MATHEMATICS

1. If $|\vec{a}| = 3$ then value of

$|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times k|^2 =$ ____________.

(A) 9 (B) 18

(C) 27 (D) 36

Answer (B)

Sol. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $|\vec{a}| = x^2 + y^2 + z^2$

Now, $|\vec{a}\times\hat{i}|^2 = y^2 + z^2$, $|\vec{a}\times\hat{j}|^2 = x^2 + z^2$

and $|\vec{a}\times\hat{k}|^2 = x^2 + y^2$

$\therefore \quad |\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2(x^2 + y^2 + z^2)$

$= 2|\vec{a}|^2 = 18$

2. The co-ordinates of the foot of perpendicular drawn from origin to the plane $2x - 3y + 4z - 6 = 0$ is ________

(A) $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$ (B) $\left(\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

(C) $\left(\frac{12}{29}, \frac{18}{29}, \frac{24}{29}\right)$ (D) $\left(-\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

Answer (A)

Sol. Let (x_1, y_1, z_1) be foot of perpendicular from origin to plane $2x - 3y + 4z - 6 = 0$.

$\therefore \quad \frac{x_1 - 0}{2} = \frac{y_1 - 0}{-3} = \frac{z_1 - 0}{4} = -\frac{(-6)}{2^2 + 3^2 + 4^2}$

$\therefore \quad \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \frac{6}{29}$

$\therefore \quad (x_1, y_1, z_1) = \left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$

3. The angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$ is ________.

(A) $\cos^{-1}\frac{8}{21}$ (B) $\tan^{-1}\frac{8}{\sqrt{377}}$

(C) $\sin^{-1}\frac{8}{\sqrt{377}}$ (D) $\sin^{-1}\left(\frac{21}{8}\right)$

Answer (B)

Sol. Let angle between line and plane be θ.

Line is $\frac{x+1}{2} = \frac{y-0}{3} = \frac{z-3}{6}$

with direction ratios (2, 3, 6)

And plane is $10x + 2y - 11z - 3 = 0$ direction ratios of normal to plane is $(10, 2, -11)$

$\therefore \quad \sin\theta = \left|\frac{20 + 6 - 66}{\sqrt{4+9+36}\sqrt{100+4+121}}\right|$

$\sin\theta = \left|\frac{-40}{7\times 15}\right|$

$\sin\theta = \frac{8}{21}$ or $\tan\theta = \frac{8}{\sqrt{377}}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{8}{\sqrt{377}}\right)$

4. If the points (1, 1, p) and (–3, 0, 1) be equidistant from the plane $\vec{r}\cdot(3\hat{i} + 4\hat{j} - 12k) + 13 = 0$ then the values of p are _____________.

(A) $1, \frac{7}{3}$ (B) $1, \frac{4}{3}$

(C) $2, \frac{4}{3}$ (D) $\frac{7}{3}, 2$

Answer (A)

Sol. Equation of plane is $3x + 4y - 12z + 13 = 0$

Now as $(1, 1, p)$ and $(-3, 0, 1)$ are equidistant from the plane

$\left|\frac{3 + 4 - 12P + 13}{\sqrt{9+16+144}}\right| = \left|\frac{-9 + 0 - 12 + 13}{\sqrt{9+16+144}}\right|$

$|20 - 12p| = -8 \Rightarrow 20 - 12p = 8 \Rightarrow p = 1$

and $20 - 12p = -8 \Rightarrow 12p = 28 \Rightarrow p = \frac{7}{3}$

5. The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \le 4, x \ge 0, y \ge 0$ is ________.

(A) 16 (B) 12

(C) 0 (D) not possible

Answer (A)

Sol. $z = 3x + 4y$

At $(0, 0)$; $z = 0$

At $(4, 0)$; $z = 12$

At $(0, 4)$; $z = 16$

$\therefore$ Maximum value = 16

6. If A and B are independent events such that $P(A) = p$, $P(B) = 2p$ and P(Exactly one of A and B) = $\frac{5}{9}$, then p = ____________.

(A) $\frac{1}{3}, \frac{5}{12}$ (B) $\frac{1}{2}, \frac{3}{4}$

(C) $\frac{1}{12}, \frac{5}{3}$ (D) $\frac{2}{15}, \frac{5}{12}$

Answer (A)

Sol. As A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = 2p^2$$

and P(exactly one of A and B)

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B) = \frac{5}{9}$$

$$\Rightarrow \quad p + 2p - 2 \cdot 2p^2 = \frac{5}{9}$$

$$\Rightarrow \quad 36p^2 - 27p + 5 = 0$$

$$\Rightarrow \quad (12p - 5)(3p - 1) = 0$$

$$\therefore \quad p = \frac{5}{12}, \frac{1}{3}$$

7. For the probability distribution

X	1	2	3	4
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ = _________.

