QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

Marks

1. $\hat{a} = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$ then $7\hat{a} = \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$ ½ + ½

2. $(\sqrt{2}\vec{a} - \vec{b}) \cdot (\sqrt{2}\vec{a} - \vec{b}) = 1 \Rightarrow \theta = \frac{\pi}{4}$ ½ + ½

3. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ ½ + ½

4. $AB = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix} \Rightarrow |AB| = -28$ ½ + ½

5. $1 \cdot y + x\frac{dy}{dx} = -c\sin x \Rightarrow x\frac{dy}{dx} + y + xy\tan x = 0$ ½ + ½

6. order = 2, degree = 3, sum = 2 + 3 = 5 ½ + ½

SECTION B

7. System of equation is

3x + y + 2z = 1100, x + 2y + 3z = 1400, x + y + z = 600 1½

(i) Matrix equation is

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$ 1

(ii) $|A| = -3 \neq 0$, system of equations can be solved. ½

(iii) Any one value with reason. 1

8. $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=0$

$\begin{bmatrix}2x-9 & 4x\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=0$ 1

$[2x^2-9x+12x]=[0] \Rightarrow 2x^2+3x=0,\ x=0 \text{ or } \frac{-3}{2}$ 1+1+1

9. $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$

$=\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ 2(a+2) & 1 & 0\\ 4a+10 & 2 & 0\end{vmatrix} \left.\begin{matrix}R_2 \to R_2 - R_1\\ R_3 \to R_3 - R_1\end{matrix}\right\}$ 1+1

$= 4a + 8 - 4a - 10 = -2.$ 1+1

10. $I=\int_0^{\pi/2}\frac{1}{1+\sqrt{\tan x}}dx$

$I=\int_0^{\pi/2}\frac{1}{1+\sqrt{\tan(\pi/2-x)}}dx=\int_0^{\pi/2}\frac{1}{1+\sqrt{\cot x}}dx=\int_0^{\pi/2}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$ 1½

$\Rightarrow 2I=\int_0^{\pi/2}\frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}dx=\int_0^{\pi/2}1\cdot dx=\frac{\pi}{2}$ 1½

$\Rightarrow I=\frac{\pi}{4}$ 1

11. $\frac{x}{(x-1)^2(x+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$ ½

$A=\frac{2}{9}, B=\frac{1}{3}, C=-\frac{2}{9}$ 1½

$$\int \frac{x}{(x-1)^2\,(x+2)}dx = \int \frac{2}{9(x-1)}dx + \int \frac{1}{3(x-1)^2}dx - \int \frac{2}{9(x+2)}dx$$ 1½

$$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$$ 1½

12. Let X be the number of defective bulbs. Then

$X = 0, 1, 2$ 1

$$P(X=0) = \frac{{}^{10}C_2}{{}^{15}C_2} = \frac{3}{7},\ P(X=1) = \frac{{}^{10}C_1 \cdot {}^{5}C_1}{{}^{15}C_2} = \frac{10}{21}$$ 1+1

$$P(X=2) = \frac{{}^{5}C_2}{{}^{15}C_2} = \frac{2}{21}$$ 1

X	0	1	2
P(X)	$\frac{3}{7}$	$\frac{10}{21}$	$\frac{2}{21}$

OR

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{3},\ P(\bar{E}_1) = \frac{1}{2},\ P(\bar{E}_2) = \frac{2}{3}$$ 1

$$P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{6}$$

$$P(\text{problem is solved}) = 1 - P(\bar{E}_1)\cdot P(\bar{E}_2) = 1 - \frac{1}{2}\cdot\frac{2}{3} = \frac{2}{3}$$ 1½

$$P(\text{one of them is solved}) = P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$$

$$= \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{2}$$ 1½

13. $\overrightarrow{AB} = -4\hat{i}-6\hat{j}-2\hat{k}$

$\overrightarrow{AC} = -\hat{i}+(\lambda-5)\hat{j}+3\hat{k}$ 1½

$\overrightarrow{AD} = -8\hat{i}-\hat{j}+3\hat{k}$

$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & \lambda-5 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ 1

