CAT 2015 based paper

1. d Let us assume that he has Rs. 100. In this he can buy 50 oranges or 40 mangoes. In other words, the price of an orange is Rs. 2 and that of a mango is Rs. 2.50. Now if he decides to keep 10% of his money for taxi fares, he would be left with Rs. 90. Now if he buys 20 mangoes, he would spend Rs. 50 and will be left with Rs. 40, in which he can buy 20 oranges.

2. a Let there be 100 voters in all. So initially there were 40 of these who promised to vote for P, while 60 of them promised to vote for Q. On the last day, (15% of 40) = 6 voters shifted their interest from P to Q and (25% of 60) = 15 voters shifted their interest from Q to P. So finally, P would end up getting (40 – 6 + 15) = 49 votes and Q would end up getting (60 – 15 + 6) = 51 votes. Hence, margin of victory for Q = (51 – 49) = 2, which matches the data given in the question. Hence, there were 100 voters in all.

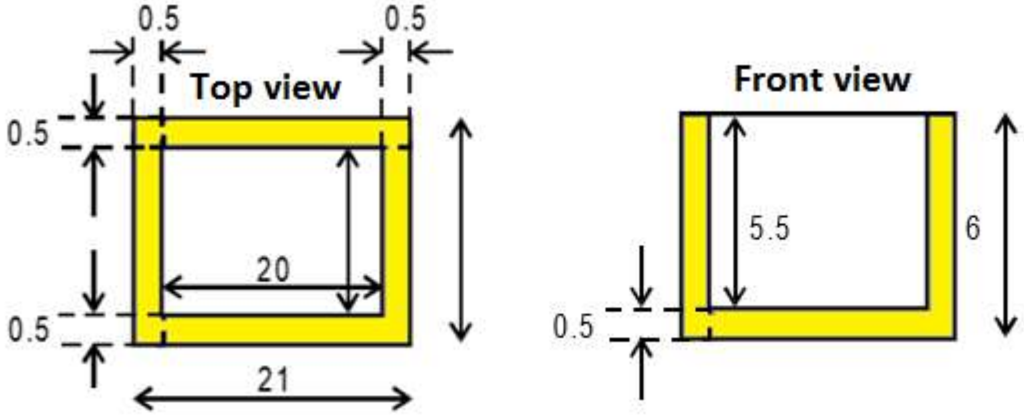
3. b Profit percentage in each case is
(i) 10%
(ii) (100 × 100)/ 900 = 100/9%
(iii) (100 - 100 / 1.1) / (100/1.1) = 10%
(iv) (100 x 100) / 9 = 200/19%

4. d Since n(n + 1) forms two consecutive integers, one of them will be even and hence the product will always be even. Also the sum of the squares of first n natural numbers is given by n(n+1)(2n+1)/6. Hence, our product will always be divisible by this. Also you will find that the product is always divisible by 3 (you can use any value of n to verify this). However, we can find that the option (d) is not necessarily true. Only under certain situation does it hold good. e.g. if n = 118, (2n + 1) = 237 or if n = 236, then (n + 1) = 237 or if n itself is 237, etc.

5. The sum of the perimeters of the triangles = (Perimetes of the square) + 2 × (Sum of its diagonals). This is so because the bases of each triangle will be counted once. But since each of the other two sides of the triangles is common to two triangles, it will be counted twice. Since area of the square = 4, its side = 2 and perimeter = 8. Also its diagonal = 2√2. So the required perimeter = (8 + 2 × 4√2) = 8(1 + √2).

6. a In the given figure, the area of the circle = πr^2. To find the area of the circle, we need to find the length of the side of the square. We know that OR = OT + TR = OT + OS = 2r. So in the right-angled triangle ORS, we have OR = 2r, OS = r. So $SR^2 = OR^2 – OS^2$. But SR2 = Area of the square = $4r^2 – r^2 = 3r^2$. So the required ratio = π/3.

7. Area of the original paper = π(20)2 = 400π cm2. The total cut portion area = 4(π)(5)2 = 100π cm2 . Therefore, area of the uncut (shaded) portion = (400 – 100) = 300π cm2. Hence, the required ratio = 300π : 100π = 3 : 1.



8. As it can be seen from the diagram, because of the thickness of the wall, the dimensions of the inside of the box is as follows: length = (21 – 0.5 – 0.5) = 20 cm, width = (11 – 0.5 – 0.5) = 10 cm and height = (6 – 0.5) = 5.5. Total number of faces to be painted = 4 walls + one base (as it is open from the top). The dimensions of two of the walls = (10 × 5.5), that of the remaining two walls = (20 × 5.5) and that of the base = (20 × 10). So the total area to be painted = 2 × (10 × 5.5) + 2 × (20 × 5.5) + (20 × 10) = 530 cm2. Since the total expense of painting this area is Rs. 70, the rate of painting = 70/530 = 0.13 = Re 0.1 per sq. cm.

