31/01/2023

Morning

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Memory Based
Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The ratio of molar specific heat capacity at constant pressure (C_P) to that at constant volume (C_V) varies with temperature (T) as : [Assume temperature to be low]

(1) T^0 (2) $T^{1/2}$

(3) T^1 (4) $T^{3/2}$

Answer (1)

Sol. $\frac{C_P}{C_V} = \frac{f+2}{f} = \nu = 1 + \frac{2}{f} = \text{constant}$

We take 'f' to be constant for molecule at low temperature.

$\boxed{\frac{C_P}{C_V} \propto T^0}$

2. If n : Number density of charge carriers

A : Cross-sectional area of conductor

q : Charge on each charge carrier

I : Current through the conductor

then the expression of drift velocity is

(1) $\frac{nAq}{I}$ (2) $\frac{I}{nAq}$

(3) $nAqI$ (4) $\frac{IA}{nq}$

Answer (2)

Sol. We Know $I = nAe\,v_d$

$\Rightarrow v_d = \frac{I}{nAq}$

3. If R, X_L and X_C denote resistance, inductive reactance and capacitive reactance respectively. Then which of the following options shows the dimensionless physical quantity.

(1) $\frac{X_L X_C}{R}$ (2) $\frac{R}{\sqrt{X_L X_C}}$

(3) $\frac{R}{X_L X_C}$ (4) $\frac{R}{(X_L X_C)^2}$

Answer (2)

Sol. X_L = Inductive reactance = [R] = dimension of R

X_C = Capacitive reactance = [R] = dimension of R

R = Resistance

$\frac{R}{\sqrt{X_L X_C}}$ = dimensionless

4. A drop of water of 10 mm radius is divided into 1000 droplets. If surface tension of water surface is equal to 0.073 J/m^2 then increment in surface energy while breaking down the bigger drop in small droplets as mentioned is equal to

(1) 8.25×10^{-5} J (2) 9.17×10^{-4} J

(3) 9.17×10^{-5} J (4) 8.25×10^{-4} J

Answer (4)

Sol. Let the radius of one small droplet is r then

$1000\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3$

$\Rightarrow r = 1$ mm

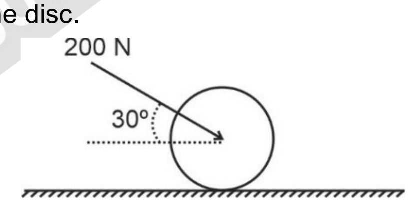
$U_f = 1000\ 4\pi r^2\,T = 1000 \times 4\pi \times 10^{-6} \times 0.073$

$= 9.17 \times 10^{-4}$ J

$U_i = 4\pi \times (10^{-2})^2\,T = 9.17 \times 10^{-5}$ J

So $\Delta U = 8.25 \times 10^{-4}$ J

5. A force 200 N is exerted on a disc of mass 70 kg as shown. Find the normal reaction given by ground on the disc.

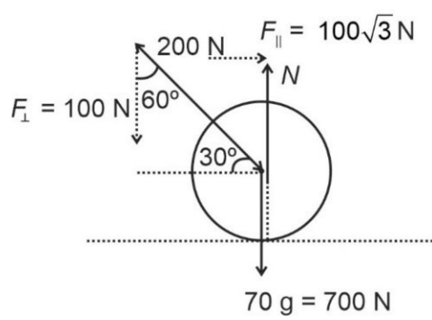


(1) 200 N (2) 600 N

(3) 800 N (4) $\frac{200}{\sqrt{3}}$N

Answer (3)

Sol.



$N = Mg + F_{\perp}$

$= 700 + 100$

$= 800$ N

6. At depth d from surface of earth acceleration due to gravity is same as its value at height d above the surface of earth. If earth is a sphere of radius 6400 km, then value of d is equal to

(1) 2975 km (2) 3955 km

(3) 2525 km (4) 4915 km

Answer (2)

Sol. $g_0\left(1-\frac{d}{R}\right)=\frac{g_0}{\left(1+\frac{d}{R}\right)^2}$

$$\left(1-\frac{d}{R}\right)\left(1+\frac{d}{R}\right)^2=1$$

On solving

$$\frac{d}{R}=0,\ -\left(\frac{\sqrt{5}+1}{2}\right),\ \frac{\sqrt{5}-1}{2}$$

So, $d=\frac{\sqrt{5}-1}{2}R$

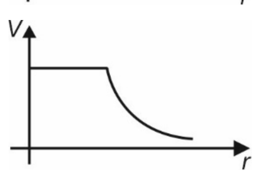
$\Rightarrow d$ = 3955 km

7. Which of the following graphs depicts the variation of electric potential with respect to radial distance from centre of a conducting sphere charged with positive charge.

(1)

(2)

(3)

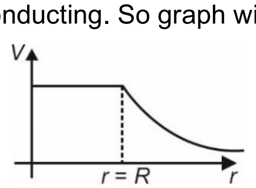


(4)

Answer (3)

Sol. $V(r)=\begin{cases}\dfrac{q}{4\pi\varepsilon_0 R} & \text{if } r<R\\[2mm] \dfrac{q}{4\pi\varepsilon_0 r} & \text{if } r>R\end{cases}$

Where r is radial distance and R is radius of sphere, as charge will be on the surface because the sphere is conducting. So graph will be



8. In a sample of hydrogen atoms, one atom goes through a transition n = 3 → ground state with emitted wavelength λ_1. Another atom goes through a transition n = 2 → ground state with emitted wavelength λ_2. Find $\frac{\lambda_1}{\lambda_2}$.

(1) $\frac{6}{5}$ (2) $\frac{5}{6}$

(3) $\frac{27}{32}$ (4) $\frac{32}{27}$

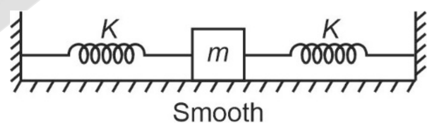
Answer (3)

Sol. $\frac{1}{\lambda_1}=RZ^2\left[1-\frac{1}{9}\right]$

$$\frac{1}{\lambda_2}=RZ^2\left[1-\frac{1}{4}\right]$$

$$\Rightarrow\frac{\lambda_1}{\lambda_2}=\frac{\frac{3}{4}}{\frac{8}{9}}=\frac{27}{32}$$

9. A block of mass m is connected to two identical springs of force constant K as shown. Find total number of oscillations of block per unit time.

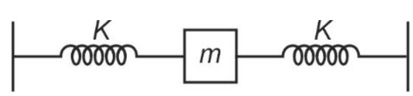


(1) $2\pi\sqrt{\frac{2m}{K}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{K}{m}}$

(3) $2\pi\sqrt{\frac{m}{2K}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{2K}{m}}$

Answer (4)

Sol.



