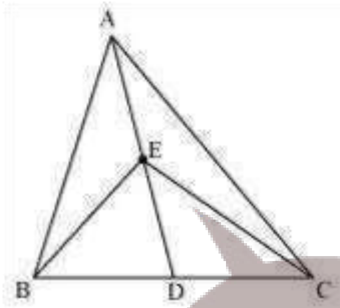
1:

In the given figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE)




Answer:

AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

∴ Area (ΔABD) = Area (ΔACD) ... (1) ED is the median of ΔEBC.

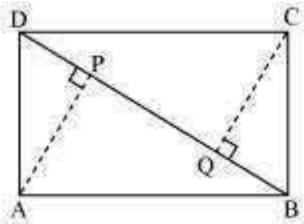
∴ Area (ΔEBD) = Area (ΔECD) ... (2)

On subtracting equation (2) from equation (1), we obtain Area

(ΔABD) − Area (EBD) = Area (ΔACD) − Area (ΔECD) Area

(ΔABE) = Area (ΔACE) Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that




(i) $\Delta APB \cong \Delta CQD$
(ii) $AP = CQ$

(i) In ΔAPB and ΔCQD,

∠APB = ∠CQD (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

∠ABP = ∠CDQ (Alternate interior angles for AB || CD)

∴ ΔAPB ≅ ΔCQD (By AAS congruency)

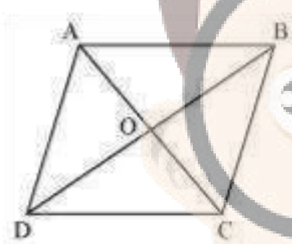
(ii) By using the above result

ΔAPB ≅ ΔCQD, we obtain AP = CQ (By CPCT)

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer:




We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

∴ Area (ΔAOB) = Area (ΔBOC) ... (1) In ΔBCD, CO is the median.

∴ Area (ΔBOC) = Area (ΔCOD) ... (2)

Similarly, Area (ΔCOD) = Area (ΔAOD) ... (3)

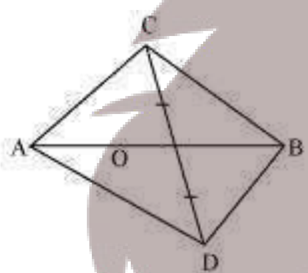
From equations (1), (2), and (3), we obtain

Area (ΔAOB) = Area (ΔBOC) = Area (ΔCOD) = Area (ΔAOD)

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If linesegment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).




Answer:

Consider ΔACD.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of ΔACD.

∴ Area (ΔACO) = Area (ΔADO) ... (1)

Considering ΔBCD, BO is the median.

∴ Area (ΔBCO) = Area (ΔBDO) ... (2)

Adding equations (1) and (2), we obtain

Area (ΔACO) + Area (ΔBCO) = Area (ΔADO) + Area (ΔBDO)

⇒ Area (ΔABC) = Area (ΔABD)

Question 6:

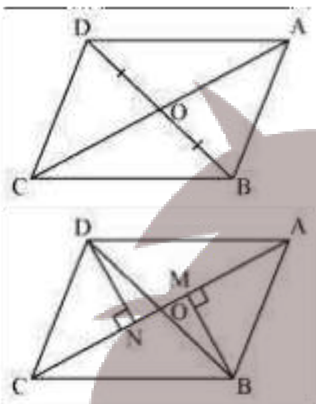
In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]




Let us draw DN ⊥ AC and BM ⊥ AC.

(i) In ΔDON and ΔBOM,

⊥ ⊥ DNO = BMO (By construction)

⊥ ⊥ DON = BOM (Vertically opposite angles)

OD = OB (Given)

By AAS congruence rule,

ΔDON ΔBOM

⊥ DN = BM ... (1)

We know that congruent triangles have equal areas.

⊥ Area (ΔDON) = Area (ΔBOM) ... (2)

In ΔDNC and ΔBMA,

⊥DNC = ⊥BMA (By construction)

CD = AB (Given)

DN = BM [Using equation (1)]

ΔDNC ı ΔBMA (RHS congruence rule)

Area (ΔDNC) = Area (ΔBMA) ... (3)

On adding equations (2) and (3), we obtain

Area (ΔDON) + Area (ΔDNC) = Area (ΔBOM) + Area (ΔBMA)

Therefore, Area (ΔDOC) = Area (ΔAOB)
(ii) We obtained,

Area (ΔDOC) = Area (ΔAOB)

⊥ Area (ΔDOC) + Area (ΔOCB) = Area (ΔAOB) + Area (ΔOCB)

(Adding Area (ΔOCB) to both sides)

⊥ Area (ΔDCB) = Area (ΔACB)

(iii) We obtained,
Area (ΔDCB) = Area (ΔACB)

If two triangles have the same base and equal areas, then these will lie between the same parallels.