9. c Let the original weight of the diamond be 10x. Hence, its original price will be $k(100x^2)$ . . . where k is a constant. The weights of the pieces after breaking are x, 2x, 3x and 4x. Therefore, their prices will be kx^2, $4kx^2$, $9kx^2$ and $16kx^2$. So the total price of the pieces = $(1 + 4 + 9 + 16)kx^2 = 30kx^2$. Hence, the difference in the price of the original diamond and its pieces = $100kx^2 – 30kx^2 = 70kx^2$ = 70000. Hence, kx^2 = 1000 and the original price = $100kx^2$ = 100 × 1000 = 100000 = Rs. 1 lakh.

10.Let radius of the semicircle be R and radius of the circle be r. Let P be the centre of semicircle and Q be the centre of the circle. Draw QS parallel to BC. Now, ΔΔ PQS ~ PBC

∴= PQ / PB = QS/ BC

⇒(R+ r) / √2R = (R – r) /R

⇒ R + r = √2R - √2r

⇒ r(1 + √2)= R(√2 - 1)

⇒ r = R (√2 - 1) / (√2 + 1) x (√2 - 1) / (√2 - 1)

⇒ r = R $(\sqrt{2} - 1)^2$

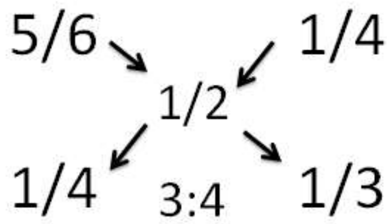
Required Ratio = $\pi r^2 / \pi R^2$ X 2

= $\pi R^2 (\sqrt{2} - 1)^4 / \pi R^2$ X 2

=$2(\sqrt{2} - 1)^4$: 1

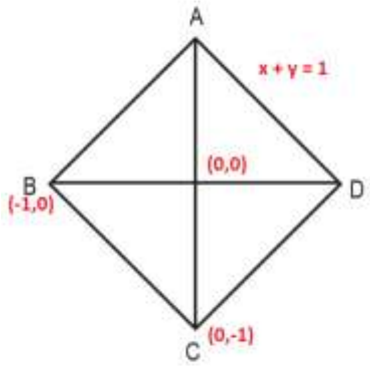
11.. d In a mile race, Akshay can be given a start of 128 m by Bhairav. This means that Bhairav can afford to start after Akshay has travelled 128 m and still complete one mile with him. In other words, Bhairav can travel one mile, i.e. 1,600 m in the same time as Akshay can travel (1600 – 128) = 1,472 m. Hence, the ratio of the speeds of Bhairav and Akshay = Ratio of the distances travelled by them in the same time = 1900/1472 = 25 : 23. Bhairav can give Chinmay a start of 4 miles. This means that in the time Bhairav runs 100 m, Chinmay only runs 96 m. So the ratio of the speeds of Bhairav and Chinmay = 100/96 = 25 : 24. Hence, we have B : A = 25 : 23 and B : C = 25 : 24. So A : B : C = 23 : 25 : 24. This means that in the time Chinmay covers 24 m, Akshay only covers 23 m. In other words, Chinmay is faster than Akshay. So if they race for 1 ½ miles = 2,400 m, Chinmay will complete the race first and by this time Aksahy would only complete 2,300 m. In other words, Chinmay would beat Akshay by 100 m = 1/ 16 mile.

12.d We can solve this by alligation. But while we alligate, we have to be careful that it has to be done with respect to any one of the two liquids, viz. either A or B. We can verify that in both cases, we get the same result. e.g. the proportion of A in the first vessel is 5/6 and that in the second vessel is 1/4, and we finally require ½ parts of A. Similarly, the proportion of B in the first vessel is 1/6 that in the second vessel is 3/4 and finally we want it to be 1/2. With respect to liquid A.



13.b $x^2 + y^2 = 0.1$
$|x - y|^2 = x^2 + y^2 - 2xy$
$(0.2)^2 = 0.1 - 2xy$ or $2xy = 0.06$ or $xy = 0.03$
Now $|x| + |y| = \sqrt{(x^2 + y^2 - 2xy)} = \sqrt{(0.1 + 0.06)}$
$|x| + |y| = 0.40$
Hence, $x = 0.3$, $y = 0.1$ or vice versa.

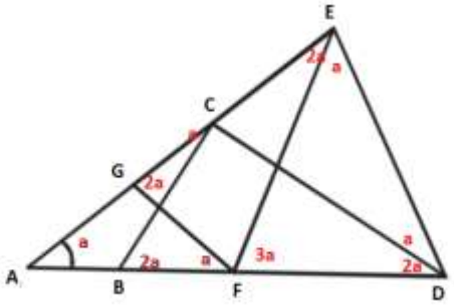
14.The gradient of the line AD is –1. Coordinates of B are (– 1, 0). Equation of line BC is $x + y = -1$.