(A) 7 (B) 5

(C) 3 (D) 10

Answer (D)

Sol. $E(X^2) = 1 \times \frac{1}{10} + 4 \times \frac{1}{5} + 9 \times \frac{3}{10} + 16 \times \frac{2}{5}$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5}$$

$$= \frac{1 + 8 + 27 + 64}{10}$$

$$= \frac{100}{10} = 10$$

8. If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$ then ________.

(A) $P\left(\frac{B}{A}\right) = 1$ (B) $P\left(\frac{A}{B}\right) = 0$

(C) $P\left(\frac{B}{A}\right) = 0$ (D) $P\left(\frac{A}{B}\right) = 1$

Answer (D)

Sol. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

9. Let $f : R \to R$ be defined by $f(x) = 2x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2 + 1}$, then gof is ____________.

(A) $\frac{2x^2 - 5}{4x^4 + 20x^2 + 26}$ (B) $\frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$

(C) $\frac{2x^2}{x^4 + 2x^2 - 4}$ (D) $\frac{2x^2}{4x^4 - 20x^2 + 26}$

Answer (B)

Sol. $g(f(x)) = \frac{f(x)}{(f(x))^2 + 1}$

$$= \frac{2x^2 - 5}{(2x^2 - 5)^2 + 1}$$

$$= \frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$$

10. Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$. Then the range of f is _________.

(A) $[1, \infty)$ (B) $[4, \infty)$

(C) R (D) $[5, \infty)$

Answer (A)

Sol. $f(x) = (x-2)^2 + 1$

As $x \in [2, \infty)$

$0 \le x - 2 < \infty$

$1 \le (x-2)^2 + 1 < \infty$

$\therefore$ Range is $[1, \infty)$

11. On R, binary operation $*$ is defined by $a * b = a + b + ab$ then identity and inverse of $*$ are .... respectively.

(A) $0, \frac{a}{1-a}$ (B) $1, \frac{a}{1+a}$

(C) $0, -\frac{a}{1+a}$ (D) $1, \frac{a}{1-a}$

Answer (C)

Sol. If $a * e = e * a = a$ then e is identity

$a + e + ae = a$

$e(1+a) = 0$

$e = 0$ (Identity)

Now if $a * b = e$ then b is inverse of a

$a + b + ab = 0$

$a = -b(1+a)$

inverse $b = \frac{-a}{1+a}$

12. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) =$ ____________.

(A) $\cos^{-1}\left(\frac{84}{85}\right)$ (B) $\cos^{-1}\left(\frac{24}{85}\right)$

(C) $\sin^{-1}\left(\frac{24}{85}\right)$ (D) $\sin^{-1}\left(\frac{84}{85}\right)$

Answer (A)

Sol. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right)$

$= \sin^{-1}\left(\frac{3}{5} \times \frac{15}{17} - \frac{4}{5} \times \frac{8}{17}\right)$

$= \sin^{-1}\left(\frac{13}{85}\right)$

$= \cos^{-1}\left(\frac{84}{85}\right)$

13. $\tan^2(\sec^{-1} 3) + \text{cosec}^2(\cot^{-1} 2) + \cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right) =$ ____________.

(A) 15 (B) 16

(C) 14 (D) 13

Answer (D)

Sol. $\tan^2(\sec^{-1} 3) + \text{cosec}^2(\cot^{-1} 2) + \cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right)$

$= \tan^2(\tan^{-1}(2\sqrt{2})) + \text{cosec}^2(\text{cosec}^{-1}\sqrt{5}) + \cos^2\left(\frac{\pi}{2}\right)$

$= 8 + 5 = 13$

14. If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$ then ____________.

(A) $1 - a^2 + bc = 0$ (B) $1 + a^2 + bc = 0$

(C) $1 + a^2 - bc = 0$ (D) $1 - a^2 - bc = 0$

Answer (D)

Sol. $A^2 = I$

$\begin{bmatrix} a & b \\ c & -a \end{bmatrix}\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1$

$\Rightarrow 1 - a^2 - bc = 0$

15. If A is a square matrix such that $A^2 = I$ then $(A-I)^3 + (A+I)^3 - 7A$ is equal to ____________.

(A) $I + A$ (B) $I - A$

(C) A (D) $3A$

Answer (C)

Sol. $(A-I)^3 + (A+I)^3 - 7A$

$= A^3 - I - 3A^2 + 3A + A^3 + I + 3A^2 + 3A - 7A$

$= 2A^3 - A$

$= A(2A^2 - I)$

$= A(2I - I) = A$

16. If $A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$ and inverse of A is $\frac{1}{2}\begin{bmatrix}1&-1&1\\-8&6&-2\\x&-3&1\end{bmatrix}$ then $x=$ ____________.