ı DA || CB ... (4)

In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other pair of opposite sides is parallel (DA || CB).

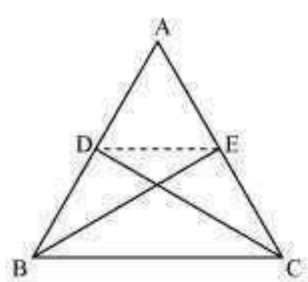
Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

Answer:

Answer:



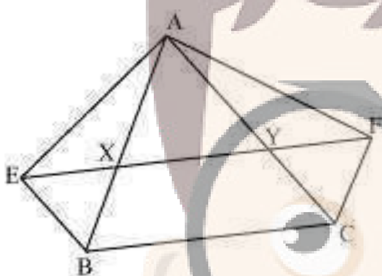
Since ΔBCE and ΔBCD are lying on a common base BC and also have equal areas,

ΔBCE and ΔBCD will lie between the same parallel lines.

⊥ DE || BC

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively, show that ar (ABE) = ar (ACF)




It is given that

XY || BC ꞁ EY || BC BE

|| AC ꞁ BE || CY

Therefore, EBCY is a parallelogram.

It is given that

XY || BC ⊁F || BC FC

|| AB ⊦C || XB

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

⊥ Area (EBCY) = Area (BCFX) ... (1)

Consider parallelogram EBCY and ΔAEB

These lie on the same base BE and are between the same parallels BE and AC.

⊥ Area (ΔABE) = $\frac{1}{2}$ Area (EBCY) ... (2)

Also, parallelogram BCFX and ΔACF are on the same base CF and between the same parallels CF and AB.

⊥ Area (ΔACF) = $\frac{1}{2}$ Area (BCFX) ... (3)

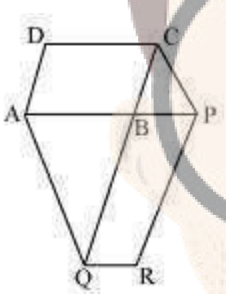
From equations (1), (2), and (3), we obtain

Area (ΔABE) = Area (ΔACF)

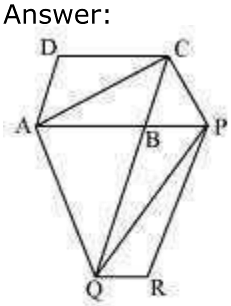
Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that ar (ABCD) = ar (PBQR).

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]




Answer:




Let us join AC and PQ.

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

∴ Area (ΔACQ) = Area (ΔAPQ)

∴ Area (ΔACQ) − Area (ΔABQ) = Area (ΔAPQ) − Area (ΔABQ)

∴ Area (ΔABC) = Area (ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

∴ Area (ΔABC) = $\frac{1}{2}$ Area (ABCD) ... (2)

Area (ΔQBP) = $\frac{1}{2}$ Area (PBQR) ... (3)

From equations (1), (2), and (3), we obtain

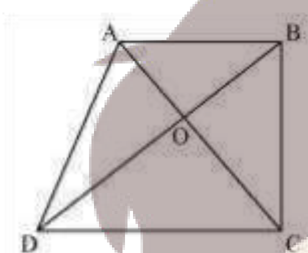
$\frac{1}{2}$ Area (ABCD) = $\frac{1}{2}$ Area (PBQR)

Area (ABCD) = Area (PBQR) Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

Prove that ar (AOD) = ar (BOC).

Answer:




It can be observed that ΔDAC and ΔDBC lie on the same base DC and between the same parallels AB and CD.

⊥ Area (ΔDAC) = Area (ΔDBC)

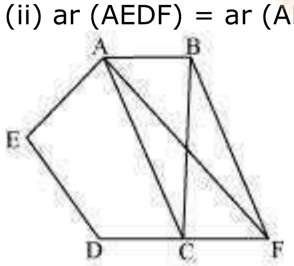
⊥ Area (ΔDAC) − Area (ΔDOC) = Area (ΔDBC) − Area (ΔDOC)

⊥ Area (ΔAOD) = Area (ΔBOC)

Question 11:

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that (i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)




Answer:

(i) ΔACB and ΔACF lie on the same base AC and are between

The same parallels AC and BF. ⇒
Area (ΔACB) = Area (ΔACF)

(ii) It can be observed that

Area (ΔACB) = Area (ΔACF)

