65/6/2

QUESTION PAPER CODE 65/6/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7.

Ans. Multiples of 7 from 1 to 2 are 7, 14, 21 $\frac{1}{2}$

P (number on each card is a multiple of 7)

$$= \frac{3}{25} \times \frac{2}{24} = \frac{1}{100}$$ $1\frac{1}{2}$

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}, \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Ans. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ 1

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\frac{1}{2}$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$ $\frac{1}{2}$

3. (a) Find the general solution of the differential equation x cos y dy = (x log x + 1) e^x dx.

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order the degree of the differential equation $x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0$.

Ans. $x \cos y\, dy = (x \log x + 1)e^x\, dx$

$\Rightarrow \int \cos y\, dy = \int \left(\log x + \frac{1}{x}\right) \cdot e^x\, dx$ $\frac{1}{2}$

$\Rightarrow \sin y = \log x . e^x + C$ $\left(\because \int [f(x) + f'(x)] \cdot e^x dx = f(x) \cdot e^x + C\right)$ $1+\frac{1}{2}$

OR

order = 2, degree = 3 $1+\frac{1}{2}$

$\therefore$ 2a – 3b = 4 – 9 = –5 $\frac{1}{2}$

4. Evaluate:

$$\int_0^5 x \cdot \sqrt{5-x}\, dx$$

Ans. $I = \int_0^5 x\sqrt{5-x}\, dx$

$= \int_0^5 (5-x)\sqrt{x}\, dx$ $\frac{1}{2}$

$= \int_0^5 (5\sqrt{x} - x^{3/2})\, dx$

$= \left. 5 \times \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} \right]_0^5$ 1

$= \frac{10}{3} \times 5\sqrt{5} - \frac{2}{5} \cdot 25\sqrt{5}$

$= \frac{20\sqrt{5}}{3}$ $\frac{1}{2}$

5. If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$. 2

Ans. $\vec{a} + \vec{b} = 3\hat{j}$, $\vec{b} - \vec{c} = 3\hat{k}$ $\frac{1}{2}+\frac{1}{2}$

Vector perpendicular to $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$

$= (3\hat{j}) \times (3\hat{k}) = 9\hat{i}$ 1

6. One bag contains 4 white abd 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Ans. **Case I:** White ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{4}{9}\times\frac{7}{14}$ $\frac{1}{2}$

Case II: Black ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{5}{9}\times\frac{6}{14}$ $\frac{1}{2}$

Total Probability = $\frac{4}{9}\times\frac{7}{14}+\frac{5}{9}\times\frac{6}{14}$ $\frac{1}{2}$

$= \frac{29}{63}$ $\frac{1}{2}$

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. (a) Find the distance between the following parallel lines:

$$\vec{r} = (2\hat{i}+\hat{j}-\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$$

$$\vec{r} = (\hat{i}-2\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points (–1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

Ans. (a) $\vec{a}_2-\vec{a}_2=-\hat{i}-3\hat{j}+2\hat{k},\ \vec{b}=\hat{i}+\hat{j}-\hat{k}$ $\frac{1}{2}$

$$(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -3 & 2\\ 1 & 1 & -1\end{vmatrix}=\hat{i}+\hat{j}+2\hat{k}$$ 1

Required distance = $\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$

$=\frac{\sqrt{1+1+4}}{\sqrt{1+1+1}}=\sqrt{\frac{6}{3}}=\sqrt{2}$ $1\frac{1}{2}$

OR

(b) Equation of line through (–1, 1, –8) and (5, –2, 10)

is $\frac{x+1}{5-(-1)} = \frac{y-1}{-2-1} = \frac{z+8}{10-(-8)}$

i.e. $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$ 1

Any point on this line is $(6\lambda - 1, -3\lambda + 1, 18\lambda - 8)$ $\frac{1}{2}$

