QUESTION PAPER CODE 65/3

**EXPECTED ANSWER/VALE POINTS**

**SECTION A**

Marks

1. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$ 1

2. For singular matrix

$4\sin^2 x - 3 = 0$ ½

$\sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3}$ ½

3. $\int e^{2y}dy = \int x^3 dx$ ½

$\Rightarrow \frac{e^{2y}}{2} = \frac{x^4}{4} + c$ ½

4. I.F. $= e^{\int \frac{1}{\sqrt{x}}dx}$ ½

$= e^{2\sqrt{x}}$ ½

5. $3\vec{a} + 2\vec{b} = 7\hat{i} - 5\hat{j} + 4\hat{k}$ ½

$\therefore$ D.R's are 7, –5, 4 ½

6. $(2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(2\hat{i} + 2\hat{j} + \hat{k}) = 12$ ½

$p = \frac{12}{|\vec{b}|} = \frac{12}{3} = 4$ ½

**SECTION B**

7. LHS $= \vec{a}\cdot\{(\vec{b}+\vec{c})\times\vec{d}\} = \vec{a}\cdot\{\vec{b}\times\vec{d}+\vec{c}\times\vec{d}\}$ 1+1

$= \vec{a}\cdot(\vec{b}\times\vec{d}) + \vec{a}\cdot(\vec{c}\times\vec{d})$ 1

$= [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$ 1

8. Here $\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k},\ \vec{a}_2 = 7\hat{i} - 6\hat{k}$

$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

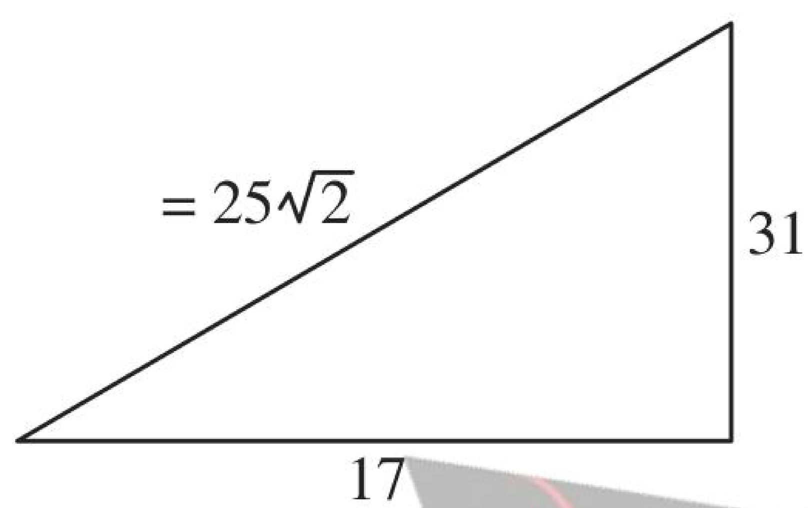
$\vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k}$ 1

$\vec{b}_1 \times \vec{b}_2 = -8\hat{i} + 4\hat{k}$ 1

$\text{SD} = \dfrac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_1|}$ 1

$= \dfrac{|-40 - 28|}{\sqrt{64 + 16}} = \dfrac{68}{\sqrt{80}} = \dfrac{17}{\sqrt{5}}$ 1

9.




$\text{LHS} = 2\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{7}\right)$

$= \tan^{-1}\dfrac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} + \tan^{-1}\left(\dfrac{1}{7}\right) = \tan^{-1}\dfrac{4}{3} + \tan^{-1}\dfrac{1}{7}$ 1½

$= \tan^{-1}\dfrac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \cdot \frac{1}{7}} = \tan^{-1}\dfrac{31}{17}$ 1½

$= \sin^{-1}\left(\dfrac{31}{25\sqrt{2}}\right) = \text{RHS}$ 1

OR

$\tan^{-1}\left(\dfrac{1 - x}{1 + x}\right) = \dfrac{1}{2}\tan^{-1} x$

$\Rightarrow \tan^{-1} 1 - \tan^{1} x = \dfrac{1}{2}\tan^{-1} x$ 1½

$\Rightarrow \dfrac{3}{2}\tan^{-1} x = \dfrac{\pi}{4} \Rightarrow \tan^{-1} x = \dfrac{\pi}{6}$ 1½

