JEE-Main-27-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: Shortest distance between lines

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$$

$$\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$$

Options:

(a) $\frac{18}{\sqrt{5}}$

(b) $6\sqrt{3}$

(c) $\frac{46}{3\sqrt{5}}$

(d) $\frac{22}{3\sqrt{5}}$

Answer: (a)

Solution:

Given, $L_1=\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$

$L_2=\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$

$\therefore$ Shortest distance $d=\left|\frac{(\bar{a}_2-\bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2)}{|\bar{b}_1\times\bar{b}_2|}\right|$

$\bar{a}_2-\bar{a}_1=\left(-3\hat{i}+6\hat{j}+5\hat{k}\right)-\left(3\hat{i}+2\hat{j}+\hat{k}\right)$

$=-6\hat{i}+4\hat{j}+4\hat{k}$

$$\bar{b}_1\times\bar{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 2 & 1 & 3\end{vmatrix}$$

$=\hat{i}(10)-\hat{j}(8)+\hat{k}(-4)$

$\Rightarrow d=\left|\frac{-60-32-16}{\sqrt{100+64+16}}\right|$

$=\left|\frac{108}{6\sqrt{5}}\right|=\frac{18}{\sqrt{5}}$

Question: $\alpha = \sin 36°$ is a root of which of the following?
Options:
(a) $16x^4 - 10x^2 + 5 = 0$

(b) $16x^4 + 20x^2 - 5 = 0$
(c) $16x^4 - 20x^2 + 5 = 0$
(d) $16x^4 - 10x^2 - 5 = 0$

Answer: (c)

Solution:
Given, $16x^4 - 20x^2 + 5 = 0$
Let $x^2 = t$
$\Rightarrow 16t^2 - 20t + 5 = 0$
$$\Rightarrow t = \frac{20 \pm \sqrt{20^2 - 4 \times 16 \times 5}}{2 \times 16}$$
$$\Rightarrow t = \frac{20 \pm \sqrt{80}}{32}$$
$$\Rightarrow t = \frac{5 \pm \sqrt{5}}{8}$$
$$x^2 = \frac{5 \pm \sqrt{5}}{8} \Rightarrow x = \frac{\sqrt{10 \pm 2\sqrt{5}}}{4}$$
$$x = \frac{\sqrt{10 - 2\sqrt{5}}}{4} = \sin 36°$$

Question: If equation of parabola whose vertex is at $(5,4)$ and directrix is $3x + y - 29 = 0$ is $x^2 + ay^2 + bxy + cx + dy + k = 0$ then $a + b + c + d + k$ is
Options:
(a) 576

(b) 575
(c) -575
(d) -576

Answer: (d)

Solution:
Given, vertex (5, 4) & directrix at $3x + y - 29 = 0$
Thus, foot of perpendicular from vertex to directrix

Thus, by calculating, focus will be (2, 3)

Now, equation of parabola

$$\sqrt{(x-2)^2+(y-3)^2} = \left|\frac{3x+y-29}{\sqrt{9+1}}\right|$$

$$10\left[x^2+4-4x+y^2+9-6y\right]=9x^2+y^2+841+6xy-174x-58y$$

$$x^2+9y^2-6xy+134x-2y-711=0$$

Comparing with

$$x^2+ay^2+bxy+cx+dy+k=0$$

$$a=9,\ b=-6,\ c=134, d=-2,\ k=-711$$

$$\therefore a+b+c+d+k=-576$$

Question: Let foot of perpendicular from point $(1,a,h)$ on line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ be P. Distance of P from the plane $3x+hy+12z+23=0$

Options:

(a) $\frac{63}{13}$

(b) 5

(c) $\frac{50}{13}$

(d) 4

Answer: (b)

Solution:

Given, $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}=k$

$\therefore$ Any point on line is

$P(4k-2,2k+1,3k-1)$

Let $(1,2,4)$

D.R. of $PQ=4k-3,\ 2k-1,\ 3k-5$

Now, PQ is perpendicular to line

$$\therefore 4(4k-3)+2(2k-1)+3(3k-5)=0$$

$$16k-12+4k-2+9k-15=0$$

$$29k-29=0$$

$$k=1$$

$$\therefore P(2,3,2)$$

Now, distance of P from $3x+4y+12z+23=0$ is

$$= \left| \frac{6+12+24+23}{\sqrt{9+16+144}} \right|$$

$$= \frac{65}{13} = 5$$

Question: If m, n are number of local maximum and local minimum points of function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + h}{2te^t} dt$. Then ordered pair (m, n) is

Options:

(a) $(3, 4)$

(b) $(2, 3)$

(c) $(3, 2)$

(d) $(2, 2)$

Answer: (b)

Solution:

Given, $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2te^t} dt$

$$f'(x) = \frac{x^4 - 5x^2 + 4}{2x^2 e^{x^2}}(2x) - 0$$

$$= \frac{(x^2-4)(x^2-1)x}{x^2 e^{x^2}}$$

$$= \frac{(x-2)(x+2)(x-1)(x+1)x}{x^2 e^{x^2}}$$

$\therefore$ Number of local maximum $(m) = 2$

Number of local minimum $(n) = 3$

$\therefore (m, n) = (2, 3)$

Question: Let $\overline{a}$ and $\overline{b}$ be two vectors along diagonals of parallelogram having area $2\sqrt{2}$. Let angle between $\overline{a}$ and $\overline{b}$ be acute. $|a| = 1$, $|\overline{a} \cdot b| = |\overline{a} \times b|$. If $\overline{c} = 2\sqrt{2}(\overline{a} \times \overline{b}) - 2\overline{b}$ then acute angle between $\overline{b}$ and $\overline{c}$ is

Options:

(a) $\frac{-\pi}{4}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{3\pi}{4}$

Answer: (d)

Solution:

$\frac{1}{2}\left|\bar{a}\times\bar{b}\right| = 2\sqrt{2}$

$\Rightarrow \left|\bar{a}\times\bar{b}\right| = 4\sqrt{2}$

$|a| = 1, \ |a\cdot b| = |a\times b|$

$|a||b|\cos\theta = |a||b|\sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

Now, $|a||b|\sin\theta = 4\sqrt{2}$

$\Rightarrow |b| = 8$

$\bar{c} = 2\sqrt{2}\left(\bar{a}\times\bar{b}\right) - 2\bar{b}$

$\bar{c}\cdot\bar{b} = 2\sqrt{2}\left(a\times b\right)\cdot b - 2|b|^2$

$c\cdot b = -2|b|^2 = -64$

$|c|\cdot|b|\cdot\cos\alpha = -64$

$|c|\cos\alpha = -8$

$|c|^2 = \left|2\sqrt{2}a\times b - 2\bar{b}\right|^2$

$= 4\left|\sqrt{2}a\times b - b\right|\cdot\left|\sqrt{2}a\times b - b\right|$

$= 4\left|4\left(a\times b\right)^2 - 2\sqrt{2}\left(a\times b\right)\cdot b + |b|^2\right|$

$= 4|4\times 32 - 0 + 64|$

$= 4\times 64\times 2$

$|c| = 16\sqrt{2}$

$\Rightarrow 16\sqrt{2}\cos\alpha = -8$

$\Rightarrow \cos\alpha = \frac{-1}{\sqrt{2}}$

$\Rightarrow \alpha = \frac{3\pi}{4}$

Question: Let A and B be two 3×3 matrices such that $AB = I$, $|A| = \frac{1}{8}$. Then $|adj(B\,adj(2A))|$

Options:
(a) 32

(b) 128
(c) 64
(d) 16

Answer: (c)

Solution:

Given, $AB = I$, $|A| = \frac{1}{8}$

$|adj\ B(adj\,2A)|$

$= |B\,adj(2A)|^2$

$= |B|^2 |adj(2A)|^2$

$= |B|^2 (|2A|)^2$

$= |B|^2 \left((2^3|A|)^2\right)^2$

$= |B|^2 (2^{12})|A|^4$

$= 2^{12} |B|^2 |A|^4$

$= 2^{12} |A|^2$

$= \frac{2^{12}}{2^6} = 2^6 = 64$

Question: Let f be a differential function is $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$. Then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to

Options:
(a) $\frac{9}{\sqrt{2}} - 6$

(b) $6 - \frac{9}{\sqrt{2}}$
(c) $6 - 9\sqrt{2}$
(d) $\frac{9}{2} - 6\sqrt{2}$

Answer: (b)

Solution:

Given, $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$

Differentiating w.r.t. x

$\cos^2 x\, f(\cos x)\cdot \sin x = 3\sin^2 x \cdot \cos x - \sin x$

$f(\cos x) = \dfrac{3\sin x \cos x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$

$f(\cos x) = 3\tan x - \sec^2 x$

Again differentiating

$-f'(\cos x)(\sin x) = 3\sec^2 x - 2\sec x \cdot \sec x \tan x$

$f'(\cos x) = \dfrac{2\sec^2 x \cdot \tan x}{\sin x} - \dfrac{3\sec^2 x}{\sin x}$

$f'(\cos x) = \dfrac{2}{\cos^2 x} - \dfrac{3}{\cos^2 x \cdot \sin x}$

Now, put $\cos x = \dfrac{1}{\sqrt{3}}$, $\sin x = \dfrac{\sqrt{2}}{\sqrt{3}}$

$f'\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{2}{\left(\dfrac{1}{\sqrt{3}}\right)^3} - \dfrac{3}{\left(\dfrac{1}{\sqrt{3}}\right)^2\left(\dfrac{\sqrt{2}}{\sqrt{3}}\right)}$

$\dfrac{1}{\sqrt{3}} f'\left(\dfrac{1}{\sqrt{3}}\right) = 6 - \dfrac{9}{\sqrt{2}}$

Question: Find the number of complex number z, show that $|z-(4+3i)| = 2$ &

$|z| + |z-4| = 6$

Options:

(a) 1

(b) 2

(c) 3

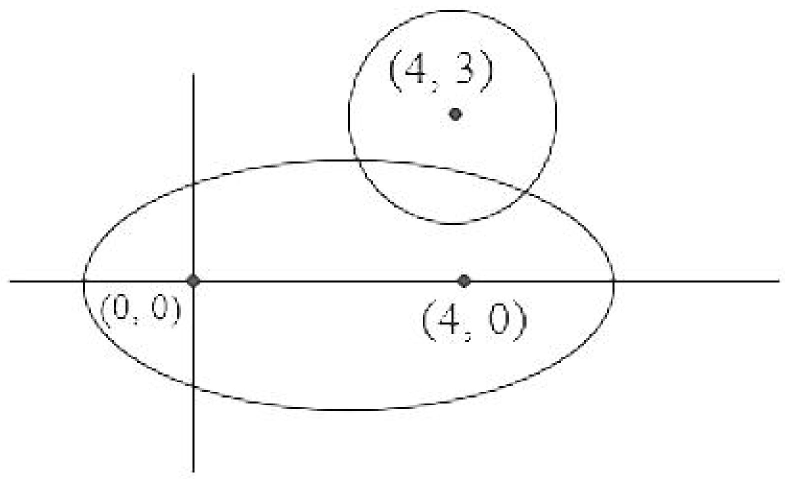
(d) 4

Answer: (b)

Solution:

$|z-(4+3i)| = 2$ and $|z| + |z-4| = 6$

$c \equiv (4,3)$, $r = 2$



Circle and ellipse intersect at 2 points

Question: Which of the following is a tautology?

