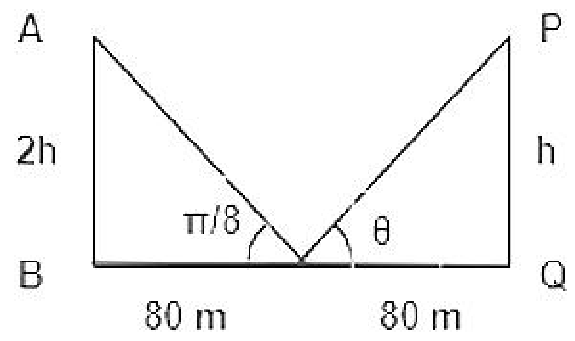
JEE-Main-28-06-2022-Shift-1 (Memory Based)

MATHEMATICS

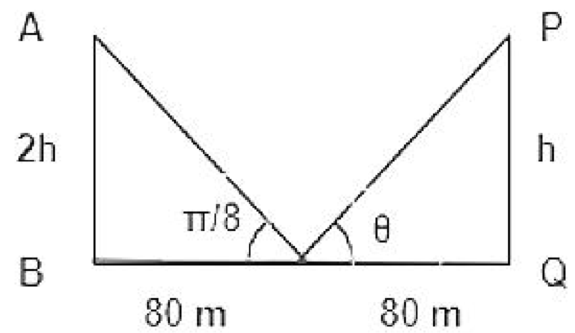
Question:



Find $\tan^2\theta$.

Answer: $4\left(3-2\sqrt{2}\right)$

Solution:



$\tan\frac{\pi}{8}=\frac{h}{80}$,

$\tan\theta=\frac{2h}{80}$

$\Rightarrow \tan\theta = 2\tan\frac{\pi}{8}$

$\Rightarrow \tan^2\theta = 4\tan^2\frac{\pi}{8}$

$=4\left(\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}\right)$

$=4\left(\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}\right)$

$=\frac{4\left(\sqrt{2}-1\right)}{\sqrt{2}+1}$

$=4\left(\sqrt{2}-1\right)^2$

$= 4\left(3-2\sqrt{2}\right)$

Question: Find probability that 3 digit number has atleast two odd numbers.

Answer: $\frac{19}{36}$

Solution:

Total 3 digit numbers $= 9\times10\times10 = 900$

Numbers with 3 odd digit $= 5\times5\times5 = 100$

Number with 2 odd, one even digit $= 4\times5\times5 + 2\times5\times5\times5$

$= 350$

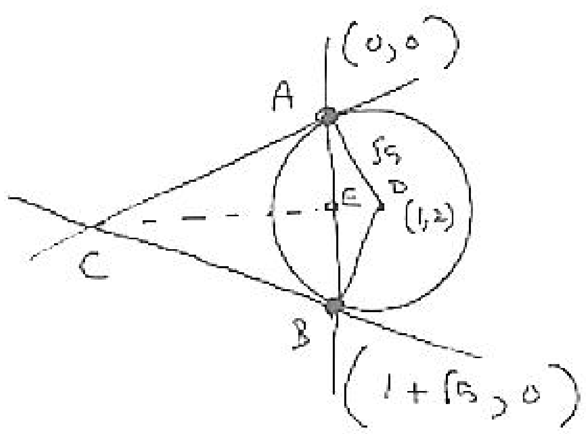
Favourable number of cases $= 125 + 350 = 475$

Probability $= \frac{475}{900} = \frac{19}{36}$

Question: Equation of circle $(x-1)^2 + (y-2)^2 = 5$ drawing 2 tangents at $(0,0)$ & $\left(1+\sqrt{5},0\right)$ they intersect. Find area of that triangle.

Answer: $\frac{1+\sqrt{5}}{4}$

Solution:



Equation of $AB \equiv y = 0$

$(x-1)^2 + (y-2)^2 = 5$

$\Rightarrow x^2 + y^2 - 2x - 4y = 0$

Let C be (α,β)

Equation of $AB = \alpha x + \beta y - (x+\alpha) - 2(y+\beta) = 0$

$\Rightarrow x(\alpha-1) + y(\beta-2) - \alpha - 2\beta = 0$

$\Rightarrow \alpha = 1,\ -\alpha - 2\beta = 0$

$\beta = \frac{-1}{2}$

$CE = \frac{1}{2}$, $AB = 1+\sqrt{5}$

Area $= \frac{1}{2}\left(\frac{1}{2}\right)\left(1+\sqrt{5}\right) = \frac{1+\sqrt{5}}{4}$

Question: $|A| = 2$. Find $\left||A| adj\left(5\, adj\, A^3\right)\right|$.

Answer: $2^{15} \times 5^6$

Solution:

$\left||A| adj\left(5\, adj\, A^3\right)\right|$

$= |A|^3 \left|adj\left(5\, adj\, A^3\right)\right|$

$= |A|^3 \left|5\, adj\, A^3\right|^2$

$= |A|^3\, 5^6 \left|adj\, A^3\right|^2$

$= |A|^3\, 5^6 \left(\left|A^3\right|^2\right)^2$

$= |A|^3\, 5^6\, |A|^{12}$

$= 2^{15} \cdot 5^6$

Question: $\overline{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\overline{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ & $\overline{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, $\overline{a}, \overline{b}, \overline{c}$ are coplanar, $\overline{a} \cdot \overline{c} = 5$ & $\overline{b}$ is perpendicular to $\overline{c}$. Find $122\left(c_1 + c_2 + c_3\right)$.