$x = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}.$ 1

10. $\text{LHL} = \lim_{x \to 0^-} f(x) = 2\lambda$

$\text{RHL} = \lim_{x \to 0^+} f(x) = 6$

$f(0) = 2\lambda$

$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3$ 2

Differentiability

$\text{LHD} = \lim_{h \to 0} \dfrac{f(0) - f(0-h)}{h} = \lim_{h \to 0} \dfrac{3(2) - 3((-h)^2 + 2)}{h} = \lim_{h \to 0} 3h = 0$ 1

$\text{RHD} = \lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \dfrac{(4h+6) - 3(2)}{h} = \lim_{h \to 0} 4 = 4$ ½

$\text{LHD} \neq \text{RHD}$ $\therefore$ f(x) is not differentiable at x = 0 ½

11. $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$

$\dfrac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)] = -y + x$ 1½

$\dfrac{dy}{dt} = a[e^t(\cos t + \sin t) + e^t(\sin t - \cos t) = x + y$ 1½

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{x+y}{x-y}$ 1

12. $y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$ 1

$\dfrac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx}$ 1

$$\text{LHS} = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\}$$ 1

$$= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn)$$

$$= 0 = \text{RHS.}$$ 1

13. $$I = \int \frac{x+3}{\sqrt{5-4x-2x^2}}\,dx = \int \frac{-\frac{1}{4}(-4-4x)+2}{\sqrt{5-4x-2x^2}}\,dx$$ 1

$$= -\frac{1}{4}\cdot 2\cdot\sqrt{5-4x-2x^2} + \frac{2}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - (x+1)^2}}$$ 1+1

$$= -\frac{1}{2}\sqrt{5-4x-2x^2} + \sqrt{2}\sin^{-1}\left(\frac{x+1}{\sqrt{7/2}}\right) + C$$ 1

14. Let investment in first type of bonds be Rs x.

∴ Investment in 2nd type = Rs (35000 – x) ½

$$\begin{pmatrix} x & 35000-x \end{pmatrix}\begin{pmatrix} \frac{8}{100} \\ \frac{10}{100} \end{pmatrix} = (3200)$$ 1½

$$\Rightarrow \frac{8}{100}x + (35000-x)\frac{10}{100} = 3200$$

$$\Rightarrow x = \text{Rs } 15000$$ 1

∴ Investment in first = Rs 15000

and in 2nd = Rs 20000 1

15. Getting $A' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 1

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{pmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix}$ 1

Since $P' = P$ $\therefore$ P is a symmetric matrix

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{pmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix}$ 1

Since $Q' = -Q$ $\therefore$ Q is skew symmetric

Also

$P + Q = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} = A$ 1

OR

$AB = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -14 \end{pmatrix}$ 1

$LHS = (AB)^{-1} = -\frac{1}{11}\begin{pmatrix} -14 & -5 \\ -5 & -1 \end{pmatrix}$ or $\frac{1}{11}\begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$ 1

$RHS = B^{-1}A^{-1} = 1\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}\frac{-1}{11}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$ 1+1

$\therefore$ LHS = RHS

16. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3,$

$$\Rightarrow \begin{vmatrix} 3a-x & 3a-x & 3a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$ 1

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 3a-x & 0 & 0 \\ a-x & 2x & 0 \\ a-x & 0 & 2x \end{vmatrix} = 0$$ 1+1

$\Rightarrow 4x^2(3a - x) = 0$

$\Rightarrow x = 0,\ x = 3a$ 1

17. $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$ ...(i)

$= \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx = \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$ 1 + ½

$= \int_0^{\pi/4} [\log 2 - \log(1+\tan x)]\,dx$ ...(ii) 1

adding (i) and (ii) to get

$2I = \log 2 \int_0^{\pi/4} 1 \cdot dx = \frac{\pi}{4}\log 2$ 1

$\Rightarrow I = \frac{\pi}{8}\log 2$ ½

18. Writing $I = \int \frac{x}{(x^2+1)(x-1)}\,dx = \int\left(\frac{A}{x-1}+\frac{Bx+C}{x^2+1}\right)dx$ 1

$= \int \frac{1/2}{x-1}\,dx + \int \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\,dx$ 1½