$-4(3\lambda-12)+6(21)-2(8\lambda-39) = 0 \Rightarrow \lambda = 9$ 1½

14. $\vec{a}_1 = \hat{i}+2\hat{j}+\hat{k}, \vec{b}_1 = \hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2 = 2\hat{i}-\hat{j}-\hat{k}, \vec{b}_2 = 2\hat{i}+\hat{j}+2\hat{k}$ 1

$\vec{a}_2-\vec{a}_1 = \hat{i}-3\hat{j}-2\hat{k}, \vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i}+3\hat{k}$ ½ + 1

$|\vec{b}_1\times\vec{b}_2| = 3\sqrt{2}$ ½

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1-\vec{b}_2) = -3-6=-9$

Shortest distance $= \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3\sqrt{2}}{2}$ 1

15. $\tan^{-1}2x+\tan^{-1}3x = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$ 1

$\frac{5x}{1-6x^2} = 1$ 1

$\Rightarrow 6x^2 + 5x - 1 = 0$ 1

$\Rightarrow x = \frac{1}{6}, x = -1$ (rejected) 1

OR

$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$ 1

$\cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3}$ 1

$$\text{R.H.S.} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$

$$= \tan\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$$ 1

$$= \tan^{-1}\frac{63}{16}$$ 1

16. $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x - \frac{x}{\sqrt{1-x^2}}\right) - \frac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2} + \frac{2x}{2(1-x^2)}$$ 1+1

$$= \frac{\sqrt{1-x^2}\,\cos^{-1}x - x + \frac{x^2\cos^{-1}x}{1-x^2}}{1-x^2} + \frac{x}{1-x^2}$$ 1

$$= \frac{(1-x^2)\cos^{-1}x + x^2\cos^{-1}x}{(1-x^2)^{3/2}} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$$ 1

17. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$\Rightarrow y = e^{x \log \sin x} + \sin^{-1}\sqrt{x}$ 1

$\Rightarrow \frac{dy}{dx} = e^{x \log \sin x}[\log \sin x + x \cot x] + \frac{1}{2\sqrt{x}\sqrt{1-x}}$ 1½

$\Rightarrow \frac{dy}{dx} = (\sin x)^x (\log \sin x + x \cot x) + \frac{1}{2\sqrt{x}\sqrt{1-x}}$ 1½

18. $x = a \sec^3\theta$

$\frac{dx}{d\theta} = 3a \sec^3\theta \tan\theta$ ½

$y = a \tan^3\theta$

$\frac{dy}{d\theta} = 3a \tan^2\theta \sec^2\theta$ ½

$\frac{dy}{dx} = \frac{3a \tan^2\theta \sec^2\theta}{3a \sec^3\theta \tan\theta} = \sin\theta$ 1

$\frac{d^2y}{dx^2} = \cos\theta \cdot \frac{d\theta}{dx} = \frac{\cos\theta}{3a \sec^3\theta \tan\theta} = \frac{\cos^4\theta}{3a \tan\theta}$ 1

$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{4}} = \frac{1}{12a}$ 1

19. $\int \frac{e^x(x^2+1)}{(x+1)^2}\,dx$

$= \int e^x\left[\frac{(x^2-1)+2}{(x+1)^2}\right]dx$ 1

$= \int e^x\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right]dx$ 1

$$= \frac{x-1}{x+1}\cdot e^x - \int \frac{2}{(x+1)^2} e^x\, dx + \int \frac{2}{(x+1)^2} e^x\, dx$$ 1

$$= \frac{e^x (x-1)}{x+1} + C$$ 1

SECTION C

20. (a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b) ∴ * is commutative 1½

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)