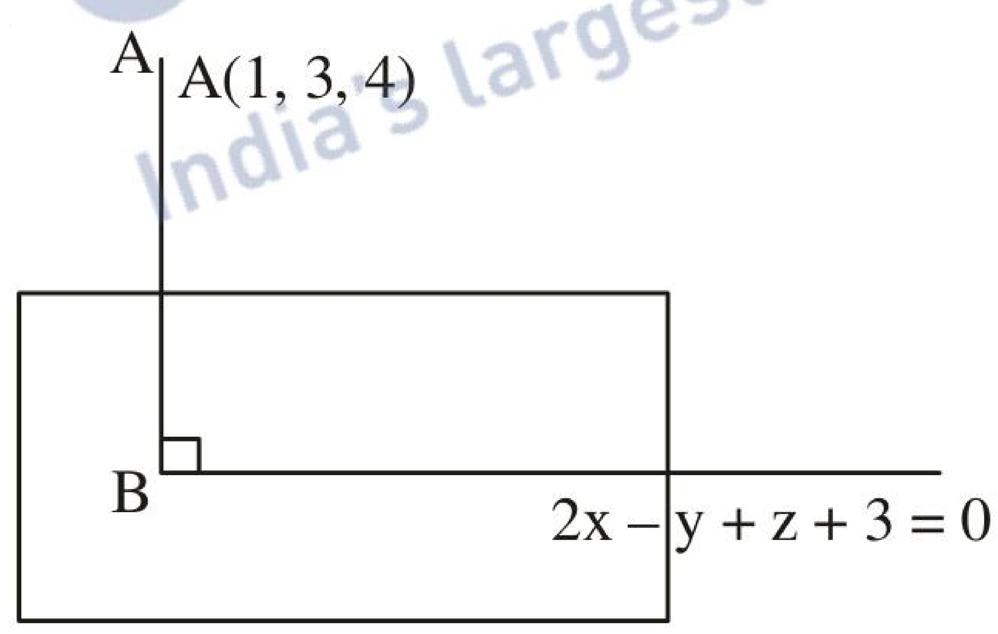
Line crosses ZX-plane i.e. $y = 0$

$\Rightarrow -3\lambda + 1 = 0 \Rightarrow \lambda = \frac{1}{3}$ 1

Required point is (1, 0, –2) $\frac{1}{2}$

8. Find the coordinates of the foot of the perpendicular drawn from the point (1, 3, 4) to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

Ans.



Equation of line AB is $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$ 1

Any point on AB is $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Let coordinates of B are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ $\frac{1}{2}$

B lies on $2x - y + z + 3 = 0$

$\Rightarrow 2(2\lambda + 1) - 1(-\lambda + 3) + 1(\lambda + 4) + 3 = 0$

$\Rightarrow \lambda = -1$ 1

$\therefore$ Required coordinates are B (–1, 4, 3) $\frac{1}{2}$

9. Find:

$$\int \sin^{-1} x \, dx$$

Ans. $I = \int \sin^{-1} x \cdot 1 \, dx$

$= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x \, dx$ $1\frac{1}{2}$

$= x \cdot \sin^{-1} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx$ $\frac{1}{2}$

$= x \cdot \sin^{-1} x + \frac{1}{2} \cdot 2\sqrt{1-x^2} + C$ 1

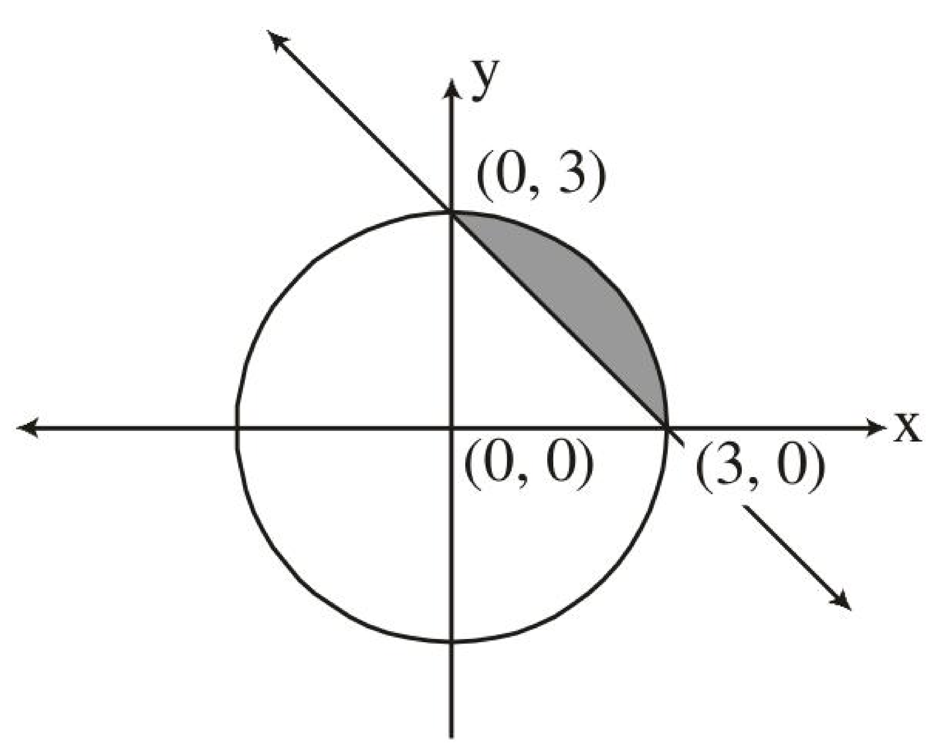
or $x \sin^{-1}x + \sqrt{1-x^2} + C$

10. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \le 9, x + y \ge 3\}$, using integration.