$K_{eq}=2K$

$$\omega=\sqrt{\frac{K_{eq}}{m}}=\sqrt{\frac{2K}{m}}$$

$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{2K}{m}}$ oscillation per second.

10. Consider the two statements:

Assertion : The beam of electrons shows wave nature and exhibits interference and diffraction.

Reason : Davisson-Germer experiment verified the wave nature of electrons.

(1) Both are correct. Reason correctly explains assertion

(2) Both are incorrect

(3) Assertion is correct but Reason is incorrect

(4) Both are correct. Reason does not explain assertion.

Answer (1)

Sol. Davisson Germer experiment verified wave nature of electrons.

Option (1) is correct.

11. A projectile is launched on horizontal surface such that if thrown with initial velocity of u, it has velocity of $\frac{\sqrt{3}u}{2}$ at maximum height. Then time of flight of the projectile is equal to

(1) $\frac{\sqrt{3}u}{g}$ (2) $\frac{2u}{g}$

(3) $\frac{u}{g}$ (4) $\frac{u}{2g}$

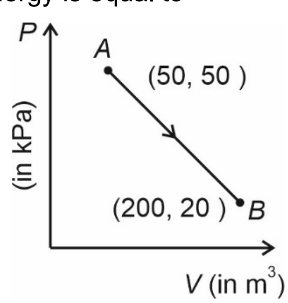
Answer (3)

Sol. $u\cos\theta = \frac{\sqrt{3}u}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$ angle of projection

$T = \frac{2u\sin\theta}{g} = \frac{u}{g}$

12. A diatomic gas is taken from point A to point B in a thermodynamic process as described in the Pressure-Volume graph shown. The change in internal energy is equal to



(1) 3.75×10^6 J

(2) 2.25×10^6 J

(3) 7.5×10^6 J

(4) 4.5×10^6 J

Answer (1)

Sol. $\Delta U = \frac{f}{2}nR\Delta T$

$= \frac{5}{2}(P_fV_f - P_iV_i)$

$= \frac{5}{2}(200\times 20\times 10^3 - 50\times 50\times 10^3)$ J

$= \frac{5}{2}\times 1500\times 10^3$ J $= 3.75 \times 10^6$ J

13. A conductor of length l and cross-sectional area A has drift velocity v_d when used across a potential difference V. When another conductor of same material and length l but double cross-sectional area than first is used across same potential difference than drift velocity is equal to

(1) $\frac{V_d}{2}$ (2) v_d

(3) $2v_d$ (4) $4v_d$

Answer (2)

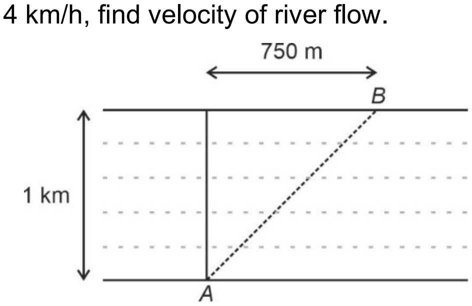
Sol. $I = \eta e v_d A$

$\frac{V}{2} = \eta e v_d A$

$\frac{VA}{\rho l} = \eta e v_d A$

$\Rightarrow$ v_d is independent of area of cross-sectional of conductor.

14. A swimmer swims perpendicular to river flow and reaches point B. If velocity of swimmer in still water is 4 km/h, find velocity of river flow.

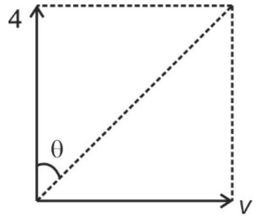


(1) 3 km/hr (2) 5 km/hr

(3) 2 km/hr (4) 6 km/hr

Answer (1)

Sol.



$\Rightarrow \quad \dfrac{v}{4} = \tan\theta = \dfrac{750}{1000}$

$\Rightarrow \quad v = 3$ km/hr

15. A bar magnet with magnetic moment of 5 Am^2 is lying at stable equilibrium in external uniform magnetic field of strength 0.4T. Work done in slowly rotating the bar magnet to the position of unstable equilibrium is equal to

(1) 1 J (2) 2 J

(3) 3 J (4) 4 J

Answer (4)

Sol. $U_i = -MB\cos 0°$

$U_f = -MB\cos 180°$

so, $W = \Delta U$

$= 2MB$

$= 2 \times 5 \times 0.4$

$= 4$ J

16. Unpolarised light (of intensity I_0) is incident on a polarizer *A* and subsequently on polarizer *B* whose pass axis is perpendicular to that of *A*. Now a polarizer *C* is introduced between *A* and *B* such that pass axis of *C* is at 45° with pass axis of *A*. Find intensity that comes out of *B*.

(1) $\dfrac{I_0}{8}$

(2) $\dfrac{I_0}{4}$

(3) Zero

(4) $\dfrac{3I_0}{8}$

Answer (1)

Sol. $I_{net} = I_0 \times \dfrac{1}{2} \times \cos^2 45° \times \cos^2 45°$

$= \dfrac{I_0}{8}$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A solid sphere is rolling on a smooth surface with kinetic energy = 7 × 10^{-3} J. If mass of the sphere is 1 kg, find the speed of centre of mass in cm/s. (Consider pure rolling)

Answer (10.00)

Sol. $\dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}\cdot\dfrac{2}{5}mv^2 = 7 \times 10^{-3}$

$\Rightarrow \dfrac{7}{10}mv^2 = \dfrac{7}{1000}$

$\Rightarrow v = \dfrac{1}{10}\text{m/s} = 10 \text{ cm/s}$

22. A lift of mass 500 kg starts moving downwards with initial speed 2 m/s and accelerates at 2 ms^{-2}. The kinetic energy of the lift when it has moved 6 m down is _____ kJ.

Answer (07.00)

Sol. $u = 2$ m/s

$a = 2$ m/s^2

$s = 6$ m

$v^2 - u^2 = 2as$

$\Rightarrow \quad v^2 = u^2 + 2as = 4 + 2 \times 2 \times 6 = 28$

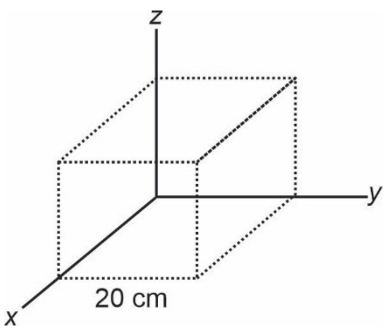
$\text{K.E.} = \dfrac{1}{2}MV^2 = \dfrac{1}{2} \times 500 \times (28)$

$= 500 \times 14$

$= 7000$ J

$= 7$ kJ

23. Electric field in a region is $4000x^2\hat{i}$ N/C. The flux through the cube is $\frac{x}{5}$ Nm^2/C. Find x.



Answer (32)

Sol. $\phi = 4000(0.2)^2 \times \text{Area}$

$= 4000(0.2)^2 \times (0.2)^2$

$= \frac{4000 \times 16}{10000}$

$= 6.4$ Nm^2/C

24. For an series LCR circuit across an A.C source, current and voltage are in same phase. Given the resistance of 20 Ω and voltage of the source is 220 V. Find current (in A) in the circuit.