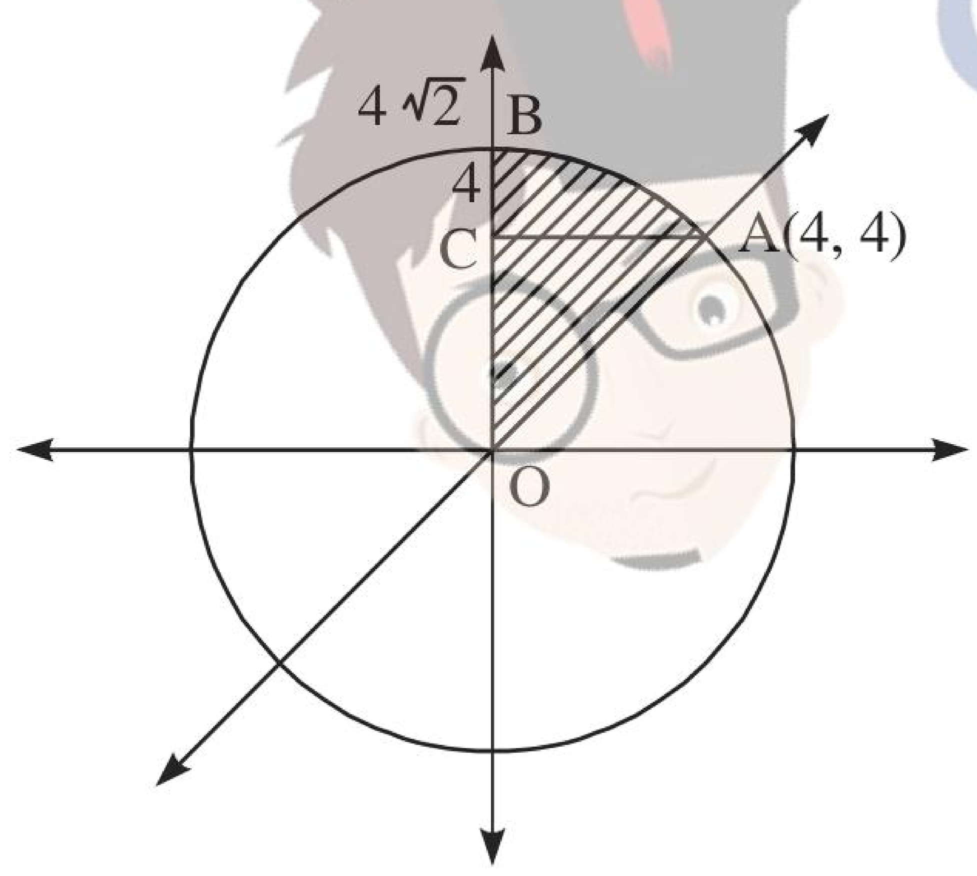
= (a + c + e, b + d + f) = (a, b) * (c + e, d + f) 1

= (a, b) * [(c, d) * (e, f) ∴ * is associate 1½

Let (e, e′) be the identity

(a, b) * (e, e′) = (a, b) ⇒ (a + e, b + e′) = (a, b) ⇒ e = 0, e′ = 0

⇒ Identity element is (0, 0) 2

21. 

$x^2 + y^2 = 32$; y = x point of intersection is y = 4 ½

Correct figure 1

$$\text{Required Area} = \int_0^4 y\, dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^2}\, dy$$ 1½

$$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32-y^2} + 16\sin^{-1}\frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$ 1½

$$= 8 + \left(0 + 16\cdot\frac{\pi}{2}\right) - \left(8 + 16\cdot\frac{\pi}{2}\right) = 4\pi$$ 1½

22. $$x\frac{dy}{dx} + y - x + xy\cot x = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) y = 1$$ ½

$$\text{I.F.} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = x \sin x$$ 1

Solution: $y \cdot x \sin x = \int 1 \cdot x \sin x \, dx$ 1½

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$ 1

when

$x = \frac{\pi}{2}, y = 0$, we have $C = -1$ 1

$yx \sin x + x \cos x - \sin x = 1$ 1

OR

$x^2 dy + (xy + y^2) dx = 0 \Rightarrow \frac{dy}{dx} = \frac{-(xy + y^2)}{x^2}$ 1