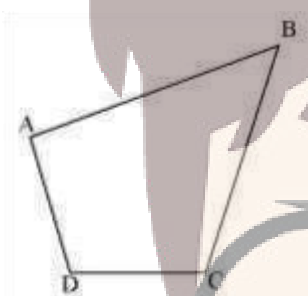
⇒ Area (ΔACB) + Area (ACDE) = Area (ACF) + Area (ACDE)

⇒ Area (ABCDE) = Area (AEDF)

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer:

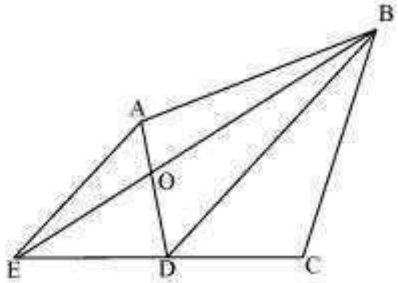



Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure)

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre) is equal to the area of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)




It can be observed that ΔDEB and ΔDAB lie on the same base BD and are between the same parallels BD and AE. ⊥ Area (ΔDEB) = Area (ΔDAB)

⊥ Area (ΔDEB) − Area (ΔDOB) = Area (ΔDAB) − Area (ΔDOB)

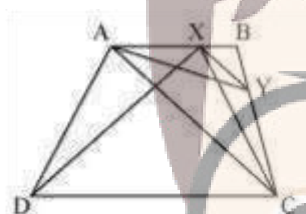
⊥ Area (ΔDEO) = Area (ΔAOB)

Question 13:

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y.

Prove that ar (ADX) = ar (ACY).

[Hint: Join CX.] Answer:




It can be observed that ΔADX and ΔACX lie on the same base AX and are between the same parallels AB and DC.

⊥ Area (ΔADX) = Area (ΔACX) ... (1)

ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and

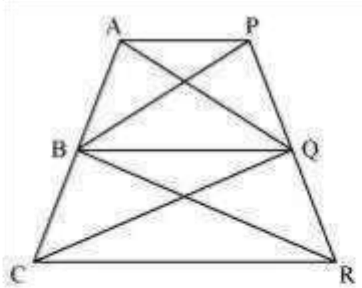
XY.

⊥ Area (ΔACY) = Area (ACX) ... (2) From equations (1) and (2), we obtain

Area (ΔADX) = Area (ΔACY) Question 14:

In the given figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

Answer:




Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,

ı Area (ΔABQ) = Area (ΔPBQ) ... (1)

Again, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels

BQ and CR.

⊥ Area (ΔBCQ) = Area (ΔBRQ) ... (2)

On adding equations (1) and (2), we obtain

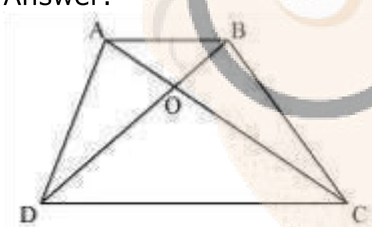
Area (ΔABQ) + Area (ΔBCQ) = Area (ΔPBQ) + Area (ΔBRQ) ⊥

Area (ΔAQC) = Area (ΔPBR)

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Answer:




It is given that

Area (ΔAOD) = Area (ΔBOC)

Area (ΔAOD) + Area (ΔAOB) = Area (ΔBOC) + Area (ΔAOB)

Area (ΔADB) = Area (ΔACB)

We know that triangles on the same base having areas equal to each other lie between the same parallels.

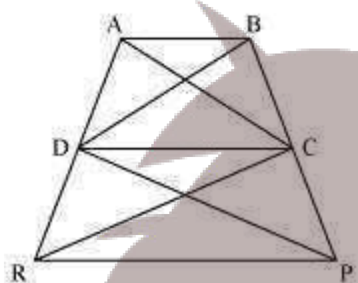
Therefore, these triangles, ΔADB and ΔACB, are lying between the same parallels. i.e., AB || CD

Therefore, ABCD is a trapezium.

Question 16:

In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Answer:




It is given that

Area (ΔDRC) = Area (ΔDPC)

As ΔDRC and ΔDPC lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines. ⊥ DC || RP

Therefore, DCPR is a trapezium. It is also given that

Area (ΔBDP) = Area (ΔARC)

⊥ Area (BDP) − Area (ΔDPC) = Area (ΔARC) − Area (ΔDRC)

⊥ Area (ΔBDC) = Area (ΔADC)

Since ΔBDC and ΔADC are on the same base CD and have equal areas, they must lie between the same parallel lines. ⊥ AB || CD

Therefore, ABCD is a trapezium.