Answer (11.00)

Sol. The given circuit is in resonance

$\therefore i = \frac{220}{20} = 11\text{ A}$

25. For a particle performing SHM, maximum potential energy is 25 J. The kinetic energy (in J) at half the amplitude is $\frac{x}{4}$. Find x

Answer (75.00)

Sol. $\text{KE} = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{A}{2}\right)^2$

$= \frac{1}{2}kA^2\left[\frac{3}{4}\right]$

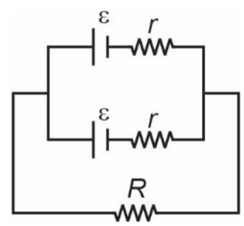
$= \frac{3}{4} \times 25\text{ J}$

$= \frac{75}{4}\text{ J}$

26. The current through a 5 Ω resistance remains same, irrespective of its connection across series or parallel combination of two identical cells. Find the internal resistance (in Ω) of the cell.

Answer (05.00)

Sol.



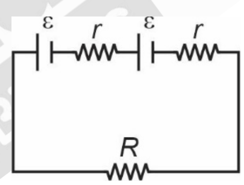
$$\frac{\varepsilon_{eq}}{\left(\frac{r}{2}\right)} = \frac{\varepsilon}{r} + \frac{\varepsilon}{r}$$

$$\varepsilon_{eq} = \varepsilon$$

$$r_{eq} = \left(\frac{r}{2}\right)$$

$$\text{current} = i = \frac{\varepsilon}{R + \left(\frac{r}{2}\right)}$$

When connected in series



$$\varepsilon_{eq} = 2\varepsilon$$

$$i = \left(\frac{2\varepsilon}{R + 2r}\right)$$

$$\Rightarrow \frac{\varepsilon}{R + \frac{r}{2}} = \frac{2\varepsilon}{R + 2r}$$

$$\Rightarrow R + 2r = 2R + r$$

$$\Rightarrow \boxed{r = R = 5\ \Omega}$$

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Electronic configuration of Nd^{2+} is

(1) $4f^2$

(2) $4f^3$

(3) $4f^4$

(4) $4f^5$

Answer (3)

Sol. Nd^{2+} = [Xe] $4f^4$

2. Following values of K (Rate constants) are given at different temperatures. Find out (E_a) Activation energy.

T = 200 K $\rightarrow$ K_1 = 0.03

T = 300 K $\rightarrow$ K_2 = 0.05

(1) 2.548 kJ

(2) 11.488 kJ

(3) 1.106 kJ

(4) 51.437 kJ

Answer (1)

Sol. $$\log\left(\frac{0.05}{0.03}\right) = \frac{E_a}{2.303 \times 8.314}\left(\frac{1}{200} - \frac{1}{300}\right)$$

$$= \frac{E_a}{2.303 \times 8.314}\left(\frac{1}{600}\right)$$

E_a = 2.548 kJ

3. Basic strength of oxides of V

V_2O_3 V_2O_5 V_2O_4

(1) $V_2O_3 < V_2O_5 < V_2O_4$

(2) $V_2O_3 < V_2O_4 < V_2O_5$

(3) $V_2O_3 > V_2O_4 > V_2O_5$

(4) $V_2O_3 = V_2O_4 = V_2O_5$

Answer (3)

Sol. As oxidation state of V increases than other acidic nature increases

Correct basic order is $\overset{+3}{V_2}O_3 > \overset{+4}{V_2}O_4 > \overset{+5}{V_2}O_5$

4. XeF_4, SF_4 and $BrCl_3$ show hybridizations respectively

(1) sp^3, sp^3, sp^3

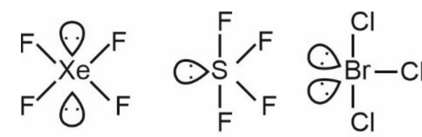
(2) dsp^2, sp^3, sp^3

(3) sp^3d^2, sp^3d, sp^3d

(4) d^2sp^2, sp^3d, sp^3d

Answer (3)

Sol.



sp^3d^2 sp^3d sp^3d

5. $Cu^{2+} + I^- \rightarrow A \xrightarrow[\Delta]{} B + C$

B and C are

(1) I_2, Cu_2I_2

(2) $[CuI_4]$

(3) CuI_3^-

(4) I^-, CuI_2

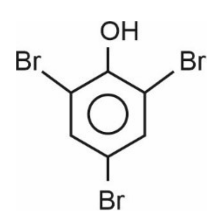
Answer (1)

Sol. $Cu^{2+} + 2I^- \rightarrow \underset{(A)}{[CuI_2]} \xrightarrow{\Delta} \underset{(B)}{\frac{1}{2}Cu_2I_2} + \underset{(C)}{\frac{1}{2}I_2}$

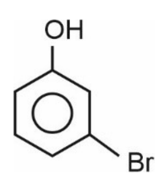
$\therefore$ Products (B) and (C) are Cu_2I_2 and I_2 respectively.

6. When phenol reacts with Br_2 in low polarity solvent, it produces as a major product ____________?

(1)

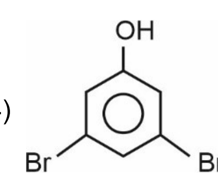


(2)



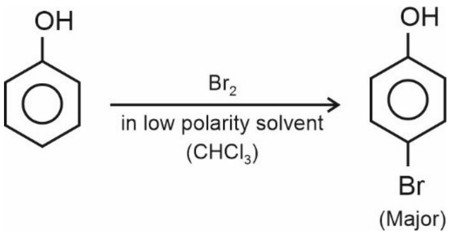
(3)

(4)



Answer (3)

Sol.