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis.

Ans. Point of intersection (3, 0) and (0, 3)



Correct figure $\frac{1}{2}$

Required Area

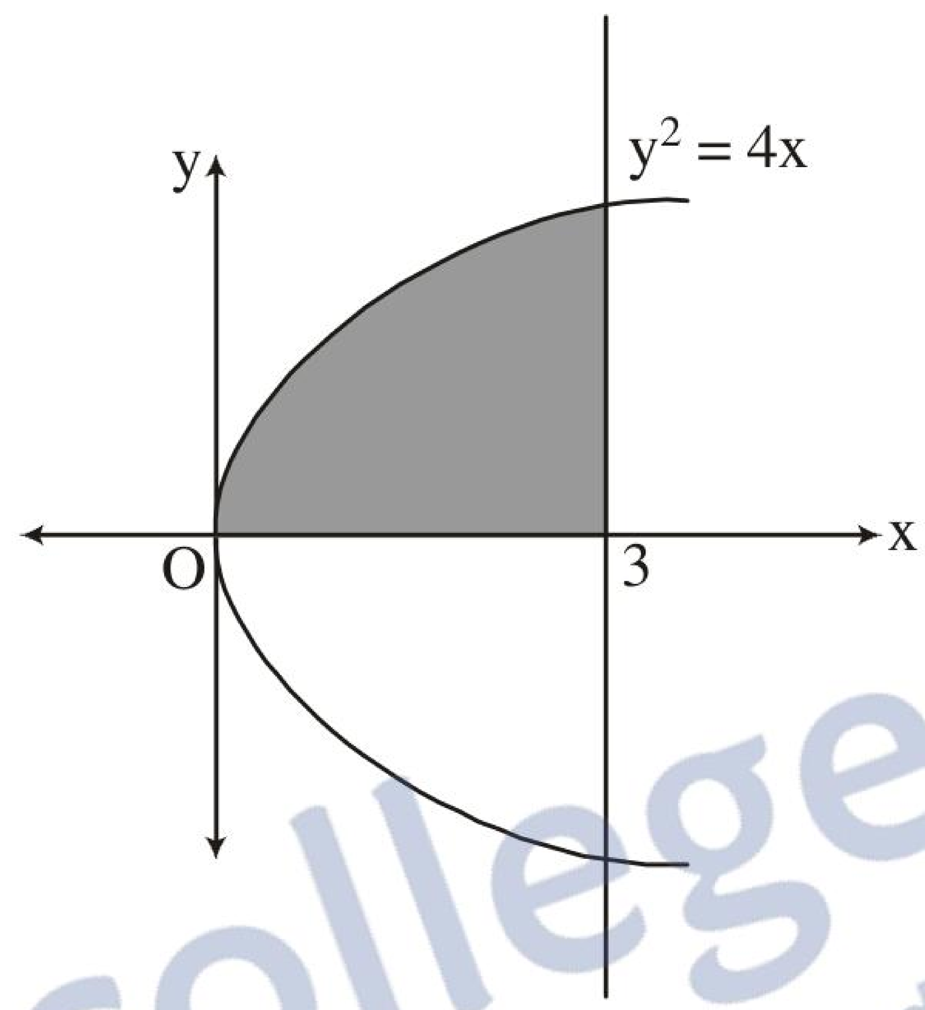
$$= \int_0^3 \sqrt{9-x^2}\, dx - \int_0^3 (3-x)dx$$ 1

$$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 - \left[\frac{(3-x)^2}{-2}\right]_0^3$$ 1

$$= \frac{9}{2}\sin^{-1}1 - \frac{9}{2} = \frac{9}{2}\left(\frac{\pi}{2}-1\right)$$ $\frac{1}{2}$

OR

Required Area



Correct Figure 1

$$= \int_0^3 2\sqrt{x}\,dx$$ 1

$$= 2\times\frac{2}{3}[x^{3/2}]_0^3$$ $\frac{1}{2}$

$$= \frac{4}{3}\times 3^{3/2} = 4\sqrt{3}$$ $\frac{1}{2}$

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. Find the particular solution of the differential equation $(1 + \sin x)\frac{dy}{dx} = -x - y\cos x$, given $y(0) = 1$.