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}x + C$$ 1½

OR

$$I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\,dx$$

Putting x = sin θ, ∴ dx = cos θ dθ and x = 0 then θ = 0

$x \Rightarrow \frac{1}{\sqrt{2}}$ then $\theta = \frac{\pi}{4}$ 1

$$I = \int_0^{\pi/4} \theta \cdot \frac{\cos\theta}{\cos^3\theta}\,d\theta = \int_0^{\pi/4} \theta \cdot \sec^2\theta\,d\theta$$ 1

$$= \left[\theta\tan\theta - \log|\sec\theta|\right]_0^{\pi/4}$$ 1

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$ 1

19. (i) P (all four spades) $= {}^4C_4\left(\frac{13}{52}\right)^4\left(\frac{39}{52}\right)^0 = \frac{1}{256}$ 2

(ii) P (only 2 are spades) $= {}^4C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 = \frac{27}{128}$ 2

OR

$n = 4,\ p = \frac{1}{6},\ q = \frac{5}{6}$

No. of successes

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 | ½ |
| P(x) | ${}^4C_0\left(\frac{5}{6}\right)^4$ | ${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$ | ${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$ | ${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)$ | ${}^4C_4\left(\frac{1}{6}\right)^4$ | 2½ |
| | $=\frac{625}{1296}$ | $=\frac{500}{1296}$ | $=\frac{150}{1296}$ | $=\frac{20}{1296}$ | $=\frac{1}{1296}$ | |
| xP(x) | 0 | $\frac{500}{1296}$ | $\frac{300}{1296}$ | $\frac{60}{1296}$ | $\frac{4}{1296}$ | |

Mean = $\sum xP(x) = \frac{864}{1296} = \frac{2}{3}$. 1

## SECTION C

20. Equation of plane is

$\left\{\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\right\}+\lambda\left\{\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\right\}=0$ 1½

$\Rightarrow \vec{r}\cdot\left\{(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}(-3+3\lambda)\hat{k}\right\} = (7+9\lambda)$ 1½

x-intercept = y-intercept $\Rightarrow \frac{7+9\lambda}{2+2\lambda} = \frac{7+9\lambda}{-3+3\lambda}$ 1

$\Rightarrow \lambda = 5$ ½

$\therefore$ Eqn. of plane is

$\vec{r}\cdot(12\hat{i}+27\hat{j}+12\hat{k}) = 52$ ½

and 12x + 27y + 12z – 52 = 0 1

21. $E_1$: student knows the answer

$E_2$: student guesses the answer

A: answers correctly.