7. Choose the correct information regarding products obtained on electrolysis of Brine solution

(1) Cl_2 at cathode (2) O_2 at cathode

(3) H_2 at cathode (4) OH^- at anode

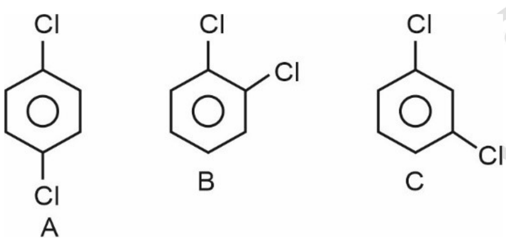
Answer (3)

Sol. Anode : $2Cl^- \longrightarrow Cl_2 + 2e^-$

Cathode : $2e^- + 2H_2O \longrightarrow H_2 + 2OH^-$

Net reaction $2Cl^- + 2H_2O \longrightarrow \underbrace{Cl_2}_{\text{anode}} + \underbrace{H_2 + 2OH^-}_{\text{cathode}}$

8. Melting point order of



(1) A > B > C

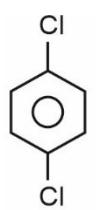
(2) C > A > B

(3) B > A > C

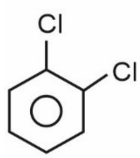
(4) A > C > B

Answer (1)

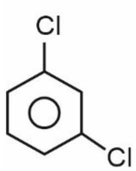
Sol.



m.p. = 323 K



m.p. = 256 K



m.p. = 249 K

9. Consider the following reaction :

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

If $K_p = 2\times10^{12}$ and $K_c = x \times 10^{13}$, the value of x in terms of RT will be

(1) $\frac{\sqrt{RT}}{4}$

(2) $\frac{\sqrt{RT}}{5}$

(3) $\frac{\sqrt{RT}}{10}$

(4) $10\sqrt{RT}$

Answer (2)

Sol. $K_p = K_c(RT)^{-1/2}$

$2\times10^{12} = x \times 10^{13}(RT)^{-1/2}$

$x = \frac{2\times10^{12}}{10^{13}\times(RT)^{-1/2}} = \frac{2\sqrt{RT}}{10} = \frac{\sqrt{RT}}{5}$

10. Arrange the following ions in the increasing order of their ionic radii.

S^{2-}, Cl^-, K^+ and Ca^{2+}

(1) $S^{2-} < Cl^- < K^+ < Ca^{2+}$

(2) $Cl^- < S^{2-} < K^+ < Ca^{2+}$

(3) $K^+ < Ca^{2+} < Cl^- < S^{2-}$

(4) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

Answer (4)

Sol. The given ionic species are isoelectronic species. The radii of isoelectronic ionic species increases as the atomic of the ion decreases. Therefore, the correct increasing order of radii of ionic species is

$Ca^{2+} < K^+ < Cl^- < S^{2-}$

11. Which of the following option contains the compound which has highest sweetening value?

(1) Aspartame

(2) Saccharin

(3) Sucralose

(4) Alitame

Answer (4)

Sol. Alitame has the highest sweetening value.

12. Which of the following method is not a concentration of ore?

(1) Electrolysis (2) Leaching

(3) Froth floatation (4) Hydraulic washing

Answer (1)

Sol. The following methods are commonly used for concentration of ore

1. Hydraulic washing

2. Leaching

3. Froth floatation

But electrolysis is used for refining of the crude metal.

13. In which of the following reactions, H_2O_2 acts as a reducing agent

(1) $H_2O_2 + Mn^{2+} \rightarrow MnO_2 + H_2O$

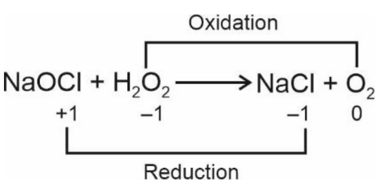
(2) $NaOCl + H_2O_2 \rightarrow NaCl + O_2$

(3) $Fe^{2+} + H_2O_2 \rightarrow Fe^{3+} + H_2O$

(4) $PbS + H_2O_2 \rightarrow PbSO_4 + H_2O$

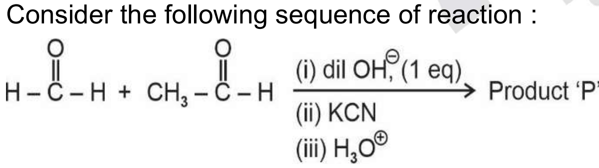
Answer (2)

Sol.



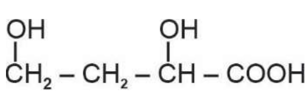
In Option (2), oxidation of H_2O_2 is taking place and hence H_2O_2 acts as a reducing agent.

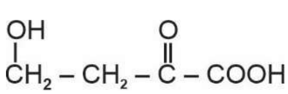
14. Consider the following sequence of reaction :

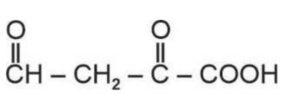


The product 'P' is?

(1) $CH_3 - CH_2 - COOH$

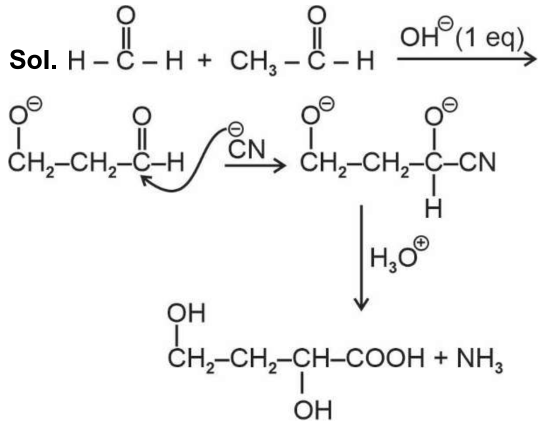
(2) 

(3) 

(4) 

Answer (2)

Sol.



15. Which of the following transition emits the same wavelength as that for (n = 4 → n = 2) for He^+ Ion

(1) H(n = 3 → n = 1)

(2) H(n = 2 → n = 1)

(3) Li^{2+}(n = 4 → n = 3)

(4) He^+(n = 6 → 3)

Answer (2)

Sol. $\dfrac{\Delta E_{(H-atom)}}{n=2\rightarrow n=1} = \dfrac{(\Delta E)_{He^+Ion}}{n=4\rightarrow n=2} = \dfrac{(\Delta E)_{Li^{++}Ion}}{n=6\rightarrow n=3}$

16. A complex compound of Co (X) is pink colour in water. On reaction with conc. HCl forms (Y) of deep blue colour and has geometry (Z). Identify (X), (Y) and (Z).