Answer: 150.00

Solution:

$\overline{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\overline{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ & $\overline{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\overline{a} \cdot \overline{c} = 5$, $\overline{b} \perp \overline{c}$, $122\left(c_1 + c_2 + c_3\right) = ?$

$a \cdot c = 2c_1 + c_2 + 3c_3 = 5 \quad ....(1)$

$b \cdot c = 3c_1 + 3c_2 + c_3 = 0 \quad ....(2)$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

$c_1(1-9) - c_2(2-9) + c_3(6-3) = 0$

$-8c_1 + 7c_2 + 3c_3 = 0 \quad ....(3)$

From (1), (2) and (3)

$c_1 = \frac{10}{122}$, $c_2 = \frac{-85}{122}$, $c_3 = \frac{225}{122}$

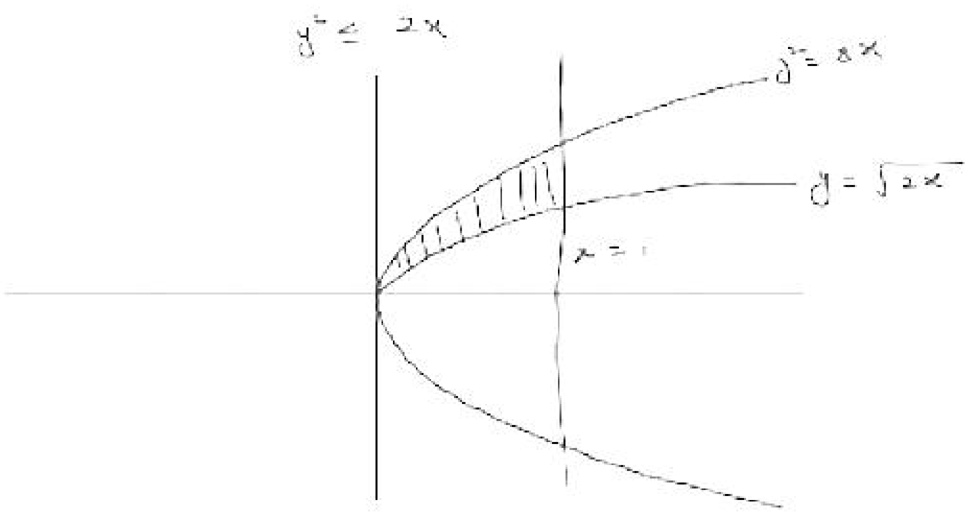
$122(c_1+c_2+c_3)=150$

Question: $y^2 \le 8x,\ y \ge \sqrt{2x}, x \le 1$. Find area.

Answer: $\frac{2\sqrt{2}}{3}$

Solution:

Given, $y^2 \le 8x,\ y \ge \sqrt{2x}, x \le 1$.



$$\text{Area } = \int_0^1 \left(\sqrt{8x}-\sqrt{2x}\right)dx$$

$$= \int_0^1 \left(2\sqrt{2}x-\sqrt{2}\sqrt{x}\right)dx$$

$$= \int_0^1 \sqrt{2}\sqrt{x}\,dx$$

$$= \sqrt{2}\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= \frac{2\sqrt{2}}{3}$$

Question: Find number of numbers divisible by 6, formed using $3,6,1,7,2,5$.

Answer: 72.00

Solution:

Given $3,6,1,7,2,5$

$3+6+1+7+2+5=24$

Case I: $3,1,7,2,5$

$\overline{4}\times\overline{3}\times\overline{2}\times\overline{1}\times\overline{1}=24$

Case II: $6,1,7,2,5$

$\overline{4}\times\overline{3}\times\overline{2}\times\overline{1}\times\overline{2}=48$

Total $=24+48=72$

Question: $\sum_{k-1}^{31} {}^{31}C_k \times {}^{31}C_{k-1} + \sum_{k-1}^{30} {}^{30}C_k \times {}^{30}C_{k-1} = \frac{\alpha\times 60!}{30!\times 30!}$. $16\alpha=?$

Answer: $\frac{2371}{8}$

Solution:

$$\sum_{k-1}^{31} {}^{31}C_k \times {}^{31}C_{k-1} + \sum_{k-1}^{30} {}^{30}C_k \times {}^{30}C_{k-1}$$

$$=\sum_{k=1}^{31} {}^{31}C_{31-k}\ {}^{31}C_{k-1} + \sum_{k=1}^{30} {}^{30}C_k\ {}^{30}C_{31-k}$$

$$={}^{62}C_{30}+\left({}^{60}C_{31}\right)$$

$$=\frac{62!}{32!30!}+\frac{60!}{31!29!}$$

$$=\frac{60!}{30!}\left(\frac{61\times 62}{32!}+\frac{1}{31\times 29!}\right)$$

$$=\frac{60!}{30!30!}\left(\frac{61\times 62}{31\times 32}+\frac{30}{31}\right)$$

$$=\frac{60!}{30!30!}\left(\frac{2371}{496}\right)$$

$$\alpha=\frac{2371}{496}$$

$$16\alpha=\frac{2371}{31}$$

Question: $x\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)\frac{dy}{dx}=x+y\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)$. Let $y=y(x)$ satisfy above differential equation such that $y(1)=0$. If $y(2\alpha)=\alpha$, find α.