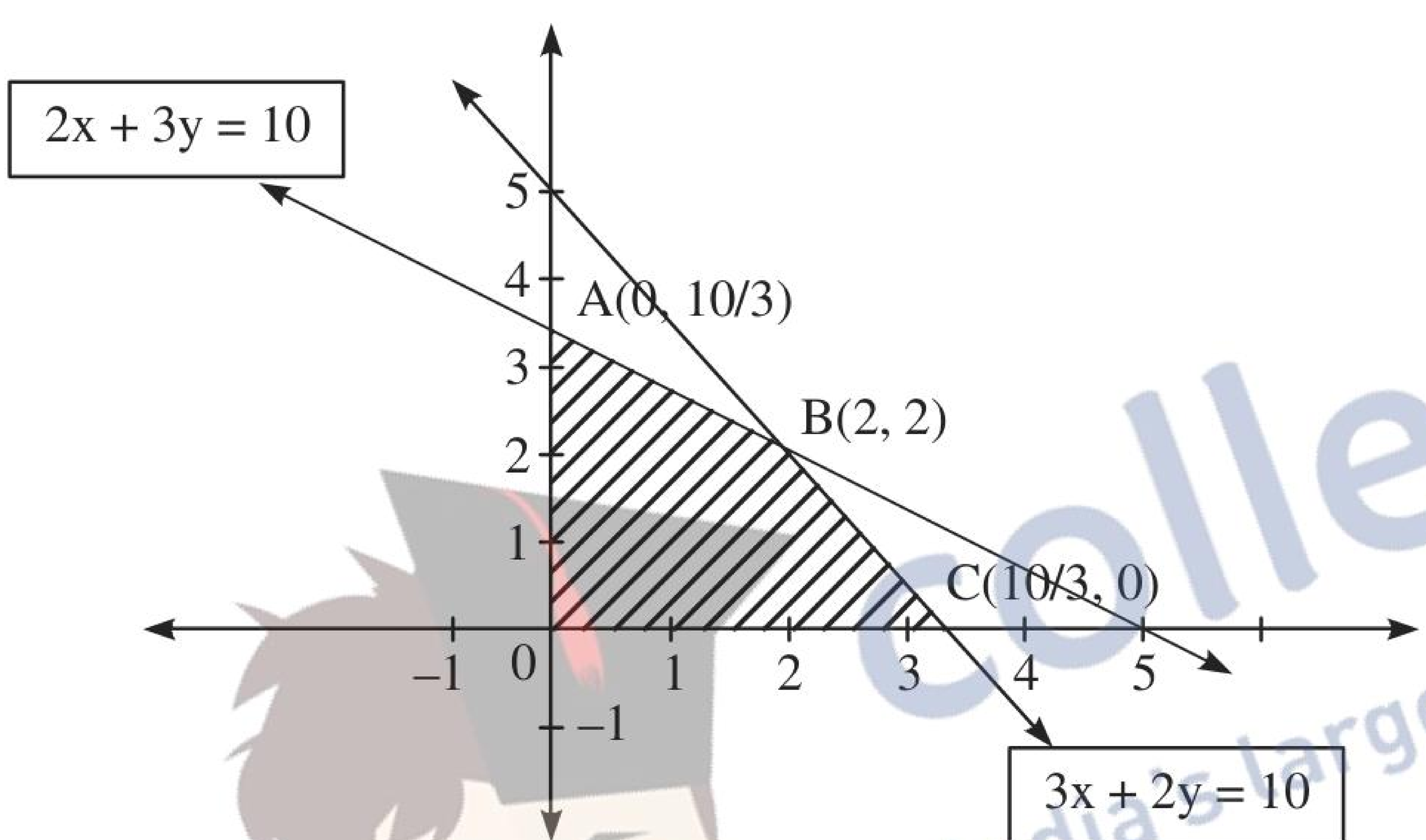
$P(E_1) = \frac{3}{5},\ \ P(E_2) = \frac{2}{5}$ 1

$P\left(\frac{A}{E_1}\right) = 1,\ \ P\left(\frac{A}{E_2}\right) = \frac{1}{3}$ 1+1

$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$ 1

$= \frac{\frac{3}{5}\cdot 1}{\frac{3}{5}\cdot 1 + \frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{11}$ 1+1

22.




L.P.P. is Maximise P = 24x + 18y ½

s.t. $2x + 3y \le 10$

$3x + 2y \le 10$ 2

$x, y \ge 0$

Correct figure 2

P(A) = Rs 60

P(B) = Rs 84 ½

P(C) = Rs 80

$\therefore$ Max. = 84 at (2, 2) 1

23. Given: $s = 4\pi r^2 + 2\left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3}\right]$

$= 4\pi r^2 + 6x^2$ 1

$V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$

$V = \frac{4}{3}\pi r^3 + \frac{2}{3}\left(\frac{S - 4\pi r^2}{6}\right)^{3/2}$ 1

$$\frac{dv}{dr} = 4\pi r^2 + \left(\frac{S-4\pi r^2}{6}\right)^{1/2}\left(\frac{-8\pi r}{6}\right)$$ 1

$$\frac{dv}{dr} = 0 \Rightarrow r = \sqrt{\frac{S}{54+4\pi}}$$ 1

showing $\frac{d^2v}{dr^2} > 0$ 1

$\therefore$ For $r = \sqrt{\frac{S}{54+4\pi}}$ volume is minimum

i.e., $(54 + 4\pi)r^2 = 4\pi r^2 + 6x^2$

$6x^2 = 54r^2 \Rightarrow x^2 = 9r^2 \Rightarrow x = 3r$ 1

24. Here,

$$R = \begin{Bmatrix} (1,1),(2,2),(3,3),(4,4),(5,5) \\ (1,3),(1,5),(2,4),(3,5) \\ (3,1),(5,1),(4,2),(5,3) \end{Bmatrix}$$ 2