(1) $[Co(H_2O)_6]^{2+}$, $[CoCl_6]^{3-}$, Octahedral

(2) $[Co(H_2O)_6]^{3+}$, $[CoCl_4]^{2-}$, Tetrahedral

(3) $[Co(H_2O)_6]^{2+}$, $[CoCl_4]^{2-}$, Tetrahedral

(4) $[Co(H_2O)_6]^{3+}$, $[CoCl_6]^{3-}$, Octahedral

Answer (3)

Sol. Co^{2+} ions in aqueous medium are pink in colour. On addition of conc. HCl to it, the solution becomes blue due to formation of $[CoCl_4]^{2-}$ which is tetrahedral.

$$Co^{2+} + 6H_2O \longrightarrow \underset{\substack{(X)\\(Pink)}}{[Co(H_2O)_6]^{2+}}$$

$$[Co(H_2O)_6]^{2+} + \underset{(Conc.)}{4HCl} \longrightarrow$$

$$\underset{\substack{(Y)\\(Blue)}}{[CoCl_4]^{2-}} + 4H^+ + 6H_2O$$

Geometry of (Y) is tetrahedral (Z).

17. Which of the following option contains the correct match?

List-I	List-II
(A) XeF_4	(P) T-shape
(B) SF_4	(Q) See-saw
(C) $NH_4^{\oplus}$	(R) Square planar
(D) BrF_3	(S) Tetrahedral

(1) $A \to P, B \to Q, C \to R, D \to S$

(2) $A \to R, B \to Q, C \to S, D \to P$

(3) $A \to Q, B \to P, C \to S, D \to R$

(4) $A \to S, B \to R, C \to P, D \to Q$

Answer (2)

Sol. $XeF_4 \to$ Square planar

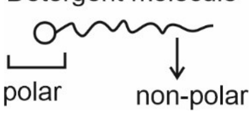
$SF_4 \to$ See-saw

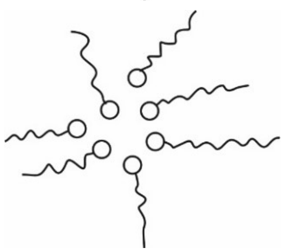
$NH_4^{\oplus} \to$ Tetrahedral

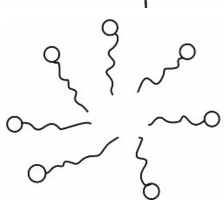
$BrF_3 \to$ T-shaped

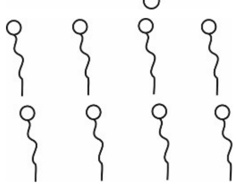
18. A detergent is dissolved in non-polar solvent. The structure of micelle in non-polar solvent

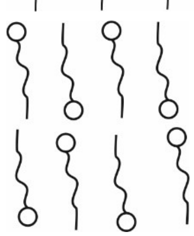
Detergent molecule



(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. In non-polar-solvent, non-polar part will be outside.

19. Consider the following reaction :

$$ClO(g) + NO_2(g) \longrightarrow A \xrightarrow{H_2O(g)} B + C$$

A, B and C are respectively

(1) $ClONO_2(g)$; $HOCl(g)$; $HNO_3(g)$

(2) $ClONO_2(g)$; $HOCl(g)$; $NO_2(g)$

(3) $ClNO_2(g)$; HCl; Cl_2

(4) $ClNO_2(g)$; $HCl(g)$; $HNO_3(g)$

Answer (1)

Sol. $$ClO(g) + NO_2(g) \longrightarrow \underset{(A)}{ClONO_2(g)}$$

$$ClONO_2(g) + H_2O(g) \longrightarrow \underset{(B)}{HOCl(g)} + \underset{(C)}{HNO_3(g)}$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 2.56 g of a non-electrolyte solute is dissolved in one litre of a solution, it has osmotic pressure (π) equal to 4 bar at 300 K temperature. Then find the molar mass of the compound.

[Given : R = 0.083 bar] (Round off to the nearest integer)

Answer (16)

Sol. p = CRT

$$4 = \frac{2.56}{M} \times 0.083 \times 300$$

$$\approx 16 \text{ g}$$

22. Weight of an organic compounds is 0.492 g, when the hydrocarbon undergoes combustion it produces 0.792 g CO_2. Find the % of carbon in the given hydrocarbon (Round off to the nearest integer)

Answer (44)

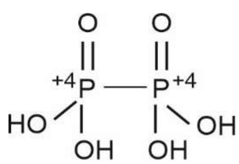
Sol. $\%C = \frac{12}{44} \times \frac{0.792}{0.492} \times 100$

= 43.90%

23. The oxidation state of phosphorus atom in the hypophosphoric acid is ______?

Answer (4)

Sol. The hypophosphoric acid is :



24. What is the volume of Hydrogen Gas produced (in litre) when 11.2 gm of Zn metal reacts with excess of dil. HCl (Closest Integer)

Given: Molar volume of H_2 = 22.7 L/mole

Molar mass of Zn is 65 gm/mole

Answer (4)

Sol. $\underset{11.2\ \text{gm}}{Zn} + 2HCl \rightarrow ZnCl_2 + H_2$

$\left(\frac{11.2}{65}\right) \qquad \frac{11.2}{65} \times 22.7$ litre

= 3.911 litre

$\approx$ 4 litre

25. The value of logarithm of the equilibrium constant of the following reaction is $\frac{X}{3}$. Then 'X' is

$$Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$$

Given : $[Pd^{2+} + 2e^- \longrightarrow Pd \quad ; \quad E^\circ = 0.83\ V$

$PdCl_4^{2-} + 2e^- \longrightarrow Pd + 4Cl^- \ ; \ E^\circ = 0.63\ V$

and $\frac{2.303\ RT}{F} = 0.06]$

Answer (20)

Sol. $\Delta G_3^\circ = \Delta G_1^\circ - \Delta G_2^\circ$

–2.303 × RTlogk = –0.83 × 2 × F + 0.63 × 2 × F

$\log k = \frac{0.2 \times 2 \times F}{2.303 \times RT}$

$= \frac{0.2 \times 2}{0.06} = \frac{20}{3}$

26. Find the value of |ΔH| in kJ for

$\frac{1}{2}Cl_2(g) \rightarrow Cl^-(aq)$

Given:

$\Delta H_{diss}\ Cl_2(g) \rightarrow 2Cl(g) \quad 240\ kJ\ mol^{-1}$

$\Delta H_{eg}\ Cl(g) + e \rightarrow Cl^-(g) \quad -320\ kJ\ mol^{-1}$

$\Delta H_{hydration}\ Cl^-(g) + aq \rightarrow Cl^-(aq) \quad -340\ kJ\ mol^{-1}$

Answer (540)

Sol.