15.b g(1) = f[f(1)] + 1 = 2 . Since f(1) has to be 1, else all the integers will not be covered. f(n) is the set of odd numbers and g(n) is the set of even numbers.

16.b f(1, 2) = f(0, f(1, 1)); Now f(1, 1) = f[0, f(1, 0)] = f[0, f(0, 1)] = f[0, 2] = 3 Hence, f(1, 2) = f(0, 3) = 4

17.Let ∠EAD = a. Then ∠AFG = a and also ∠ACB = a. Therefore, ∠CBD = 2a (exterior angle to ΔABC). Also ∠CDB = 2a (since CB = CD). Further, ∠FGC = 2a (exterior angle to ΔAFG). Since GF = EF, ∠FEG = 2a. Now ∠DCE = ∠DEC = b (say). Then ∠DEF = b – 2a. Note that ∠DCB = 180 – (a + b). Therefore, in ΔDCB, 180 – (a + b) + 2a + 2a = 180 or b = 3a. Further ∠EFD = ∠EDF = γ (say). Then ∠EDC = γ – 2a. If CD and EF meet at P, then ∠FPD = 180 – 5a (because b = 3a). Now in ΔPFD, 180 – 5a + γ + 2a = 180 or γ = 3a. Therefore, in ΔEFD, a + 2γ = 180 or a + 6a = 180 or a = 26 or approximately 25.



18.b Since a bucket holds 5 litres of water, Tap A discharges 20 litres of water in 24 min or 5/6 litres of water in 1 minute. Tap B discharges 40 litres in 1 hours or 2/3 litres in 1 minute. Tap C discharges 10 litres in 20 min. or ½ litres in 1 minute If A, B & C are all opened simultaneously, total discharge = (5/6 + 2/3 + ½) = 2 litres in 1 minute. So in 2 hours, the discharge would be 240 litres, which should be the capacity of the tank.

19.c It is clear that the ratio of the distances between (Delhi-Chandigarh) : (Chandigarh-Shimla) = 3 : 4. The ratio of the speeds between (Delhi-Chandigarh) : (Chandigarh-Shimla) = 3 : 2. Let the distances be 3x & 4x respectively and speeds be 3y and 2y. So the time taken will be (x/y) and (2x/y) respectively. Since average speed is given as (Total Distance) / (Total Time) = (7x)/(x/y + 2x/y) = 7y/3 = 49. Hence y = 21. So the average speed from Chandigarh to Shimla = 2y = 42 kmph.

20.c HINT : Students please note that you need not apply any formula in this case. The middle term of an AP is always the average of all the terms. Hence, if we multiply the middle term by the number of terms, we should get the sum of all the terms of that AP. In our problem, we have to find the sum of first 7 terms and we have been given the 4th term (which is the middle term). Hence the required answer is 8 x 7 = 56.

21.d

Option	Location	Expenditure of Town A students	Expenditure of Town B students	Total Expenditure
(a)	33 km from A	33x1.2x30 = 1188	67x1.2x100 = 8040	1188+8040 = 9228
(b)	33 km from B	67x1.2x30= 2412	33x1.2x100 = 3960	2412+3960 = 6372
(c)	Town A	0	100x100x1.2 = 12000 12000	12000
(d)	Town B	30x100x1.2 = 3600	0	3600

Hence we find that the least expenditure will be incurred if the school is located in town B. HINT : Students please note that since there are more number of students from Town B, to minimise the total expenditure the school should be located as closer to town B as possible.
Ganashakti

22.d It is clear that since there are 39 people in the ratio 6 : 5 : 2, there are 18 men, 15 women and 6 children. Ratio of the work done by a man : woman = 2 : 1. The ratio of the work done by a woman : child = 3 : 1. Hence the ratio of work done in a day by a man : a woman : a child = 6 : 3 : 1. So the ratio of the work done in a day by 18 men, 15 women and 6 children would be (18x6) : (15x3) : (6x1) = 108:45:6. Hence the daily wage of Rs.1113 should be divided in this ratio. That makes it, Rs.756 for men, 315 for women and Rs.42 for children. Hence 6 children earn Rs.42 in a day. So the daily wage of a child should be equal to 42/6 = Rs.7

23.a $U_0 = 2^0 – 1 = 0$
$U_1 = 2^1 – 1 = 1$
$U_2 = 2^2 – 1 = 3$
$U_3 = 2^3 – 1 = 7$ and so on.
$\therefore U_{10} = 2^{10} – 1 = 1023$.