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow v + x\frac{dv}{dx} = -(v + v^2) \Rightarrow \frac{dv}{v^2 + 2v} = -\frac{dx}{dx}$

$\Rightarrow \int \frac{dv}{(v+1)^2 - (1)^2} - \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -\log x + \log C$ 1

$\Rightarrow \frac{C}{x} = \sqrt{\frac{y}{y+x}}$ 1

If $x = 1, y = 1$, then $C = \frac{1}{\sqrt{3}}$ 1

$\Rightarrow \frac{1}{\sqrt{3}\,x} = \sqrt{\frac{y}{y+x}}$ 1

23. Plane passing through the intersection of given planes:

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$ 1½

Now $(1 + 2\lambda)\,1 + (1 + 3\lambda)\,(-1) + (1 + 4\lambda)1 = 0$ 1½

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Equation of required plane is

$\Rightarrow x - z + 2 = 0$ 1

24. E_1: First bag is selected.

E_2: Second bag is selected. } 1

A: both balls are red.

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{12}{56}, P\left(\frac{A}{E_2}\right) = \frac{2}{56}$ ½ + ½ + 1 + 1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{12}{56}}{\frac{1}{2}\times\frac{12}{56} + \frac{1}{2}\cdot\frac{2}{56}} = \frac{6}{7}$$ ½ + 1½

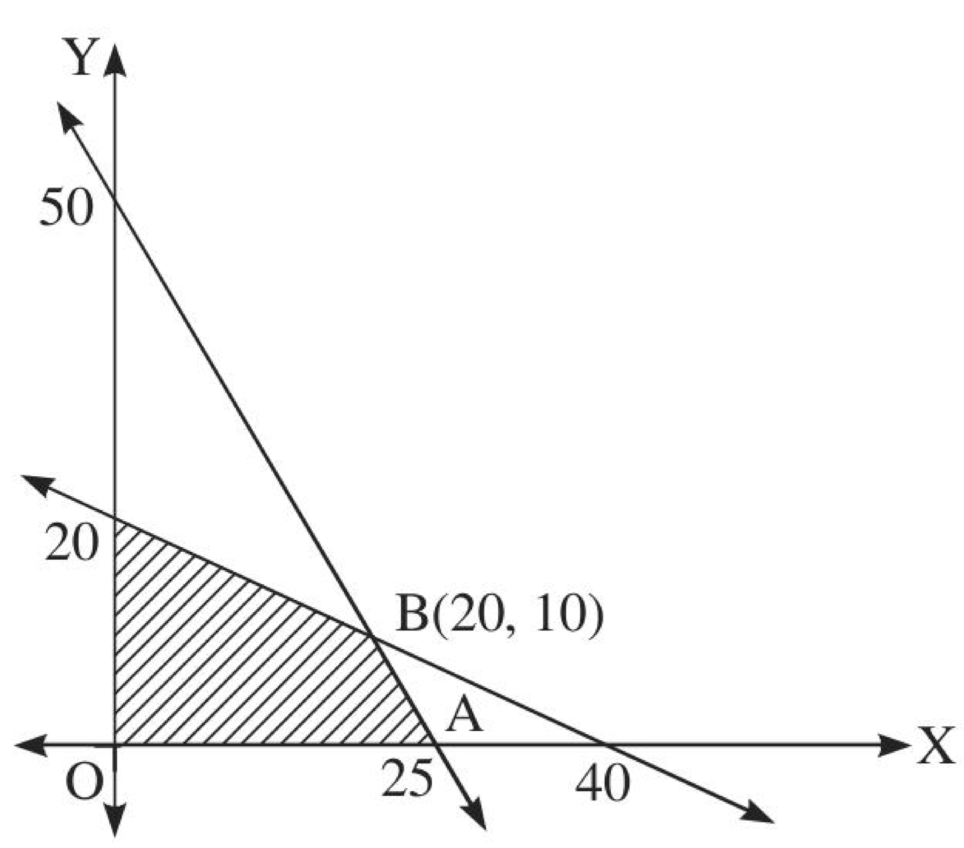
25. Let x and y be the number of takes. Then

