MATHEMATICS

1. The vertices of a triangle are A(0, 0), B(0, 2) and C(2, 0); then find the distance between its Ortho-centre and Circum-centre.

   (A) 0 (B) $\sqrt{2}$

   (C) $\frac{1}{\sqrt{2}}$ (D) None of these

2. The vertices of ΔABC are A(2, 2), B(– 4, – 4) and C(5, – 8). Find the length of a median of a triangle, which is passing through the point C.

   (A) $\sqrt{65}$ (B) $\sqrt{117}$

   (C) $\sqrt{85}$ (D) $\sqrt{116}$

3. The equation of the lines with slope -2, and intersecting X-axis at a distance of 3 units from the origin is ....

   (A) $2x + y \pm 6 = 0$ (B) $x + 2y \pm 6 = 0$

   (C) $2x + y \pm 3 = 0$ (D) $x + 2y \pm 3 = 0$

4. If the distance of a line from the origin is $\sqrt{5}$ and having intercepts in the ratio of 1 : 2 on axes, then the equations of lines are ....

   (A) $2x - y \pm 5 = 0$ (B) $2x + y \pm 5 = 0$

   (C) $x - 2y \pm 5 = 0$ (D) $x + 2y \pm 5 = 0$

5. Ortho-centre of the triangle formed by the lines $x - y = 0,\ x + y = 0,\ x = 3$ is .....

   (A) (0, 0) (B) (3, 0)

   (C) (0, 3) (D) can't be found

6. If the line $y = mx + c$ is passing through origin and away from the circle $4x^2 + 4y^2 - 80y + 360 = 0$, then .....

(A) $|m| > 3$ (B) $m > 3$

(C) $m < -3$ (D) $|m| < 3$

7. The centre of the circle $2x^2 + 2y^2 + \frac{3}{2}x + 9 = 0$ is ....

(A) $\left(\frac{3}{8}, 0\right)$ (B) $\left(-\frac{3}{8}, 0\right)$

(C) $\left(0, \frac{3}{8}\right)$ (D) $\left(0, -\frac{3}{8}\right)$

8. The equation of tangent of $y^2 = 12x$ and making an angle $\frac{\pi}{3}$ with X-axis is ......

(A) $\pm y - \sqrt{3}x + \sqrt{3} = 0$ (B) $\pm y + \sqrt{3}x + 3 = 0$

(C) $\pm y - \sqrt{3}x - \sqrt{3} = 0$ (D) $\pm y + \sqrt{3}x - 3 = 0$

9. A focal chord of the Parabola $y^2 = 4x$ makes an angle of measure θ with the positive direction of the X-axis. If the length of the focal chord is 8,

then $\theta =$ .... $\left(0 < \theta < \frac{\pi}{2}\right)$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) None of them

10. If a line $x + 2y = k$ touches to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, then find k.

(A) $k = \pm 2$ (B) $k = \pm 5$

(C) $k = 25$ (D) $k = -25$

11. Equation of auxiliary circle of $\frac{x^2}{16} - \frac{y^2}{25} = -1$ is ......

(A) $x^2 + y^2 = 16$ (B) $x^2 + y^2 = 25$

(C) $x^2 + y^2 = 9$ (D) $x^2 + y^2 = 41$

12. Find the measure of angle between the asymptotes of $x^2 - y^2 = 16$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

13. If $|\bar{x}| = 13$ and direction angles of $\bar{x}$ are $\cos^{-1}\frac{3}{13}$, $\cos^{-1}\frac{4}{13}$ and $\cos^{-1}\frac{12}{13}$, then find $\bar{x}$.

(A) $3\bar{i} + 4\bar{j} - 12\bar{k}$ (B) $3\bar{i} + 4\bar{j} + 12\bar{k}$

(C) $3\bar{i} - 4\bar{j} + 12\bar{k}$ (D) $3\bar{i} - 4\bar{j} - 12\bar{k}$

14. In R^2, find the unit vector orthogonal to unit vector $\bar{x} = (\cos\alpha, \sin\alpha)$.

(A) $\left(\cos \alpha/2, \sin \alpha/2\right)$ (B) $(-\cos\alpha, -\sin\alpha)$

(C) $(-\sin\alpha, \cos\alpha)$ (D) $(\cos\alpha, \sin\alpha)$

15. If $\bar{a} = 3\bar{i} + 4\bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + \bar{j} - \bar{k}$, then $\text{Comp}_{\bar{b}}\, \bar{a}$ = ...