Clearly

(i) $\forall a \in A, (a, a) \in R \quad \therefore$ R is reflexive 1

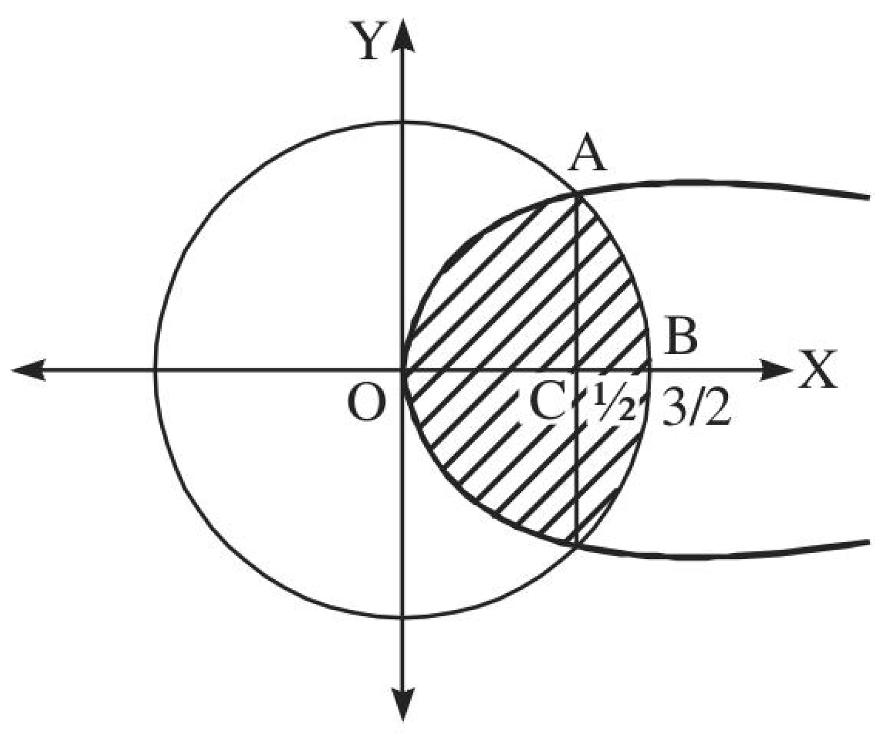
(ii) $\forall (a,b) \in A, (b, a) \in R \quad \therefore$ R is symmetric 1

(iii) $\forall (a,b),(b,c) \in R, (a,c) \in R \quad \therefore$ R is transitive 1

$\therefore$ R is an equivalence relation.

