tan^{-1}\left(\frac{s}{a}\right)$

D. $\tan\left(\frac{s}{a}\right)$

150. The inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$ is

A. $e^{2t}\cos 3t$

B. $\frac{4}{3}e^{2t}\sin 3t$

C. $e^{2t}\cos 3t+\frac{4}{3}e^{2t}\sin 3t$

D. $e^{2t}\cos 3t-\frac{4}{3}e^{2t}\sin 3t$

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