$\frac{1}{2}Cl_2(g) \rightarrow Cl(g) \qquad \Delta H_1 = \frac{240}{2} = 120\ kJ$

$Cl(g) + e \rightarrow Cl^-(g) \qquad \Delta H_2 = -320\ kJ$

$Cl^-(g) + aq \rightarrow Cl^-(aq) \qquad \Delta H_3 = -340\ kJ$

$\frac{1}{2}Cl_2(g) + e^- + aq \rightarrow Cl^-(aq) \ \ \Delta H$

$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$

= 120 – 320 – 340

= –540 kJ

|ΔH| = 540 kJ

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $\int_{\pi/6}^{\pi/3} \frac{2+3\sin x}{\sin x(1+\cos x)}dx$ is equal to

(1) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}+6\sqrt{3}-\frac{28}{3}$

(2) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}$

(3) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}-\frac{28}{3}$

(4) $6\sqrt{3}-\frac{28}{3}$

Answer (1)

Sol. $I=\int \frac{2}{\sin x(1+\cos x)}dx+\int \frac{3}{1+\cos x}dx$

$$=\underbrace{\int \frac{2\sin x}{\sin^2 x(1+\cos x)}dx}_{I_1}+\underbrace{\int \frac{3}{2\cos^2 \frac{x}{2}}dx}_{I_2}$$

Let $\cos x = t$

$$I_1=\int \frac{-2dt}{(1-t^2)(1+t)}$$

$$=-2\left(\frac{\ln|t+1|}{4}+\frac{1}{2t+2}-\frac{\ln|t-1|}{4}\right)+C$$

$$=-2\left(\frac{\ln|\cos x+1|}{4}+\frac{1}{2\cos x+2}-\frac{\ln|\cos x-1|}{4}\right)+C$$

$$I_2=\frac{3}{2}2\tan\frac{x}{2}+C$$

So,

$$\int_{\pi/6}^{\pi/3}\frac{2+3\sin x}{\sin x(1+\cos x)}dx=\ln(2+\sqrt{3})-\ln\frac{3}{2}+6\sqrt{3}-\frac{28}{3}$$

2. The product and sum of first four terms of G.P. is 1296 and 126 respectively, then sum of the possible values of common difference is ___.

(1) 14 (2) $\frac{10}{3}$

(3) $\frac{7}{2}$ (4) 3

Answer (4)

Sol. $\frac{a}{r^3}\cdot\frac{a}{r}\cdot ar\cdot ar^3=1296$

$\Rightarrow$ $a = 6$

Now, $\frac{a}{r^3}+\frac{a}{r}+ar+ar^3=126$

$\Rightarrow$ $\frac{1}{r^3}+\frac{1}{r}+r+r^3=21$

$\Rightarrow$ $\left(r+\frac{1}{r}\right)\left(\left(r+\frac{1}{r}\right)^2-3\right)+\left(r+\frac{1}{r}\right)=21$

Let $r+\frac{1}{r}=t$

$t^3-3t+t=21$

$\Rightarrow$ $t^3-2t-21=0$

$\Rightarrow$ $t = 3$

$\Rightarrow$ $r+\frac{1}{r}=3$

$\Rightarrow$ $r^2-3r+1=0$

$r_1+r_2=3$

Sum of possible values of r is 3

3. If $B = \ln(1-a)$ and $P(a)$

$$=\left(a+\frac{a^2}{2}+\frac{a^3}{3}+.....+\frac{a^{50}}{50}\right)$$

then $\int_0^a \frac{t^{50}}{1-t}dt$ equals

(1) $-(B+P(a))$ (2) $-B+P(a)$

(3) $B-P(a)$ (4) $B+P(a)$

Answer (3)

Sol. $\int_0^a \left(\frac{t^{50}-1}{1-t}+\frac{1}{1-t}\right)dt$

$\Rightarrow \int_0^a \left(-(1+t+t^2+....+t^{49})+\frac{1}{1-t}\right)dt$

$\Rightarrow \ln(1-t)-\left(t+\frac{t^2}{2}+\frac{t^3}{3}+.....\frac{t^{50}}{50}\right)\Big|_0^a$

$\Rightarrow \ln(1-a)-\left(a+\frac{a^2}{2}+\frac{a^3}{3}+....+\frac{a^{50}}{50}\right)$

$\Rightarrow B-P(a)$

4. $\sin^{-1}\left(\frac{a}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{77}{36}\right)=0$, then the value of $\sin^{-1}(\sin a)+\cos^{-1}(\cos a)$ is

(1) 0 (2) $16-2\pi$

(3) π (4) 5

Answer (3)

Sol. $\sin^{-1}\left(\frac{a}{17}\right)=-\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{77}{36}\right)$

Let $\cos^{-1}\frac{4}{5}=\beta$ and $\tan^{-1}\left(\frac{77}{36}\right)=\alpha$

$\Rightarrow \sin\left(\sin^{-1}\frac{a}{17}\right)=\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\Rightarrow \frac{a}{17}=\frac{77}{85}\cdot\frac{4}{5}-\frac{36}{85}\cdot\frac{3}{5}$

$\Rightarrow a=\frac{200}{25} \Rightarrow a=8$

$\therefore \sin^{-1}\sin 8+\cos^{-1}\cos 8=3\pi-8+8-2\pi$

$=\pi$

5. The range of $\frac{[x]}{x^2+1}$ if domain is [–2, 6] is ([.] represents G.I.F)

(1) $\left[-4,\frac{-5}{2}\right]\cup\left[\frac{5}{37},\frac{1}{2}\right]\cup\{0\}$

(2) $\left(-1,\frac{-2}{5}\right)\cup\left[0,\frac{5}{37}\right]$

(3) $\left(-1,\frac{-2}{5}\right]\cup\left[\frac{5}{37},\frac{1}{2}\right)\cup\{0\}$

(4) $\left[-1,\frac{-2}{5}\right)\cup\left[0,\frac{5}{37}\right)$

Answer (3)

Sol. $x\in[-2,-1), f(x)=\frac{-2}{x^2+1}, f(x)\in\left[\frac{-2}{5},-1\right]$

$x\in[-1,0), f(x)=\frac{-1}{x^2+1}, f(x)\in\left[\frac{-1}{2},-1\right]$

$x\in[0,1), f(x)=0$

$x\in[1,2), f(x)=\frac{1}{x^2+1}, f(x)\in\left(\frac{1}{5},\frac{1}{2}\right]$

$x\in[2,3), f(x)=\frac{2}{x^2+1}, f(x)\in\left(\frac{1}{5},\frac{2}{5}\right]$

$x\in[3,4), f(x)=\frac{3}{x^2+1}, f(x)\in\left(\frac{3}{17},\frac{3}{10}\right]$

$x\in[4,5), f(x)=\frac{4}{x^2+1}, f(x)\in\left(\frac{4}{26},\frac{4}{17}\right]$

$x\in[5,6), f(x)=\frac{5}{x^2+1}, f(x)\in\left(\frac{5}{37},\frac{5}{26}\right]$

$x=6, f(x)=\frac{6}{37}$

$\therefore$ range of $f(x)$ is $\left(-1,\frac{-2}{5}\right]\cup\{0\}\cup\left(\frac{5}{37},\frac{1}{2}\right]$

6. If maximum distance of a normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$ from (0, 0) is 1, then find the eccentricity of the ellipse.