(A) $3\sqrt{2}$ (B) $2\sqrt{3}$

(C) $-3\sqrt{2}$ (D) $-2\sqrt{3}$

16. By vector method, find the co-ordinates of a point which divides $\overline{AB}$ from A in the ratio $-3:2$; where A(1, 2, 3), B(5, 6, 7).

(A) (13, 14, 15) (B) (–13, –14, –15)

(C) (–13, 14, 15) (D) (–13, –14, 15)

17. Find the shortest distance between the lines $\dfrac{x-3}{2} = \dfrac{y+15}{-7} = \dfrac{z-9}{5}$ and

$\dfrac{x+1}{2} = \dfrac{y-1}{1} = \dfrac{z-9}{-3}$

(A) $2\sqrt{3}$ (B) $3\sqrt{3}$

(C) $4\sqrt{3}$ (D) None of these

18. Find the distance between the planes $\bar{r} \cdot \left(2\bar{i} - \bar{j} + 3\bar{k}\right) = 4$ and

$\bar{r} \cdot \left(6\bar{i} - 3\bar{j} + 9\bar{k}\right) + 13 = 0.$

(A) $\dfrac{5}{3\left(\sqrt{14}\right)}$ (B) $\dfrac{10}{3\left(\sqrt{14}\right)}$

(C) $\dfrac{25}{3\left(\sqrt{14}\right)}$ (D) None of these

19. Find the direction of intersecting line of two planes

$2x + y + z = 1$ and $3x + 2y - z = 3$

(A) (3, 5, 1) (B) (3, 5, –1)

(C) (– 3, 5, 1) (D) None of these

20. The equation of a Sphere having centre (1, 2, 3) and radius 3 units is ......

(A) $x^2 + y^2 + z^2 - 2x - 4y - 6z = 0$

(B) $x^2 + y^2 + z^2 - 2x - 4y - 6z + 5 = 0$

(C) $x^2 + y^2 + z^2 - 2x - 4y - 6z - 5 = 0$

(D) None of these.

21. $\lim\limits_{x \to \infty} \left(1 + \frac{4}{x-1}\right)^{x+3} =$ ....

(A) e^4 (B) e^2

(C) e^3 (D) e

22. $N(4, \delta) \cap N(6, \delta) \neq \phi$, then the value of δ is ....

(A) 1 (B) > 1

(C) < 1 (D) δ is not possible.

23. $\lim\limits_{x \to 2} \frac{x^n - 2^n}{x - 2} = 32$, then $n =$ .... ; $n \in N$

(A) 3 (B) 4

(C) 5 (D) 2

24. If $f(x)=\begin{cases} cx+1 & ; \quad x \le 3 \\ cx^2-1 & ; \quad x > 3 \end{cases}$ is continuous at $x = 3$, then $c =$ ....

(A) $1/3$ (B) $2/3$

(C) $3/2$ (D) 3

25. If $f'(x)=f(x)$, $f(0) = 1$, then $\lim_{x\to 0} \frac{f(x)-1}{x}$ = ...

(A) 0 (B) 1

(C) – 1 (D) 2

26. $\frac{d}{dx}\left[x^3\left(\cos^{-1}x+\sin^{-1}x\right)\right]=$ .... ; $|x|<1$

(A) 0 (B) $\frac{\pi}{2}\cdot x^3$

(C) $3x^2\cdot\frac{\pi}{2}$ (D) $3x^2$

27. $\frac{d}{dx}\left[\sin^{-1}\frac{x}{a}\right]$ = ... ; $a<0$ and $\left|\frac{x}{a}\right|<1$

(A) $\frac{1}{\sqrt{a^2-x^2}}$ (B) $\frac{1}{\sqrt{x^2-a^2}}$

(C) $\frac{-1}{\sqrt{a^2-x^2}}$ (D) $\frac{-1}{\sqrt{x^2-a^2}}$

28. If $\pi < x < 2\pi$, then $\frac{d}{dx}\left[\tan^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)^{1/2}\right]$ is ....

