QUESTION PAPER CODE 65/1/G

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $3\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix} - \begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$ ½ m

$= \begin{bmatrix}8 & 7\\6 & 2\end{bmatrix}$ ½ m

2. $y = e^{-x} + ax + b \Rightarrow y' = -e^{-x} + a$ ½ m

$y'' = e^{-x}$ or $\frac{d^2y}{dx^2} = e^{-x}$ ½ m

3. Order = 2, degree = 2 (any one correct) ½ m

Sum = 2 + 2 = 4 ½ m

4. d.r's of $\vec{AB}$: 1, –5 – a, b – 3 ; d.r's of $\vec{BC}$ are – 4, 16, 9 – b or d.r's of $\vec{AC}$: – 3, 11 – a, 6 ½ m

getting $a = -1, \quad b = 1, \quad a + b = 0$ ½ m

5. $|\vec{a}|\,|\vec{b}|\sin\theta = 16 \Rightarrow \sin\theta = \frac{16}{20} = \frac{4}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$ ½ m

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = \pm 12$ ½ m

6. $d = \frac{|9-6|}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$ ½ m

$= 1$ ½ m

SECTION - B

7. $\text{LHS} = \sin\left[\cot^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$ 1 m

$= \sin\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}x\right]$ 1 m

$= \sin\left[\frac{\pi}{2} - 2\tan^{-1}x + 2\tan^{-1}x\right]$ 1 m

$= \sin\frac{\pi}{2} = 1 = \text{R.H.S}$ 1 m

OR

$\tan^{-1}\left(\frac{\frac{x-5}{x-6}+\frac{x+5}{x+6}}{1-\frac{x-5}{x-6}\cdot\frac{x+5}{x+6}}\right) = \frac{\pi}{4}$ 2 m

$\Rightarrow \quad \frac{(x-5)(x+6)+(x+5)(x-6)}{x^2-36-x^2+25} = \tan\frac{\pi}{4}$ 1 m

$\Rightarrow \quad 2x^2 = 49$ ½ m

$\Rightarrow \quad x = \pm\frac{7}{\sqrt{2}}$ ½ m

8. $\text{L.H.S.} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$R_1 \to R_1 + b\cdot R_3, \quad R_2 \to R_2 - a R_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1+1 m

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ 1 m

Expanding and getting

$$\Delta = (1+a^2+b^2)^3 = \text{R.H.S.}$$ 1 m

9. $$A^2 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= O$$ 1 m

Pre multiplying by A^{-1} and getting $A^{-1} = \frac{1}{4}(5I - A)$ ½ m

and $$A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$ 1 m

OR

$$A = IA$$ 1 m

$$\Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2, \quad R_3 \rightarrow R_3 + 5R_2$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{pmatrix} A$$

$R_1 \rightarrow R_1 + R_3, \quad R_2 \rightarrow R_2 - 2R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix} A$$ [operating Row wise to reach at this step] 2½ m

$$A^{-1} = \begin{pmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$ ½ m

10. A Candidate who has made an attempt to solve the question to be given 4 marks 4 m

11. $y = -x^3 \log x$ ½ m

$$\frac{dy}{dx} = -x^2(1 + 3\log x)$$ 1 m

$$\frac{d^2y}{dx^2} = -\ (5x + 6x \log x)$$ 1 m

L.H.S. $= x\,[-(5x + 6x \log x)] + 2x^2 (1 + 3 \log x) + 3x^2$ 1 m

$= 0$ ½ m

$=$ R.H.S.

OR

$f(x) = (x-4)]\,(x-6)\,(x-8)$

$= x^3 - 18\,x^2 + 104\,x - 192$

Being a polynomial function f (x) is continuous

in [4, 10] and differentiable in (4, 10) with

$f'(x) = 3x^2 - 36\,x + 104$ 1+1 m

$\exists\ c \in (4, 10) \cdot$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow\quad 3\,c^2 - 36c + 104 = 8$ 1½ m