(A) 5 (B) 3

(C) 2 (D) 4

Answer (A)

Sol. We Know that $AA^{-1}=I$

Solving, we get $x=5$

17. Let $f(t)=\begin{vmatrix}\cos t&t&1\\2\tan t&t&2t\\\tan t&t&t\end{vmatrix}$. Then $\lim\limits_{t\to 0}\frac{f(t)}{t^2}$ is equal to ____________.

(A) 3 (B) 1

(C) –1 (D) 0

Answer (D)

Sol. $f(t)=\begin{vmatrix}cost&t&1\\2\tan&t&2t\\\tan t&t&t\end{vmatrix}$

$\Rightarrow f(t)=t\left[-t\cos t-(2t\tan t-2t\tan t)+1(\tan t)\right]$

$\Rightarrow f(t)=-t^2\cos t+t\tan t$

$\therefore \lim\limits_{t\to 0}\frac{f(t)}{t^2}\ \lim\limits_{t\to 0}\left(-\cos t+\frac{\tan t}{t}\right)=0$

18. If $x, y\in R$ and $\begin{vmatrix}(a^x+a^{-x})^2&(a^x-a^{-x})^2&1\\(b^x+b^{-x})^2&(b^x-b^{-x})^2&1\\(c^x+c^{-x})^2&(c^x-c^{-x})^2&1\end{vmatrix}=2y+6$, then $y=$ ____________.

(A) 0 (B) 3

(C) –3 (D) 6

Answer (C)

Sol. Applying $C_1\to C_1-C_2$, we get

$\begin{vmatrix}4&(a^x-a^{-x})^2&1\\4&(b^y-b^{-x})^2&1\\4&(c^x-c^{-x})^2&1\end{vmatrix}=2y+6$

$\Rightarrow 2y+6=0$

$\Rightarrow y=-3$

19. For ΔABC, the value of $\begin{vmatrix}0&\sin A&\tan B\\-\sin(B+C)&0&\cos C\\\tan(A+C)&-\cos C&0\end{vmatrix}=$ ____________.

(A) –1 (B) 0

(C) 1 (D) $\sin A\cos C$

Answer (B)

Sol. Given determinant is of skew symmetric matrix of odd order so value = 0

20. If function $f(\alpha)=\begin{cases}\frac{1-\cos 6\alpha}{36\alpha^2} & \text{if } \alpha\neq 0\\ k & \text{if } \alpha=0\end{cases}$ is continuous at $\alpha=0$ then $k=$ ____________.

(A) $-\frac{1}{2}$ (B) 1

(C) $\frac{1}{2}$ (D) 0

Answer (C)

Sol. $k=\lim\limits_{\alpha\to 0}\frac{1-\cos 6\alpha}{36\alpha^2}=\frac{1}{2}$

21. If $y=\sin^{-1}\left(\frac{2^{x-1}}{1+4^x}\right)$ and $\frac{dy}{dx}=\frac{2^{x-1}\log 2}{f(x)}$ then $f(0)=$ ____________.

(A) 0 (B) –2

(C) 2 (D) 2 log 2

Answer (C)

Sol. Now $y=\sin^{-1}\left(\frac{2\cdot 2^x}{(2^x)^2+1}\right)$

Let $2^x=\tan\theta$

$=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \sin^{-1}(\sin 2\theta)=2\theta$

$\Rightarrow y=2\tan^{-1}(2^x)$

$\therefore \frac{dy}{dx}=\frac{2.2^x\log 2}{1+4^x}=\frac{2^{x+1}\log 2}{4^x+1}$

$\therefore f(x)=4^x+1$

$\Rightarrow f(0)=2$

22. For function $f(x) = x + \frac{1}{x},\ x \in [1, 2]$, the value of C for mean value theorem is ____________.

(A) 2 (B) $\sqrt{2}$

(C) 1 (D) $\sqrt{3}$

Answer (B)