Options:

(a) $(\sim p \wedge q) \vee (p \vee \sim p)$

(b) $(p \rightarrow q) \vee q$

(c) $(p \leftrightarrow q) \vee (p \wedge q)$

(d) $p \wedge (p \leftrightarrow q)$

Answer: (a)

Solution:

$(\sim p \wedge q) \vee (p \vee \sim p)$

$\equiv (\sim p \wedge q) \vee T$

$\equiv T$

Hence $(\sim p \wedge q) \vee (p \vee \sim p)$ is a tautology

Question: For some real numbers α and β, $a = \alpha - i\beta$. If system of equations $hix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is

Answer: $2-\sqrt{3}$

Solution:

For more than one solution

$$\begin{vmatrix} 4i & (1+i) \\ 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) & \bar{a} \end{vmatrix} = 0$$

$$\Rightarrow 4i\bar{a} - (1+i)8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 0$$

$$\Rightarrow 4i\bar{a} - 4(1+i)\left(-1+\sqrt{3}i\right) = 0$$

$$\Rightarrow i\bar{a}-\left(-1+\sqrt{3}i-i-\sqrt{3}\right)=0$$

$$\Rightarrow i\bar{a}-\left(-1-\sqrt{3}+\sqrt{3}i-i\right)=0$$

$$\Rightarrow -\bar{a}-\left(-i-\sqrt{3}i-\sqrt{3}+1\right)=0$$

$$\Rightarrow \bar{a}=\sqrt{3}-1+i\left(1+\sqrt{3}\right)$$

$$\alpha=\sqrt{3}-1,\ \beta=-1-\sqrt{3}$$

$$\frac{\alpha}{\beta}=\frac{\sqrt{3}-1}{-1-\sqrt{3}}=\frac{\left(\sqrt{3}-1\right)\left(1-\sqrt{3}\right)}{-\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}$$

$$=\frac{-\left(3+1-2\sqrt{3}\right)}{-\left(1+\sqrt{3}\right)}$$

$$=2-\sqrt{3}$$

Question: If $y(x)=\left(x^{x}\right)^{x}, x>0, \frac{d^2x}{dy^2}+20$ at $x=1$ is ___

Answer: 24.00

Solution:

Given, $y=x^{x^2}$

$$\log y=x^2\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\left(2x\log x+x\right)$$

$$\frac{dy}{dx}=y\left(2x\log x+x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{dy}{dx}\left(2x\log x+x\right)+y\left(2+2\log x+1\right)$$

Now, at $x=1,\ \frac{dy}{dx}=1$

$$\frac{d^2y}{dx^2}=1+1(3)=4$$

$$\therefore \frac{d^2y}{dx^2}+20=4+20=24$$

Question: $S = 2 + \frac{6}{7} + \frac{12}{49} + \frac{20}{343} + \ldots$ Find $4S$.

Answer: $\frac{343}{27}$

Solution:

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \ldots$$

$$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots$$

$$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \ldots$$

$$\frac{6S}{49} = \quad \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \ldots$$

$$\left(\frac{6}{7} - \frac{6}{49}\right)S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots$$

$$\frac{36}{49}S = 2\left(1 + \frac{1}{7} + \frac{1}{7^2} + \ldots\infty\right)$$

$$\Rightarrow S = 2 \cdot \frac{1}{1 - \frac{1}{7}} \cdot \frac{49}{36}$$

$$\Rightarrow 4S = \left(\frac{7}{3}\right)^3$$

Question: $\cot\left(\sum_{x=1}^{50} \tan^{-1}\left(\frac{1}{1+x+x^2}\right)\right)$