Maximise:

$z = x + y$ 1

Subject to:

$$\left.\begin{aligned} 200x + 100y &\le 5000 \\ 25x + 50y &\le 1000 \\ x \ge 0,\ y &\ge 0 \end{aligned}\right\}$$ 2



Correct figure 2

at (20, 10), z = 20 + 10 = 30 is maximum.

at (25, 0), z = 25 + 0 = 25 } 1

at (0, 20), z = 20

26. $l \times b \times 3 = 75 \Rightarrow l \times b = 25$ 1

Let C be the cost. Then

$C = 100\,(l \times b) + 100\,h(b + l)$ 1

$$C = 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$$ 1

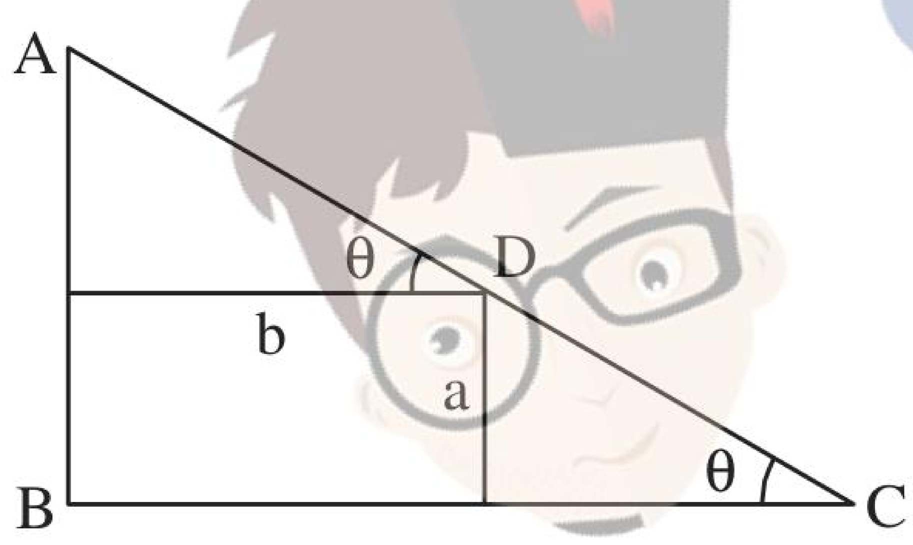
$$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$$

$$\frac{dC}{dl} = 0 \Rightarrow l = 5$$ 1

$$\frac{d^2C}{dl^2} > 0 \Rightarrow \text{C is maximum when } l = 5 \Rightarrow b = 5$$ 1

$C = 100\,(25) + 300(10)) = \text{Rs. } 5500$ 1

OR



Correct figure 1

$AD = b \sec\theta,\ DC = a \operatorname{cosec}\theta$ 1

$L = AC = b\sec\theta + a\operatorname{cosec}\theta$ 1

$$\frac{dL}{d\theta} = b\sec\theta\tan\theta - a\operatorname{cosec}\theta\cot\theta$$ 1

$$\frac{dL}{d\theta} = 0 \Rightarrow \tan^3\theta = \frac{a}{b}$$ 1

$$\frac{d^2L}{d\theta^2} > 0 \Rightarrow \text{minima}$$

$$L = \frac{b\cdot\sqrt{a^{2/3} + b^{2/3}}}{b^{1/3}} + \frac{a\sqrt{a^{2/3} + b^{2/3}}}{a^{1/3}}$$

$$\Rightarrow L = (a^{2/3} + b^{2/3})^{2/3}$$ 1