$\Rightarrow\quad c = 4, 8$; $c = 4 \notin (4, 10)$

$\therefore\ c = 8$: verifies the theoren ½ m

12. Given $\dfrac{x}{x-y} = \log a - \log(x-y)$ ½ m

Differentiating both sides and getting $[\because\ x \neq y]$

$$x - 2y + y\frac{dy}{dx} = 0$$ 2½ m

$$\Rightarrow \frac{dy}{dx} = \frac{2y - x}{y}$$

$$\Rightarrow \frac{dy}{dx} = 2 - \frac{x}{y}$$ 1 m

13. $I = \int \frac{dx}{x^3 (x^5+1)^{3/5}}$

$= \int \frac{dx}{x^3 \cdot x^3 \left(1+\frac{1}{x^5}\right)^{3/5}}$ 1½ m

Put $1+\frac{1}{x^5} = t$

$\Rightarrow \frac{dx}{x^6} = -\frac{dt}{5}$ 1 m

$\therefore \quad I = -\frac{1}{5}\int t^{-3/5} dt = -\frac{1}{2} t^{2/5} + C$ 1 m

$= -\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{2/5} + C$ ½ m

14. $I = \int_2^4 |x-2|\, dx + \int_2^4 |x-3|\, dx + \int_2^4 |x-4|\, dx$ ½ m

$= \int_2^4 (x-2)\, dx + \int_2^3 -(x-3)\, dx + \int_3^4 (x-3)\, dx + \int_2^4 -(x-4)\, dx$ 2 m

$= \left[\frac{x^2}{2} - 2x\right]_2^4 - \left[\frac{x^2}{2} - 3x\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^4 - \left[\frac{x^2}{2} - 4x\right]_2^4$ 1 m

$= 5$ ½ m

OR

$I = \int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x} dx = \int_0^{\pi/4} \frac{\cos x}{\cos^2 x (1+2\sin^2 x)} dx$

$= \int_0^{\pi/4} \frac{\cos x}{(1-\sin x)(1+\sin x)(1+2\sin^2 x)} dx$ 1 m

Put $\sin x = t \quad \Rightarrow \quad \cos x\, dx = dt$, when $x = 0, t = 0$

$$x = \frac{\pi}{4}, \quad t = \frac{1}{\sqrt{2}}$$ ½ m

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{(1-t)(1+t)(1+2t^2)}$$

$$\therefore \quad I = \int_0^{1/\sqrt{2}} \frac{dt}{6(1-t)} + \int_0^{1/\sqrt{2}} \frac{dt}{6(1+t)} + \int_0^{1/\sqrt{2}} \frac{2\ dt}{3(1+2t^2)}$$ 1 m

$$= \left[\frac{1}{6}\log\left|\frac{1+t}{1-t}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\left(\sqrt{2}\,t\right)\right]_0^{\frac{1}{\sqrt{2}}}$$ 1 m

$$= \frac{1}{6}\log\left|\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right| + \frac{\sqrt{2}}{3}\tan^{-1}(1)$$

$$= \frac{1}{3}\log\left|\sqrt{2}+1\right| + \frac{\pi}{6\sqrt{2}} \quad or \quad \frac{1}{6}\log\left(3+2\sqrt{2}\right) + \frac{\pi}{6\sqrt{2}}$$ ½ m

15. $$I = \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-2\sin\cos x}{2\sin^2 x}\right)dx$$

$$= \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1}{2}\operatorname{cosec}^2 x - \cot x\right)dx$$ 1½ m

Put $2x = t \quad \Rightarrow \quad dx = \frac{dt}{2}$

when $x = \frac{\pi}{4}$, $t = \frac{\pi}{2}$; $x = \frac{\pi}{2}$, $t = \pi$ 1 m

$$\therefore \quad I = \frac{1}{2}\int_{\pi/2}^{\pi} e^t\left(\frac{1}{2}\,\text{cosec}^2\frac{t}{2} - \cot\frac{t}{2}\right)dt$$

$$= -\frac{1}{2}\left[\cot\frac{t}{2}\cdot e^t\right]_{\frac{\pi}{2}}^{\pi}$$ 1 m

$$= \frac{e^{\pi/2}}{2}$$ ½ m

16. Let $\overrightarrow{OA} = 4\hat{i}+8\hat{j}+12\hat{k}$, $\overrightarrow{OB} = 2\hat{i}+4\hat{j}+6\hat{k}$

$\overrightarrow{OC} = 3\hat{i}+5\hat{j}+4\hat{k}$, $\overrightarrow{OD} = 5\hat{i}+8\hat{j}+5\hat{k}$