(A) 0 (B) 1

(C) $1/2$ (D) $-1/2$

29. A tangent to the curve $y = \log_e x$ at point P is passing through the point (0, 0), then the co-ordinates of point P are ....

(A) $(0, e)$ (B) $(e, 0)$

(C) $(e, 1)$ (D) $(1, e)$

30. Find the point on the Parabola $y^2 = 8x$ such that $\frac{dx}{dt} = \frac{dy}{dt}$

(A) (0, 0) (B) $\left(\frac{1}{2}, 2\right)$

(C) (4, 2) (D) (2, 4)

31. The equation of the normal to the curve $x^2 = 4y$ passing through the point (1, 2) is ....

(A) $x + y - 3 = 0$ (B) $x - y - 3 = 0$

(C) $x + y + 3 = 0$ (D) $x - y + 3 = 0$

32. $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e}\, dx = \ldots\ldots\ldots + c$

(A) $\log\left|e^x + x^e\right|$ (B) $e \log\left|e^x + x^e\right|$

(C) $\frac{1}{e}\log\left|e^x + x^e\right|$ (D) $\frac{1}{e}\log\left|e^{x-1} + x^{e-1}\right|$

33. $\int \frac{x^3}{x-1}\, dx + \int \frac{1}{1-x}\, dx = \ldots\ldots\ldots + c$

(A) $\frac{x}{6}\left(2x^2 - 3x + 6\right)$

(B) $\frac{x}{6}\left(2x^2 + 3x + 6\right)$

(C) $\frac{x}{3}\left(2x^2 - 3x + 6\right)$

(D) $\frac{x}{3}\left(2x^2 + 3x + 6\right)$

34. $\int \frac{x^2}{\left(x^3 - 1\right)\left(x^3 + 4\right)}\, dx = \ldots\ldots\ldots + c$

(A) $\frac{1}{15} \log \left| \frac{x^3 - 1}{x^3 + 4} \right|$

(B) $-\log \left| \frac{x^3}{x^3 - 1} \right|$

(C) $\frac{1}{2} \log \left| \frac{\left(x^3 - 1\right)\left(x^3 + 4\right)}{x^3} \right|$

(D) $\log \left| \frac{x^3}{\left(x^3 - 1\right)\left(x^3 + 4\right)} \right|$

35. $\int e^{e^{e^x}} \cdot e^{e^x} \cdot e^x \, dx = \ldots\ldots\ldots + c$

(A) e^{e^x}

(B) $\frac{1}{2} e^2 \cdot e^x$

(C) $e^{e^{e^x}}$

(D) $\frac{1}{2} e^{e^x}$

36. $\int_{-\pi/2}^{\pi/2} \frac{\sin^3 x . \cos^3 x}{\cos^2 x - \sin^2 x} \, dx =$ ....

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) 0 (D) $-\pi$

37. $\int_{0}^{\pi/4} |\sin x - \cos x| \, dx =$ ....

(A) $\sqrt{2}+1$ (B) $\sqrt{2}-1$

(C) 0 (D) $2\sqrt{2}$

38. Find the area, if a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(A) log 12 (B) log 64

(C) log 81 (D) log 27

39. The integrating factor (I.F.) of the differential equation

$\cos x \frac{dy}{dx} = y \sin x + e^x \cos x$ is ....

(A) $e^{-\cos x}$ (B) $\cos x$

(C) $e^{\cos x}$ (D) $\sec x$

40. A body projected in vertical direction attains maximum height 50 m. Find its velocity at 25 m. height.

(A) $2\sqrt{10}$ m/s (B) $3\sqrt{10}$ m/s

(C) $5\sqrt{10}$ m/s (D) $7\sqrt{10}$ m/s