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Answer (4)

Sol. Equation of normal is

$(2\sec\theta)x-(b\operatorname{cosec}\theta)y=4-b^2$

Perpendicular distance from (0, 0) is

$$D=\left|\frac{4-b^2}{\sqrt{4\sec^2\theta+b^2\operatorname{cosec}^2\theta}}\right|$$

$$=\frac{4-b^2}{\sqrt{(4+b^2)+4\tan^2\theta+b^2\cot^2\theta}}$$

$$\le\frac{4-b^2}{\sqrt{b^2+4+4b}} \quad \text{[Using } AM\ge GM\text{]}$$

$$=\frac{4-b^2}{(2+b)}=2-b$$

$D_{max} = 2 - b = 1 \Rightarrow b = 1$

$e^2 = 1 - \frac{1}{4}$

$e = \frac{\sqrt{3}}{2}$

7. Let the curve C_1 be represented by $|z| = 2$ and C_2 by $\left|z + \frac{1}{z}\right| = \frac{15}{4}$ then

(1) C_1 lies inside C_2

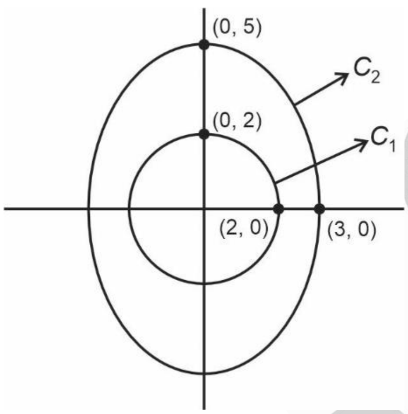
(2) C_2 lies inside C_1

(3) C_1 and C_2 has 2 points of intersection

(4) C_1 and C_2 has 4 points of intersection

Answer (1)

Sol. Let $z = x + iy$



$C_1 \Rightarrow x^2 + y^2 = 4$

$C_2 \Rightarrow \left|x + iy + \frac{1}{x + iy}\right| = \frac{15}{4}$

OR $\left|x + iy + \frac{x - iy}{4}\right| = \frac{15}{4}$

OR $\left(\frac{5x}{4}\right)^2 + \left(\frac{3y}{4}\right)^2 = \frac{225}{16}$

OR $\frac{x^2}{9} + \frac{y^2}{25} = 1$

$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

8. Find the number of real solution(s) of $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol. $x^2 - 4x + 3 \geq 0$

$(x - 1)(x - 3) \geq 0$

$x \in (-\infty, 1] \cup [3, \infty)$ ...(i)

$x^2 - 9 \geq 0 \Rightarrow x \in (-\infty, -3] \cup [3, \infty)$ ...(ii)

$4x^2 - 14x + 6 \geq 0 \Rightarrow (2x - 1)(x - 3) \geq 0$

$\Rightarrow \quad x \in \left(-\infty, \frac{1}{2}\right] \cup [3, \infty)$ ...(iii)

(i) $\cap$ (ii) $\cap$ (iii)

$x \in (-\infty, -3] \cup [3, \infty)$

Now squaring both sides of given equation:

$(x^2 - 4x + 3) + (x^2 - 9) + 2\sqrt{(x^2 - 4x + 3)(x^2 - 9)} = 4x^2 - 14x + 6$

$\Rightarrow \quad 2\sqrt{(x^2 - 4x + 3)(x - 3)(x + 3)} = 2(x^2 - 5x + 6)$

$\Rightarrow \quad (x^2 - 4x + 3)(x - 3)(x + 3) = (x - 3)^2 (x - 2)^2$

$x = 3$ is one solution

$\Rightarrow \quad (x^2 - 4x + 3)(x + 3) = (x^2 - 4x + 4)(x - 3)$

$\Rightarrow \quad x^3 - 4x^2 + 3x + 3x^2 - 12x + 9$

$= x^3 - 4x^2 + 4x - 3x^2 + 12x - 12$

$\Rightarrow \quad 6x^2 - 25x + 21 = 0$

$\Rightarrow \quad x = 3, \frac{7}{6}$

$\therefore$ Only one real solution $x = 3$ as $x = \frac{7}{6}$, is not is the domain.

9. If $f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$ then $f'(1)$ is

(1) $\frac{3\pi^2}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{16}$ (4) $\frac{\pi^2}{2}$

Answer (3)

Sol. $f'(x) = 3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$

$\cdot \cos\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$

$\cdot \frac{\pi}{3}\left(-\sin\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$

$$\cdot\frac{\pi}{3\sqrt{2}}\cdot\frac{3}{2}\left(-4x^3+5x^2+1\right)^{\frac{1}{2}}\cdot\left(-12x^2+10x\right)$$

$$f'(1)=3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)\cdot\cos\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)$$

$$\cdot\frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right)\cdot\frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2)$$

$$=3\sin^2\left(\frac{-\pi}{6}\right)\cdot\cos\left(\frac{-\pi}{6}\right)\cdot\frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)(-\pi)$$

$$=\frac{3}{4}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\cdot\pi=\frac{3\pi^2}{16}$$

10. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\left|\vec{a}+\vec{b}+\vec{c}\right|=\left|\vec{a}+\vec{b}-\vec{c}\right|$ and $\vec{b}\cdot\vec{c}=0$ then

Statement-I : $|\vec{a}+\lambda\vec{c}|\geq 0$ for all $\lambda\in \boldsymbol{R}$.

Statement-II : $\vec{a}$ is always parallel to $\vec{c}$.

(1) Statement-I is true, statement-II is false

(2) Statement-I is true, statement-II is true

(3) Statement-I is false, statement-II is true

(4) Statement-I is false, statement-II is false

Answer (1)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}+\vec{b}-\vec{c}|^2$

$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}-\vec{c}\cdot\vec{a}$

$\Rightarrow\ \vec{c}\cdot\vec{a}=0$

$\therefore\ \vec{a}$ is perpendicular to $\vec{c}$

$|\vec{a}+\lambda\vec{c}|\geq 0$ (However, it is always true)

and statement-II is false.