Sol. We know

$$f'(C) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{C^2} = \frac{1}{2} = C = \pm\sqrt{2}$$

Now as $C \in (1, 2)$

$$\Rightarrow C = \sqrt{2}$$

23. The interval in which $y = x^2 e^{-x}$ is increasing is ____________.

(A) $(0, 2)$

(B) $(2, \infty)$

(C) $(-\infty, \infty)$

(D) $(-2, 0)$

Answer (A)

Sol. The interval in which $y = x^2 \cdot e^{-x}$ is increasing

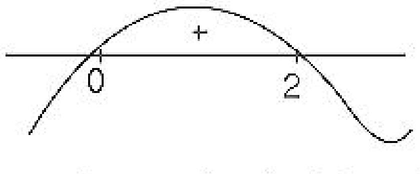
$$y = x^2 \cdot e^{-x}$$

$$\Rightarrow \frac{dy}{dx} = 2xe^{-x} + x^2\left(-e^{-x}\right)$$

$$= e^{-x}\left[2x - x^2\right]$$

$$= -xe^{-x}\left[x - 2\right]$$

For increasing function $\frac{dy}{dx} > 0$



$\therefore$ Increasing in interval (0, 2).

24. The rate of change of volume of sphere with respect to its radius r at $r = 2$ is ____________.

(A) 24π

(B) 32π

(C) 16π

(D) 8π

Answer (C)

Sol. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2$$

$$= 4\pi r^2$$

Now at $(r = 2) = 16\pi$

25. The tangent to the curve given by $x = e^{\theta} \cdot \cos\theta,\ y = e^{\theta} \cdot \sin\theta$ at $\theta = \frac{\pi}{4}$ makes an angle with X-axis is ____________.

(A) $\frac{\pi}{2}$ (B) 0

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Answer (A)

Sol. $x = e^{\theta}\cos\theta$

$$\Rightarrow \frac{dx}{d\theta} = e^{\theta}\cos\theta - e^{\theta}\sin\theta$$

$$= e^{\theta}(\cos\theta - \sin\theta)$$

$$y = e^{\theta}\sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = e^{\theta}\sin\theta + e^{\theta}\cos\theta$$

$$= e^{\theta}(\sin\theta + \cos\theta)$$

$$= \frac{dy}{dx} = \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}$$

Not defined at $\theta = \frac{\pi}{4}$

$\therefore$ $\theta = \frac{\pi}{2}$ (Angle formed by tangent with X-axis)

26. The minimum value of $f(x) = x\log x$ is ________.

(A) 0 (B) $-\frac{1}{e}$

(C) $\frac{1}{e}$ (D) e

Answer (B)

Sol. $y = x\log x$

$$\Rightarrow \frac{dy}{dx} = \log x + \left(x \times \frac{1}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \log x$$

$\therefore$ Minima at $x = \frac{1}{e}$

$$\therefore \quad f\left(\frac{1}{e}\right) = \frac{-1}{e}$$

27. If $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx = \frac{x^3}{3} + f(x) + C$, then $f(1)$ = ________.

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{1}{2}$

Answer (B)

Sol. $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$

$$\int \left(x^2 + \frac{1}{x^2 + 1}\right) dx$$

$$\frac{x^3}{3} + \tan^{-1}(x) + c$$

$$\therefore \quad f(x) = \tan^{-1}(x)$$

$$f(1) = \tan^{-1}(1)$$

$$\frac{\pi}{4}$$

28. $\int \frac{x+100}{(x+101)^2} e^x dx =$ ________ $+ C$.

(A) $\frac{1}{x+101} e^x$ (B) $\frac{x}{x+101} e^x$

(C) $\frac{1}{x+100} e^x$ (D) $(x+101) e^x$

Answer (A)

Sol. $\int \frac{(x+101) - 1}{(x+101)^2} e^x \, dx$

Now we know $\int e^x (f(x) + f'(x)) dx$

$$= e^x f(x) + c$$

$$= \frac{e^x}{x+101} + c$$

29. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx =$ ______ $+ C$.