$\overrightarrow{AB} = -2\hat{i}-4\hat{j}-6\hat{k}$, $\overrightarrow{AC} = -\hat{i}-3\hat{j}-8\hat{k}$, $\overrightarrow{AD} = \hat{i}-7\hat{k}$ 1½ m

Now, $$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = 0$$ 2 m

$\therefore$ A, B, C, D are coplanar ½ m

17. Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black

$P(E_1) = \frac{5}{9}$, $P(E_2) = \frac{4}{9}$ 1 m

$P(A/E_1) = \frac{{}^5C_2}{{}^8C_2} = \frac{5}{14}$, $P(A/E_2) = \frac{{}^4C_2}{{}^8C_2} = \frac{3}{14}$ 1 m

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$$ ½ m

$$= \frac{\frac{5}{9} \times \frac{5}{14}}{\frac{5}{9} \times \frac{5}{14} + \frac{4}{9} \times \frac{3}{14}}$$ 1 m

$$= \frac{25}{37}$$ ½ m

18. Equation of line joining (4, 3, 2) and (1, – 1, 0) is

$$\frac{x-4}{-3} = \frac{y-3}{-4} = \frac{z-2}{-2}$$ ½ m

Equation of line joining (1, 2, – 1) and (2, 1, 1) is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$ ½ m

Let equation of the required line be

$$\frac{x-1}{a} = \frac{y+1}{b} = \frac{z-1}{c} = \lambda \quad .... (i)$$ ½ m

According to the question $3a + 4b + 2c = 0$

$$a - b + 2c = 0$$ 1 m

Solving, $\frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \mu$

$\Rightarrow \quad a = 10\mu,\ b = -4\mu,\ c = -7\mu$ ½ m.

(i) $\Rightarrow$ Equation of the line is

$$\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7} \quad \text{[cartesian form]}$$ ½ m

Vector form, $\vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 7\hat{k}\right)$ ½ m

19. $\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} H & R & HW \end{array} \\ \begin{bmatrix} 3 & 4 & 6 \\ 4 & 5 & 3 \\ 6 & 2 & 4 \end{bmatrix} \end{array} \begin{bmatrix} 2500 \\ 3100 \\ 5100 \end{bmatrix}$ 1 m

$= \begin{bmatrix} 50500 \\ 40800 \\ 41600 \end{bmatrix}$ 1 m

Hence money awarded by A = Rs. 50500

money awarded by B = Rs. 40800 1 m

money awarded by C = Rs. 41600

Respect for elders or Any relevant value 1 m

SECTION - C

20. $(a, b) * (c, d) = (a + c, b + d) \quad \forall\ a, b, c, d \in R$

Since $a + c \in R$ and $b + d \in R \Rightarrow (a + c, b + d) \in R \times R$ 1½ m

i.e. '*' is binary operation

For commutative

consider $(c, d) * (a, b) = (c + a, d + b)$

$= (a + c, b + d)$

$= (a, b) * (c, d)$ 1½ m

$\Rightarrow$ '*' is commutative

For Associative

Let $(a, b), (c, d), (e, f) \in R \times R = A$

$[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)$

$= (a + c + e, b + d + f)$ ....(i)

again $(a, b) * [(c, d)] * (e, f)] = (a, b) * (c + e, d + f)$

$= (a + c + e, b + d + f)$ ....(ii) 1½ m

(i) & (ii) $\Rightarrow$ '*' is associative

For identity element

Let $(e_1, e_2) \in R \times R$ be the identity element (if exists)

then $(a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (e_1, e_2) = (0, 0) \in R \times R$ 1½ m

OR

$f(x) = x^2 - x; \quad x \in \{-1, 0, 1, 2\}$

$f(-1) = 2,\ f(0) = 0,\ f(1) = 0,\ f(2) = 2$

$\therefore\ f = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$g(x) = 2\left|x - \frac{1}{2}\right| - 1 \quad \forall\ x \in \{-1, 0, 1, 2\}$