11. $A=\begin{bmatrix}2&0&0\\0&3&0\\0&1&1\end{bmatrix}$, then find sum of diagonal elements of $(A-I)^{11}$

(1) 4096 (2) 4097

(3) 2048 (4) 2049

Answer (2)

Sol. $A-I=\begin{bmatrix}1&0&0\\0&2&0\\0&0&0\end{bmatrix}$

$$(A-I)^{11}=\begin{bmatrix}\underline{1^{11}}&&\\&\underline{2^{11}}&\\&&\underline{0^{11}}\end{bmatrix}$$

(Calculating only for diagonal elements)

trace $(A-I)^{11}=2^{11}+1^{11}$

$=4097$

12. Circle $x^2+y^2-4x-6y+11=0$ is rolled up by 4 units along a tangent to it at the point (3, 2).

Let this be circle C_1, C_2 is the mirror image of circle C_1 about the tangent. A and B are centres of circles C_1 and C_2. C and D are the feet of perpendicular from A and B respectively upon X-axis. The area of the trapezium $ABCD$ equals to

(1) $4\left(1+\sqrt{2}\right)$ (2) $2\left(1+\sqrt{2}\right)$

(3) $3\left(1+\sqrt{2}\right)$ (4) $1+\sqrt{2}$

Answer (1)

Sol. Given circle is

$x^2+y^2-4x-6y+11=0$, centre $E(2, 3)$

Tangent at (3, 2) is

$x-y-1=0$

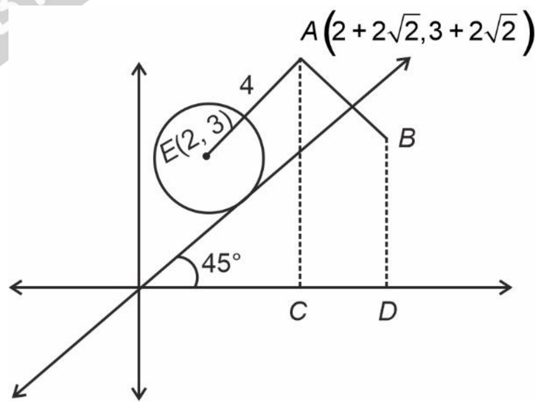
After rolling up by 4 units,

Centre of C_1 is A

Where $A\equiv\left(2+4\times\frac{1}{\sqrt{2}}, 3+4\times\frac{1}{\sqrt{2}}\right)$

$\equiv\left(2+2\sqrt{2}, 3+2\sqrt{2}\right)$

B is image of A



$$\frac{x-\left(2+\sqrt{2}\right)}{1}=\frac{y-\left(3+2\sqrt{2}\right)}{-1}=-2\left(\frac{-2}{2}\right)=2$$

$B\left(4+2\sqrt{2}, 1+2\sqrt{2}\right)$

Area of $ABCD$

$=\frac{1}{2}\times\left(4+4\sqrt{2}\right)\times\left(\left(4+2\sqrt{2}\right)-(2+2\sqrt{2})\right)$

$=4\left(1+\sqrt{2}\right)$

13. Let the relation R, $(a, b)\ R\ (c, d)$ be such that $ab(d - c) = cd(a - b)$, then R is

(1) Reflexive only

(2) Symmetric

(3) Transitive but not symmetric

(4) Reflexive and symmetric but not transitive

Answer (2)

Sol. Checking for reflexive

$\therefore\ (a, b)\ R\ (a, b)$

$\Rightarrow\ ab(b - a) = ab(a - b)$

$1 = -1 \qquad \therefore$ Not reflexive

Checking for $(a, b)\ R\ (c, d)$ then $(c, d)\ R\ (a, b)$

$\Rightarrow\ cd(b - a) = ab(c - d)$

$\Rightarrow\ ab(d - c) = cd(a - b)\ \therefore$ Symmetric

$\therefore\ R$ is symmetric

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the number of 5-digit numbers formed using the digits 0, 3, 4, 7, 9 when repetition of digits is allowed is

Answer (2500)

Sol. $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

$4 \times 5 \times 5 \times 5 \times 5$

Total number of numbers = $4 \times 5 \times 5 \times 5 \times 5$

$= 2500$

22. Find remainder when 5^{99} is divided by 11.

Answer (09)

Sol. $5^{99} = (5^5)^{19} \cdot 5^4$

$= (3125)^{19} \cdot 5^4$

$= (11\lambda + 1)^{19} \cdot 5^4$

$= (11K + 1)\ 5^4$

$= 11K_1 + 5^4$

When 5^4 is divided by 11, we get remainder = 9

23. If $f(x) + \int_3^x \frac{f(t)}{t}dt = \sqrt{x+1}$ then value of $12f(8)$ equals.

Answer (17)

Sol. Differentiating both sides we get

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\Rightarrow\ \frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = x$$

$$\therefore\ xy = \frac{1}{2}\int \frac{x}{\sqrt{x+1}}dx$$

$$\Rightarrow\ xy = \frac{1}{2}\int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right)dx$$

$$\Rightarrow\ xy = \frac{1}{2}\left(\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right) + c$$

Put $x = 3$ in $f(x) + \int_3^x \frac{f(t)}{t}dt = \sqrt{x+1}$ we get $f(3) = 2$

$$\therefore\ c = \frac{16}{3}$$

$$\Rightarrow\ 8f(8) = \frac{1}{2}\left(\frac{27}{3}\cdot 2 - 2\cdot 3\right) + \frac{16}{3}$$

$$\Rightarrow\ 12f(8) = 17$$

24. $y = f(x)$ is a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$

Given that $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$

Then number of solutions for x is

Answer (02)

Sol. $SP = SQ$

$$\left(x+\frac{1}{2}\right)^2 + y^2 = \left(y+\frac{1}{2}\right)^2$$

$$x^2 + \frac{1}{4} + x + y^2 = y^2 + \frac{1}{4} + y$$

$\Rightarrow$ Eqn. of parabola $y = x^2 + x$

$f(x) = x^2 + x$

$$\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\tan^{-1}\sqrt{x^2+x} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\cos^{-1}\frac{1}{\sqrt{x^2+x+1}} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\Rightarrow \frac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$$

$$\Rightarrow x^2+x+1 = 1$$

Or $x = 0, -1$

25. The direction ratio's of two lines which are parallel are given by < 2, 1, –1> and < $\alpha + \beta$, $1 + \beta$, 2 >. Find $|2\alpha + 3\beta|$

Answer (11)

Sol. $\dfrac{\alpha+\beta}{2} = \dfrac{1+\beta}{1} = \dfrac{2}{-1}$

$\Rightarrow \alpha + \beta = -4,\ 1 + \beta = -2$

So, $\beta = -3,\ \alpha = -1$

$|2\alpha + 3\beta| = 11$

26. Given $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}, |\vec{a}\times\vec{b}| = \sqrt{48}$. Find $(\vec{a}\cdot\vec{b})^2$

Answer (36.00)

Sol. $(\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\times\vec{b})^2$

$= 14\cdot6 - 48$

$= 36$

27.

28.

29.

30.

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