(A) $-2\sqrt{\cot x}$ (B) $-2\sqrt{\tan x}$

(C) $2\sqrt{\cot x}$ (D) $\frac{1}{\sqrt{\cot x}}$

Answer (A)

Sol. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx$

$$= \int \frac{\sqrt{\cos x}}{\sqrt{\sin x} \sin x \cos x} dx$$

$$= \int (\cos x)^{\frac{-1}{2}} (\sin x)^{\frac{-3}{2}} dx$$

As $m + n$ negative even integer put

$\therefore \quad \tan x = t$

$$= \int (\tan x)^{\frac{-3}{2}} (\cos x)^{-2} dx$$

$$= \int (t)^{\frac{-3}{2}} dt$$

$$= \frac{t^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + c$$

$$= \frac{-2}{\sqrt{\tan x}} + c$$

$$= -2\sqrt{\cot x} + c$$

30. $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right)dx =$ ____________.

(A) π (B) 0

(C) $\frac{\pi}{2}$ (D) 1

Answer (B)

Sol. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right)dx$

as we know $\int_{-a}^{a} f(x) = 0$

if $f(x)+f(-x)=0$ (odd function)

$\therefore$ 0

31. $\int_{4}^{9} \frac{\sqrt{x}}{\left(30-x^{\frac{3}{2}}\right)^2}dx =$ ____________.

(A) $\frac{19}{66}$ (B) $\frac{19}{33}$

(C) $\frac{38}{99}$ (D) $\frac{19}{99}$

Answer (D)

Sol. $\int_{4}^{9} \frac{\sqrt{x}}{\left(30-x^{\frac{3}{2}}\right)^2}dx$ ...(1)

Let $30 - x^{\frac{3}{2}} = t$

$\Rightarrow \frac{-3}{2}(x)^{\frac{1}{2}} dx = dt$

$\Rightarrow \sqrt{x}dx = \frac{-2}{3}dt$

Now using (1),

$= \frac{-2}{3}\int_{22}^{3} \frac{1}{t^2}dt$

$= \frac{2}{3}\left[\frac{1}{t}\right]_{22}^{3}$

$= \frac{2}{3}\left[\frac{1}{3} - \frac{1}{22}\right]$

$= \frac{19}{99}$

32. If $f(a+b-x) = f(x)$ then $\int_a^b x \cdot f(x)dx$ is equal to ____________.

(A) $\frac{a+b}{2}\int_a^b f(x)dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(C) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (D) $\frac{b-a}{2}\int_a^b f(x)dx$

Answer (A)

Sol. $I = \int_a^b x \cdot f(x)dx$

$= \int (a+b-x) f(a+b-x) = I$

add both

$\Rightarrow 2I = \int_a^b (a+b) f(x)dx$

$= \left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

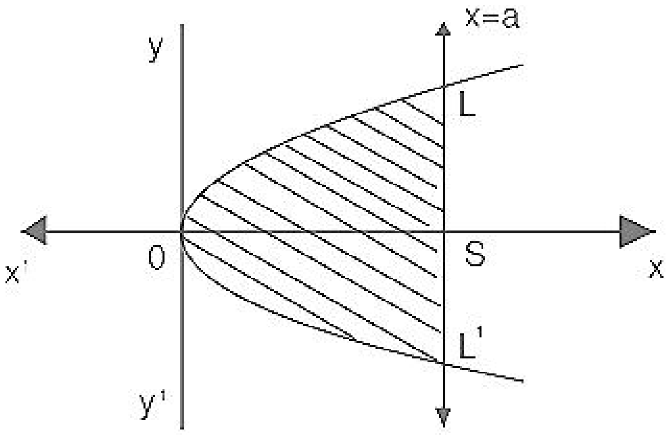
33. The area of the parabola $y^2 = 4ax$ bounded by its latus rectum is ____________.

(A) $\frac{16}{3}a^2$ (B) $\frac{4}{3}a^2$

(C) $\frac{8}{3}a^2$ (D) $4a^2$

Answer (C)

Sol. For parabola $y^2 = 4ax$



Area required = area OLSL'

= 2x Area OSL

$= 2 \times \int_0^a y dx$

Now parabola equation is

$y^2 = 4ax$

$\Rightarrow y = \pm\sqrt{4ax}$

Since OSL is in 1st quadrant

$y = \sqrt{4ax}$

Area required $= 2\times\int_0^a \sqrt{4ax}dx$

$= 2\sqrt{4a}\int_0^a \sqrt{x}dx$

$= 4\sqrt{a}\int_0^a \sqrt{x}dx$

$= \frac{8}{3}a^2$

34. The area enclosed by the curve $x = 4\cos\theta$, $y = 3\sin\theta$ is ____________.

(A) 4π (B) 6π

(C) 8π (D) 12π

Answer (D)