$[1] = \{1, 3, 5\}, \quad [2] = \{2, 4\}$ 1

25. $\{(x, y) : y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$




Correct figure — 1

Getting $x = \frac{1}{2}$ as point of intersection — ½

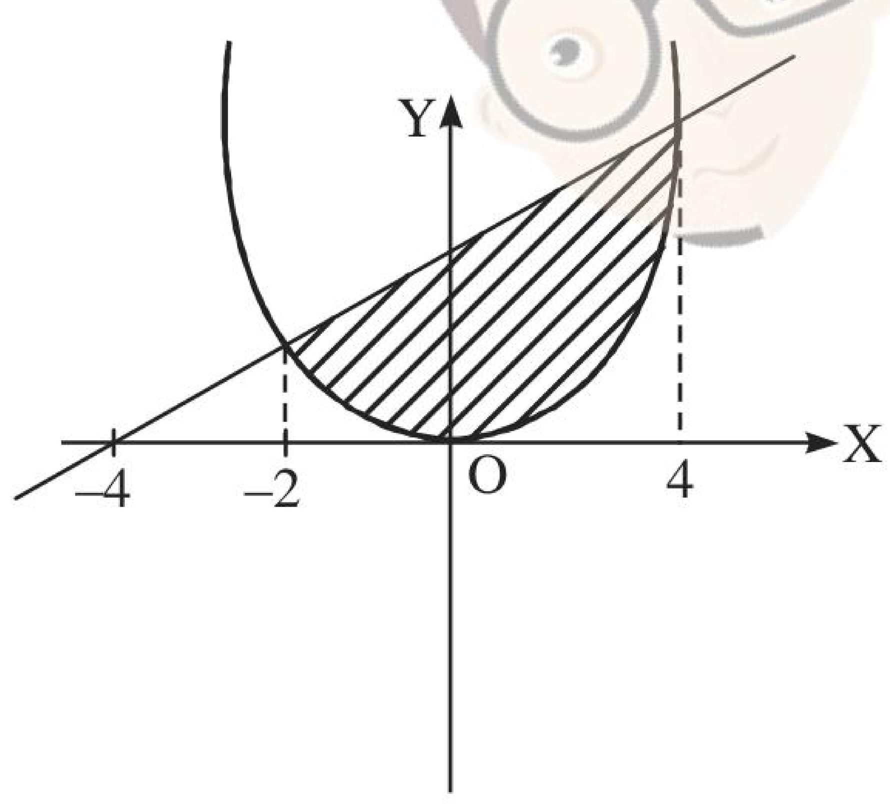
$$A = 2\left[2\int_0^{1/2}\sqrt{x}\,dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4} - x^2}\,dx\right] \quad 1$$

$$= 2\left[\left(\frac{4}{3}x^{3/2}\right)_0^{1/2} + \left(\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\right)_{1/2}^{3/2}\right] \quad 1½$$

$$= 2\left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right] \quad 1$$

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} \text{ sq. unit} \quad 1$$

OR




Correct figure — 1

Getting x = 4, –2 as points of intersection — ½

$$A = \int_{-2}^{4}\frac{1}{2}(3x + 12)\,dx - \int_{-2}^{4}\frac{3}{4}x^2\,dx \quad 1$$

$$= \frac{1}{2}\left(\frac{3x^2}{2} + 12x\right)_{-2}^{4} - \frac{1}{4}(x^3)\Big]_{-2}^{4} \quad 1½$$

$$= \frac{1}{2}(24 + 48 - 6 + 24) - \frac{1}{4}(64 + 8) \quad 1½$$

$$= 45 - 18 = 27 \text{ sq. units} \quad ½$$

26. $\left(x\sin^2\left(\frac{y}{x}\right) - y\right)dx + x\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x\sin^2(y/x)}{x} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$$ 1

$$v + x\frac{dv}{dx} = v - \sin^2 v \quad \text{where } \frac{y}{x} = v.$$ 1

$$\Rightarrow \int -\frac{dv}{\sin^2 v} = \int \frac{dx}{x} \quad \text{or} \quad \int -\text{cosec}^2 v\, dv = \int \frac{dx}{x}$$ 1½

$$\cot v = \log x + C \text{ i.e., } \cot\frac{y}{x} = \log x + C$$ 1½

$$y = \frac{\pi}{4}, x = 1, \Rightarrow C = 1$$ ½

$$\Rightarrow \cot\frac{y}{x} = \log x + 1$$ ½

OR

$$\frac{dy}{dx} - 3\cot x \cdot y = \sin 2x$$

$$\text{IF} = e^{\int -3\cot x\, dx} = e^{-3\log \sin x} = \text{cosec}^3 x$$ 1

∴ Solution is

$$y \cdot \text{cosec}^3 x = \int \sin 2x\, \text{cosec}^3 x\, dx$$ 1½

$$= \int 2\text{cosec}\, x \cot x\, dx$$ ½

$$y \cdot \text{cosec}^3 x = -2\,\text{cosec}\, x + C$$ 1½

or $y = -2\sin^2 x + C\sin^3 x$

$$x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4$$ 1

$$\Rightarrow y = -2\sin^2 x + 4\sin^3 x$$ ½