24.b Since there are two numbers which are < 1 (viz. x & y), it is obvious that the median will be less than 1. Hence (c) cannot be the answer. Since x < 0.5 and 0 < y < 1, the median will not be < 0. Hence the answer is (b) between 0 and 1.

25.d Since, $0 < x < 1$, so $0 < x^2 < 1$ or $0 < 5x^2 < 5$. Similarly, as $0 < x < 1$, so $0 < x2 < 1$ or $0 < 3x^2/4 < 3/4$ or $0 > –3x^2/4 > –3/4$ or $½ > (½ –3x^2/4) > ½ - ¾$ i.e. $½ > 5x^2 > -¼$. So, we can see that $5x^2$ varies between 0 & 5, while $½ –3x^2/4$ varies between ½ & -1/4. Hence there is a common zone of 0 to ½ between the two. Let us check for some key values of x. If x = 0, then $(½ –3x^2/4) > 5x^2/4$. If x = 1, then $(1/2–3x^2/4) < 5x^2/4$. Hence between x = 0 & x = 1, there has to be some value of x for which $(½ – 3x^2/4) = 5x^2/4$, and this will be the maximum value of the given expression. Let us check for the same. If $(½ –3x^2/4) = 5x^2/4$, then $2x^2 = ½$. Or $x^2 = 1/4$. For Or $x^2 = 1/4$, the value of $5x^2/4 = 5/16 = (½ – 3x^2/4)$.

26.a Let us evaluate each option. (b) since 0 < y < 1 and z > 1, yz will always be < 1. (c) Since both x & y are not equal to 0, xy will never be 0. (d) y is a positive number < 1 and z is a positive number > 1, hence $(y^2 – z^2)$ is always negative. Since, (b), (c) and (d) are always true, the answer has to be (a). And this can be verified. For eg. If x = -2 and z = 3, then $(x^2 – z^2) = 4 – 9 = -5$, not a positive number.

27.b If you were to run two of three cycles of how she is counting, you will observe that the number that she counts on thumb are 1, 9, 17, 25 and so on. So it forms a pattern such that all the numbers that are 1 more than the multiples of 8 are counted on thumb. The closest multiple of 8 near 1994 is 1992. In other words she would count 1993 on thumb. So she would count 1994 on the index finger.

28.4 Let number of elements in progression be n, then 1000 = 1 + (n-1) d
$\Rightarrow$ (n-1) d = 999 = 3^3 x 37
Possible values of d = 3, 37, 9, 111, 27, 333, 999 Hence 7 progressions.

29.2 $2x + y = 40$ $x \le y \Rightarrow y = 40 – 2x$ Values of x and y that satisfy the equation

X	Y
1	38
2	36
3	34
4	32
5	30
6	28
7	26
8	24
9	22
10	20
11	18

12	16
13	14

∴ 13 values of (x, y) satisfy the equation such that $x \le y$

30. 4
$\log_y x = (a \cdot \log_z y) = (b \cdot \log_x z) = ab$
$a = \log_y x / \log_z y$ and similarly $b = \log_y x / \log_x z$
$a \times b = \log_y x / \log_z y \times \log_y x / \log_x z = (\log_y x)^3$
$\Rightarrow ab - a^3b^3 = 0$
Or, $a \times b (1 - a^2b^2) = 0$
Ab = +- 1
Only option (4) does not satisfy. Hence (4).

31.2 Let the number be 10x + y so when number is reversed the number because 10y + x. So, the number increases by 18 Hence (10y + x) - (10x + y) = 9 (y - x) = 18 y - x = 2 So, the possible pairs of (x, y) is (3, 1) (4, 2) (5, 3) (6, 4), (7, 5) (8, 6) (9, 7) But we want the number other than 13 so, there are 6 possible numbers are there i.e. 24, 35, 46, 57, 68, 79. So total possible numbers are 6.

32.So, total people reading the newspaper in consecutive months i.e. July and August and August and Sept. is 2 + 7 = 9 people.

33.2 Arithmetic mean is more by 1.8 means sum is more by 18. So ba – ab = 18 b > a because sum has gone up, e.g. 31 – 13 = 18 Hence, b – a = 2

34.Let OT be te tower.
Therefore, Height of tower = OT = 30 m
Let A and B be the two points on the level ground on the opposite side of tower OT.
Then, angle of elevation from A = ∠TAO = 45o
and angle of elevation from B = ∠TBO = 60o
Distance between AB = AO + OB = x + y (say)
Now, in right triangle ATO,
tan 45o = OT/AO = 30/x
=> x = 30/tan45 = 30 m
and in right traingle BTO
tan 60o =OT/OB = 30/y
=> y = 30/tan60 = 30/√3 = 30√3 / 3 = 17.32 m

Hence, the required distance = x + y = 30 + 17.32 = 47.32 m