Sol. $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

And area of ellipse $= \pi ab$

$= \pi\times 4\times 3$

$= 12\pi$

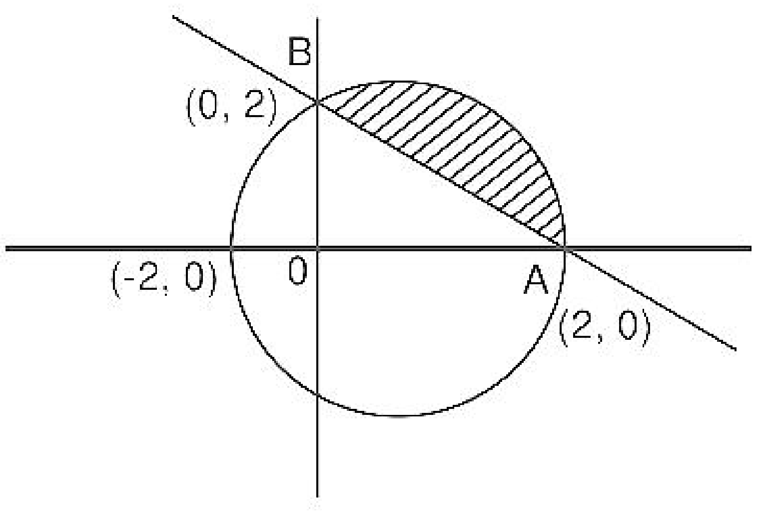
35. The smallest area enclosed by circle $x^2 + y^2 = 4$ and line $x + y = 2$ is ____________.

(A) $\pi + 2$ (B) $\pi - 2$

(C) π (D) 2π

Answer (B)

Sol. The Smallest area enclosed by circle $x^2 + y^2 = 4$ and line $x + y = 2$



Required area

$= \frac{1}{4}$(Area of circle) – area of triangle ΔOAB

$=$ Area $= \frac{\pi}{4}\times(2)^2 - \frac{1}{2}\times 2\times 2 = \pi - 2$

36. The order and degree of differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ are p and q respectively then $p + q =$ __________.

(A) 6 (B) 4

(C) 2 (D) 5

Answer (B)

Sol. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx}\right)^2$

$\therefore$ Order $= 2 = p$

Degree $= 2 = q$

$\therefore \quad p + q = 4$

37. Integrating factor of differential equation $(\tan^{-1}y - x)dy = (1+y^2)dx$ is __________.

(A) e^{1+y^2} (B) e^y

(C) $e^{\tan^{-1}x}$ (D) $e^{\tan^{-1}y}$

Answer (D)

Sol. $\frac{dy}{dx} = \frac{(1+y^2)}{(\tan^{-1}y - x)}$

$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2}\ \frac{-x}{1+y^2}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

Now integrating factor $= e^{\int\frac{dy}{1+y^2}}$

$e^{\tan^{-1}y}$

38. The differential equation $y\frac{dy}{dx} + x = k$ represents ____________.

(A) circles (B) hyperbolas

(C) parabolas (D) ellipses

Answer (A)

Sol. $\frac{dy}{dx} = \frac{k-x}{y}$

$y\,dy = dx(k-x) \quad \frac{y^2}{2} = kx - \frac{x^2}{2} + c$

$x^2 + y^2 = 2kx + 2c$

$\therefore$ Circle

39. If $\vec{a} = 2\hat{i} - \hat{j} + k, \vec{b} = \hat{i} + \hat{j} - 2k, \vec{c} = \hat{i} + 3\hat{j} - k$, if $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$, then the value of λ is ____________.

(A) 0 (B) 2

(C) –2 (D) 3

Answer (C)

Sol. As $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$\Rightarrow \vec{a}.(\lambda\vec{b} + \vec{c}) = 0$

$\Rightarrow (2\hat{i} - \hat{j} + k).(\lambda[\hat{i} + \hat{j} - 2k] + [\hat{i} + 3\hat{j} - k]) = 0$

$\Rightarrow 2(\lambda+1) - (\lambda+3) + 1(-2\lambda-1) = 0$

$\Rightarrow 2\lambda + 2 - \lambda - 3 - 2\lambda - 1 = 0$

$\Rightarrow \lambda = -2$

40. For three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfies $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 2$ then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} =$ __________.

(A) 29

(B) $\frac{29}{2}$

(C) $-\frac{9}{2}$

(D) $-\frac{29}{2}$

Answer (D)

Sol. $(\vec{a} + \vec{b} + \vec{c})^2 =$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow (9+16+4) + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = \frac{-29}{2}$

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