Ans. $(1 + \sin x)\frac{dy}{dx} = -x - y\cos x$

Given equation can be writte as

$$\frac{dy}{dx}+\frac{\cos x}{1+\sin x}\cdot y = \frac{-x}{1+\sin x}$$ $\frac{1}{2}$

I.F. $= e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1 + \sin x$ 1

Solution is given by

$$y\cdot(1+\sin x) = \int \frac{-x}{(1+\sin x)}\cdot(1+\sin x)\,dx$$ 1

$$= \int -x\,dx = -\frac{x^2}{2}+C$$ $\frac{1}{2}$

Now, when $x = 0,\ y = 1 \Rightarrow C = 1$ $\frac{1}{2}$

$\therefore$ Required particular solution is $y\cdot(1+\sin x) = -\frac{x^2}{2}+1$ $\frac{1}{2}$

12. Find the equation of the plane passing through the intersection of the planes $\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k}) = 7$ and $\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k}) = 9$ and through the point (2, 1, 3).

Ans. Equation of required plane is

$$\vec{r}\cdot[(2\hat{i}+2\hat{j}-3\hat{k})+\lambda(2\hat{i}+5\hat{j}+3\hat{k})] = 7+9\lambda$$ 1

or $\vec{r}\cdot[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k})] = 7+9\lambda$

As the plane passes through (2, 1, 3), we have

$$(2\hat{i}+\hat{j}+3\hat{k})\cdot[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}] = 7+9\lambda$$ 1

$\Rightarrow 2(2 + 2\lambda) + 1(2 + 5\lambda) + 3(-3 + 3\lambda) = 7 + 9\lambda$

$\Rightarrow 9\lambda = 10 \Rightarrow \lambda = \frac{10}{9}$ 1

Required plane is $\vec{r}\cdot\left(\frac{38}{9}\hat{i}+\frac{68}{9}\hat{j}+\frac{3}{9}\hat{k}\right) = 17$ 1

or $\vec{r}\cdot(38\hat{i}+68\hat{j}+3\hat{k}) = 153$

13. (a) Find:

$$\int \cos x \bullet \tan^{-1} (\sin x)\, dx$$

OR

(b) Find:

$$\int \frac{e^x}{(e^x + 1)(e^x + 3)} dx$$

Case-Study Based Question

Ans. $I = \int \cos x \cdot \tan^{-1}(\sin x)\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$ 1

$\therefore\ I = \int \tan^{-1} t \cdot 1\, dt$

$= \tan^{-1} t \cdot t - \frac{1}{2}\int \frac{2t}{1+t^2} dt$ $1\frac{1}{2}$

$= t \cdot \tan^{-1} t - \frac{1}{2} \log |1 + t^2| + C$ 1

$= \sin x \cdot \tan^{-1}(\sin x) - \frac{1}{2} \log |1 + \sin^2 x| + C$ $\frac{1}{2}$

OR

$I = \int \frac{e^x}{(e^x + 1)(e^x + 3)} dx$

Put $e^x = t \Rightarrow e^x\, dx = dt$ 1

$I = \int \frac{dt}{(t+1)(t+3)}$

$= \frac{1}{2}\int \left(\frac{1}{t+1} - \frac{1}{t+3}\right) dt$ $1\frac{1}{2}$

$= \frac{1}{2}[\log |t+1| - \log |t+3|] + C$ 1

$= \frac{1}{2}[\log |e^x + 1| - \log |e^x + 3|] + C$ $\frac{1}{2}$

or $\frac{1}{2}\log\left|\frac{e^x+1}{e^x+3}\right|+C$

14. A biased die is tossed and respective probabilities for various faces to turn up are the following:

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	K

Based on the above information, answer the following questions:

(a) What is the value of K?

(b) If a face showing an even number has turned up, then what is the probability that is the fae with 2 or 4?

Ans. (a) $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$ 1

$\Rightarrow$ $0.1 + 0.24 + 0.19 + 0.18 + 0.15 + K = 1$

$\Rightarrow$ $K = 0.14$ 1

(b) A: face shows 2 or 4

B: even face have turned up

Required probability = $P(A/B) = \frac{P(A \cap B)}{P(B)}$

here $P(A \cap B) = P(2) + P(4)$

$= 0.24 + 0.18 = 0.42$ 1

$P(B) = P(2) + P(4) + P(6) = 0.56$ $\frac{1}{2}$

$\therefore$ $P(A/B) = \frac{0.42}{0.56} = \frac{3}{4}$ $\frac{1}{2}$