$g(-1) = 2,\ g(0) = 0,\ g(1) = 0,\ g(2) = 2$

$\therefore\ g = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

$(g o f)(x) = g(f(-1), g(f(0), g(f(1), g(f(2) \quad \forall\ x \in A$

$= 2, 0, 0, 2$

$\therefore\ g o f = \{(-1, 2), (0, 0), (1, 0), (2, 2)\}$ 2 m

Hence $f = g = g o f$

21. Given curve cuts the x – axis when y = 0 ½ m

when y = 0, x = 7, hence point is (7, 0) ½ m

$$\frac{dy}{dx} = \frac{1 - y(2x - 5)}{x^2 - 5x + 6}$$ 2½ m

$$\left.\frac{dy}{dx}\right]_{(7,0)} = \frac{1}{20}$$ ½ m

Equation of the tangent is $y - 0 = \frac{1}{20}(x - 7)$ 1 m

$\Rightarrow x - 20y = 7$

Equation of the normal is $y - 0 = -20(x - 7)$ 1 m

$$\Rightarrow \quad 20x + y = -7$$

OR

$f(x) = \cos^2 x + \sin x, \quad x \in [0, \pi]$

$f'(x) = \cos x\,(-2\sin x + 1)$ 1 m

For extremum, $f'(x) = 0 \Rightarrow x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$ 1½ m

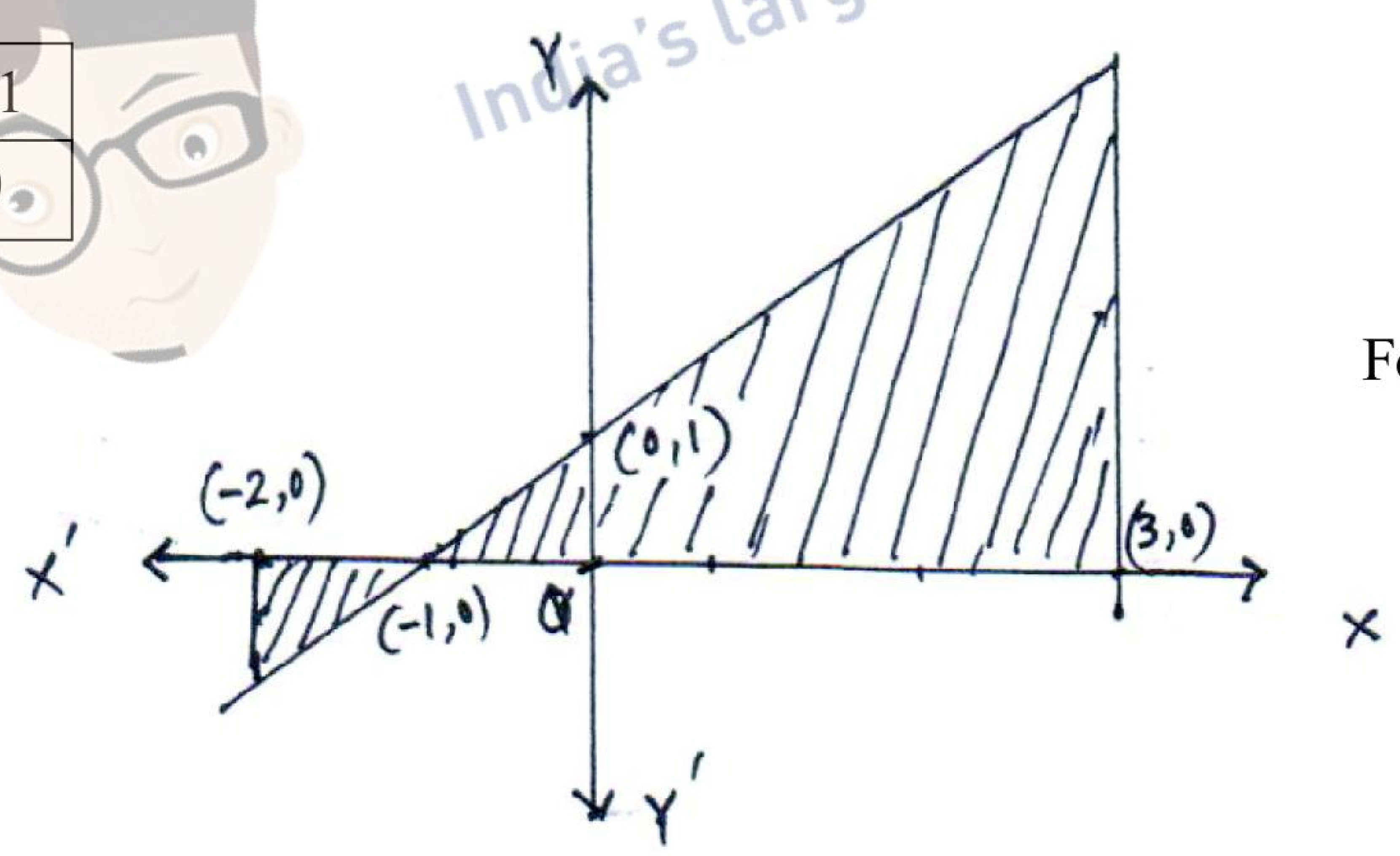
Now $f(0) = 1,\ f\left(\frac{\pi}{6}\right) = \frac{5}{4},\ f\left(\frac{\pi}{2}\right) = 1,\ f\left(\frac{5\pi}{6}\right) = \frac{5}{4},\ f(\pi) = 1$ 1½ m