Answer: ()

Solution:

$$x\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)\frac{dy}{dx}=x+y\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)$$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\left[\frac{x}{\sqrt{x^2-v^2x^2}}+e^v\right]\left(v+x\frac{dv}{dx}\right)=v\left[\frac{x}{\sqrt{x^2-v^2x^2}}+e^v\right]+1$$

$$\left[\frac{1}{\sqrt{1-v^2}}+e^v\right]\left(v+x\frac{dv}{dx}\right)=v\left[\frac{1}{\sqrt{1-v^2}}+e^v\right]+1$$

$$=\frac{v}{\sqrt{1-v^2}}+ve^v+\frac{x}{\sqrt{1-v^2}}\frac{dv}{dx}+xe^v\frac{dv}{dx}=\frac{v}{\sqrt{1-v^2}}+ve^v+1$$

$$\Rightarrow x\frac{dv}{dx}\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)=1$$

$$\Rightarrow \int\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)dv=\int\frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right)+e^{\frac{y}{x}}=\ln x+c$$

$$\because\ y(1)=0\Rightarrow 0+1=0+c\Rightarrow c=1$$

$$\therefore\ y(2\alpha)=\alpha\Rightarrow \sin^{-1}\left(\frac{1}{2}\right)+e^{\frac{1}{2}}=\sin 2\alpha+1$$

$$\frac{\pi}{6}+e^{\frac{1}{2}}-1=\ln 2\alpha$$

$$\Rightarrow 2\alpha=e^{\frac{\pi}{6}+\sqrt{e}-1}$$

$$\Rightarrow \alpha=\frac{1}{2}\cdot e^{\frac{\pi}{6}+\sqrt{e}-1}$$

Question: Find the number of real solution of $4x^7+3x^3+5x+1=0$

Answer: 1.00

Solution:

Let $f(x)=4x^7+3x^3+5x+1$

$f'(x)=28x^6+9x^2+5>0$

$f(x)$ is increasing

No. of roots = 1

Question: $(a_n)\rightarrow GP,\ a_1\times a_3\times a_5\times a_7=\frac{1}{1296},\ a_2+a_4=\frac{7}{36},$ then $a_6+a_8+a_{10}$

Answer: 43.00

Solution:

$$a\cdot ar^2\cdot ar^4\cdot ar^6=\frac{1}{1296}$$

$$a^4\cdot r^{12}=\frac{1}{64}$$

$ar^3 = \frac{1}{6}$

$a_2 + a_4 = \frac{7}{36}$

$ar + ar^3 = \frac{7}{36}$

$ar = \frac{7}{36} - \frac{1}{6} = \frac{1}{36}$

$\Rightarrow r^2 = 6$

$a_6 + a_8 + a^{10}$

$ar^5 + ar^7 + ar^9$

$= \frac{1}{36} \times 6^2 + \frac{1}{36} \times 6^3 + \frac{1}{36} \times 6^4$

$= 1 + 6 + 36 = 43$

Question: Mean & variance of 15 observations is 8 & 3 respectively. One observation is wrongly taken as 5 in place of 20. Find correct variance.

Answer: 11.00

Solution:

$$\text{Variance} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$3 = \frac{\sum x^2}{15} - 8^2$$

$$\frac{\sum x^2}{15} = 67$$

$$\sum x^2 = 1005$$

$$\text{New } \sum x^2 = 1005 - 5^2 + 20^2$$

$$= 1380$$

$$\text{New } \sum x = 8 \times 15 - 5 \times 20 = 135$$

$$\text{Variance} = \frac{1380}{15} - \left(\frac{135}{15}\right)^2$$

$$= 92 - 81 = 11$$

Question: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $e = \sqrt{\frac{5}{2}}$ & LR $= 6\sqrt{2}$. If $y = 2x + c$ is tangent, find c.

Answer: ± 2

Solution:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ e=\sqrt{\frac{5}{2}}$$

$$\frac{2b^2}{a}=6\sqrt{2}$$

$$b^2=a^2\left(\frac{5}{2}-1\right)$$

$$b^2=\frac{3}{2}a^2$$

$$\Rightarrow \frac{2\times\frac{3}{2}a^2}{a}=6\sqrt{2}$$

$$\Rightarrow a=2\sqrt{2}$$

$$\Rightarrow a^2=8$$

$$\Rightarrow b^2=12$$

$$y=2x+c$$

$$c^2=a^2m^2-b^2$$

$$=8\times 2-12=4$$

$$\Rightarrow c=\pm 2$$