Answer: $\frac{26}{25}$

Solution:

$$\tan^{-1}\left(\frac{1}{1+x+x^2}\right) = \tan^{-1}\left(\frac{(x+1)-x}{1+x(x+1)}\right)$$

$$= \tan^{-1}(x+1) - \tan^{-1}(x)$$

$$\therefore \sum_{x=1}^{50} \tan^{-1}\left(\frac{1}{1+x+x^2}\right) = \tan^{-1}2 - \tan^{-1}(1) + \tan^{-1}3 - \tan^{-1}(2) + \ldots + \tan^{-1}(51) - \tan^{-1}(50)$$

$$= \tan^{-1}(51) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{50}{52}\right)$$

$$= \cot^{-1}\left(\frac{26}{25}\right)$$

$\therefore \cot\left(\sum_{x=1}^{50} \tan^{-1}\left(\frac{1}{1+x+x^2}\right)\right) = \frac{26}{25}$

Question: If $a_1, a_2, ...$ & $b_1, b_2....$ are two AP's such that $a_1 = 2$, $a_{10} = 3$ & $a_1b_1 = 1 = a_{10}b_{10}$, then find a_4b_4.

Answer: $\frac{28}{27}$

Solution:

$a_1, a_2, ... \Rightarrow$ A.P.

$b_1, b_2, ... \Rightarrow$ A.P.

$a = 2,$

$a_{10} = 3 = a + 9d_1$

$\Rightarrow 3 = 2 + 9d_1$

$\Rightarrow d_1 = \frac{1}{9}$

$a_1b_1 = 1$

$\Rightarrow b_1 = \frac{1}{2}$

$b_{10} = \frac{1}{3} = \frac{1}{2} + 9d_2$

$\Rightarrow \frac{-1}{6} = 9d_2$

$\Rightarrow d_2 = \frac{-1}{54}$

$a_4b_4 = (a + 3d_1)(b + 3d_2)$

$= \left(2 + 3 \times \frac{1}{9}\right)\left(\frac{1}{2} + 3 \times \left(\frac{-1}{54}\right)\right)$

$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$

$= \left(\frac{7}{3}\right)\left(\frac{16}{2 \times 18}\right)$

$= \frac{7}{3} \times \frac{4}{9} = \frac{28}{27}$

Question: A is 2×2 matrix each element of A is picked from the set $\{0,1,2,3,4,5\}$. How many matrices A are possible if sum of its elements is a prime member p, $2 < p < 8$

Answer: 180.00

Solution:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a+b+c+d=3$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 3 & \rightarrow & 4 \\ 0 & 0 & 1 & 2 & \rightarrow & 12 \\ 0 & 1 & 1 & 1 & \rightarrow & 4 \\ & & & & & \text{total}=20 \end{array}$$

Or $a+b+c+d=5$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 5 & \rightarrow & 4 \\ 0 & 0 & 1 & 4 & \rightarrow & 12 \\ 0 & 0 & 2 & 3 & \rightarrow & 12 \\ 0 & 2 & 2 & 1 & \rightarrow & 12 \\ 0 & 3 & 1 & 1 & \rightarrow & 12 \\ 1 & 1 & 1 & 2 & \rightarrow & 4 \\ & & & & & \text{total}=56 \end{array}$$

Or $a+b+c+d=7$

$$\begin{array}{ccccccc} 0 & 0 & 1 & 6 & \rightarrow & 12 \\ 0 & 0 & 3 & 4 & \rightarrow & 12 \\ 0 & 1 & 1 & 5 & \rightarrow & 12 \\ 0 & 1 & 2 & 4 & \rightarrow & 24 \\ 0 & 1 & 3 & 3 & \rightarrow & 12 \\ 0 & 2 & 2 & 3 & \rightarrow & 12 \\ 1 & 1 & 1 & 4 & \rightarrow & 4 \\ 1 & 1 & 2 & 3 & \rightarrow & 12 \\ 1 & 2 & 2 & 2 & \rightarrow & 4 \\ & & & & & \text{total}=104 \end{array}$$

Answer = 104 + 56 + 20 = 180