Absolute max. is $\frac{5}{4}$ at $x = \frac{\pi}{2}$ and $\frac{5\pi}{6}$ 1 m

Absolute min. is 1 at $x = 0, \frac{\pi}{6}$ and π 1 m

22. $y = x + 1, \quad x = -2, \quad x = 3$

x	0	–1
y	1	0



For correct figure 1 m

Reqd area $= \left|\int_{-2}^{-1}(x+1)\,dx\right| + \int_{-1}^{3}(x+1)\,dx$ 2 m

$= \left|\left(\frac{x^2}{2} + x\right)_{-2}^{-1}\right| + \left(\frac{x^2}{2} + x\right)_{-1}^{3}$ 2 m

$= \frac{17}{2}$ sq. units 1 m

23. $(y - \sin x)\, dx + (\tan x)\, dy = 0 \Rightarrow \frac{dy}{dx} + \cot x\, y = \cos x$ 1 m

Linear diff. equ. with $P = \cot x,\ Q = \cos x$

I.F. $= \sin x$ 1 m

Solution is $y \cdot \sin x = \int \cos x \cdot \sin x\, dx + c$

$$= -\frac{1}{4} \cos 2x + c$$ 2 m

when $x = 0,\ y = 0 \Rightarrow c = \frac{1}{4}$ 1 m

Particular solution is

$$y \sin x = \frac{1}{4}(-\cos 2x + 1) = \frac{\sin^2 x}{2}$$

$\Rightarrow y = \frac{1}{2} \sin x$ 1 m

24. $d \cdot r's$ of first line : $k - 5,\ 1,\ 2k + 1$ 1 m

$d \cdot r's$ of 2nd line : $-1,\ k,\ 5$ 1 m

$\because$ lines are $\perp$ $\therefore$ $-1\,(k - 5) + k\,(1) + 5\,(2k + 1) = 0$

$\Rightarrow k = -1$ 1 m

Eqns of lines become $\frac{x+3}{-6} = \frac{y-1}{-1} = \frac{z-5}{-1}$ and $\frac{x+2}{-1} = \frac{y-2}{-1} = \frac{z}{5}$ 1 m

Eqn of plane containing these two lines is

$$\begin{vmatrix} x+2 & y-2 & z \\ -6 & 1 & -1 \\ -1 & -1 & 5 \end{vmatrix} = 0$$ 1 m

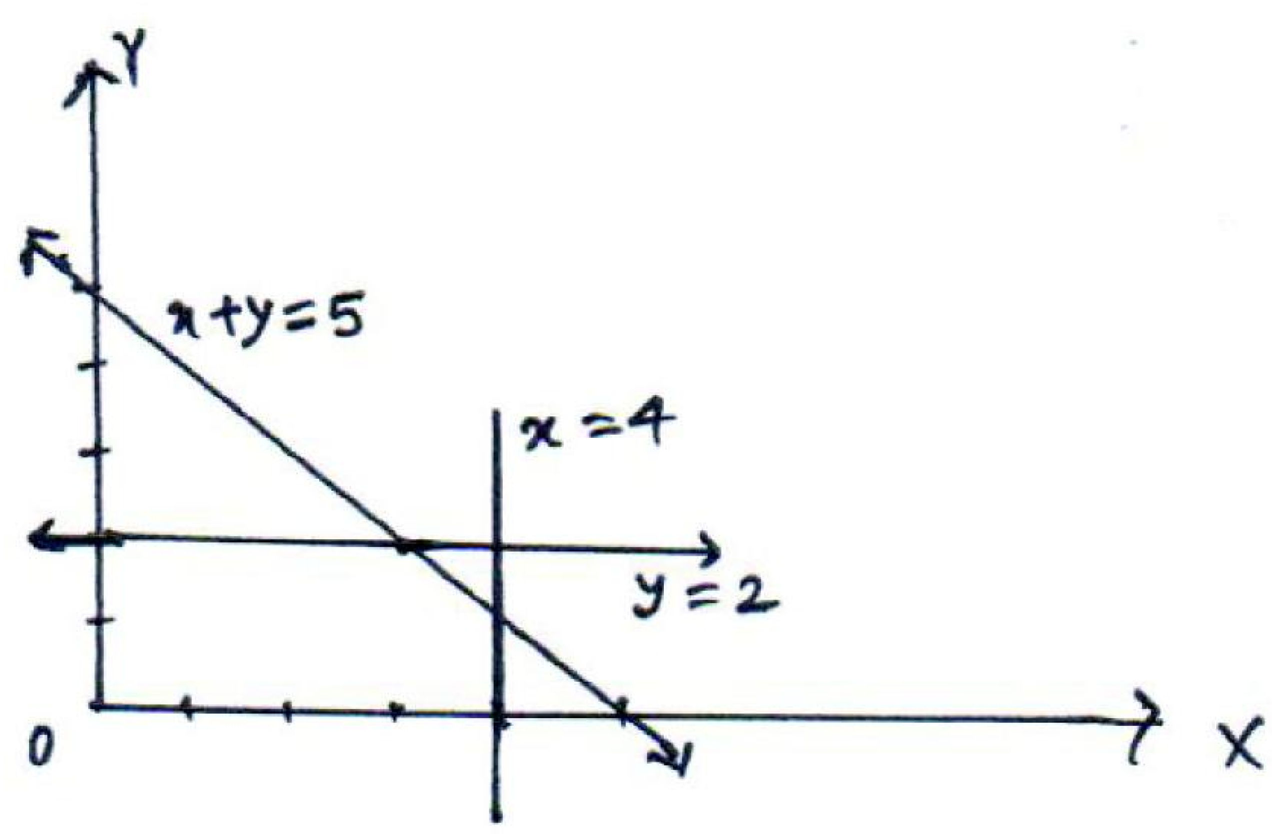
$\Rightarrow 4x + 31y + 7z = 54$ 1 m

25. Let x kg of B_1 and y kg of B_2 is taken

then to minimize $Z = 5x + 8y$ 1 m

subject to the following constraints 3 m

$x + y = 5, \quad x \leq 4, \quad y \geq 2$



Graph 2 m

26. Let x denote no. of heads

here $p = \frac{1}{2}, \quad q = \frac{1}{2}$ 1 m

$P(x = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$

$= {}^nC_r \left(\frac{1}{2}\right)^n$ 1 m

Now $P(x = 1) = {}^nC_1 \left(\frac{1}{2}\right)^n$

$P(x = 2) = {}^nC_2 \left(\frac{1}{2}\right)^n$ 1½ m

$P(x = 3) = {}^nC_3 \left(\frac{1}{2}\right)^n$

According to the question

$2.\ {}^nC_2 \left(\frac{1}{2}\right)^n = \left({}^nC_1 + {}^nC_3\right)\left(\frac{1}{2}\right)^n$ 2 m

$\Rightarrow \quad n = 2$ or 7 ½ m

n can not be 2 Hence $n = 7$