DU MPhil Phd in Mathematics

Topic:- DU_J18_MPHIL_MATHS_Topic01

1) The mathematician who was awarded Abel's prize for a proof of Fermat's Last Theorem is [Question ID = 19249]

1. Andrew Wiles. [Option ID = 46987]
2. Johan F. Nash. [Option ID = 46988]
3. S. R. Srinivasa Varadhan. [Option ID = 46989]
4. Lennart Carleson. [Option ID = 46990]

Correct Answer :-

- Andrew Wiles. [Option ID = 46987]

2) Founder of Indian Mathematical Society(IMS) was [Question ID = 19252]

1. Asutosh Mukherjee. [Option ID = 47000]
2. S. Narayana Aiyer. [Option ID = 47001]
3. M.T. Narayaniyengar. [Option ID = 47002]
4. V. Ramaswamy Aiyer. [Option ID = 46999]

Correct Answer :-

- V. Ramaswamy Aiyer. [Option ID = 46999]

3) Let R be a commutative ring with identity. If R is an Artinian domain, then the total number of prime ideals in R is [Question ID = 19280]

1. 1 [Option ID = 47111]
2. infinite. [Option ID = 47114]
3. 3 [Option ID = 47113]
4. 2 [Option ID = 47112]

Correct Answer :-

- 1 [Option ID = 47111]

4) Riemann hypothesis is associated with the function [Question ID = 19250]

1. $f(s) = \int_0^\infty t^{s-1} e^{-t}\, dt.$ [Option ID = 46991]
2. $f(x,\, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt$ [Option ID = 46992]
3. Hermite polynomial [Option ID = 46994]
4. $f(s) = \sum_{n=1}^\infty \frac{1}{n^s},\ s \in \mathbb{C}$ [Option ID = 46993]

Correct Answer :-

- $f(s) = \sum_{n=1}^\infty \frac{1}{n^s},\ s \in \mathbb{C}$ [Option ID = 46993]

5) For the stream function of a two dimensional motion, which of the following is not true

[Question ID = 19297]

1. Stream function is constant along a stream line. [Option ID = 47181]
2. Stream function is harmonic. [Option ID = 47180]
3. Stream function exists for steady motion of compressible fluid. [Option ID = 47179]
4. Stream function has dimension L^2T^{-2}. [Option ID = 47182]

Correct Answer :-

- Stream function has dimension L^2T^{-2}. [Option ID = 47182]

6) The famous Indian mathematician Srinivas Ramanujan passed away in the year [Question ID = 19248]

1. 1920 [Option ID = 46984]
2. 1922 [Option ID = 46985]
3. 1921 [Option ID = 46983]
4. 1919 [Option ID = 46986]

Correct Answer :-

- 1920 [Option ID = 46984]

7) Let F be a finite field with 9 elements. How many elements of F have order 8? [Question ID = 19287]

1. 1 [Option ID = 47142]
2. 4 [Option ID = 47140]
3. 8 [Option ID = 47139]
4. 2 [Option ID = 47141]

Correct Answer :-
- 4 [Option ID = 47140]

8) For a viscous compressible fluid Consider the following statements:
(I) Stress matrix is symmetric.
(II) Kinematic coefficient of viscosity is dependent on the mass.
(III) Rate of dilatation is $\nabla . \bar{q}$.
Then

[Question ID = 19293]

1. all of I, II and III are true. [Option ID = 47163]
2. only I and III are true. [Option ID = 47164]
3. only I and II are true. [Option ID = 47165]
4. only II and III are true. [Option ID = 47166]

Correct Answer :-
- only I and III are true. [Option ID = 47164]

9) Let $f : R \to R'$ be a ring homomorphism. Assume that 1 and $1'$ are multiplicative identities of the rings R and R' respectively. Then $f(1) = 1'$ if
I f is onto.
II f is one-one.
III R is a domain.
IV R' is a domain.
The correct options are

[Question ID = 19276]

1. III and IV only. [Option ID = 47096]
2. II and III only [Option ID = 47098]
3. I and IV only. [Option ID = 47097]
4. I and II only. [Option ID = 47095]

Correct Answer :-
- I and IV only. [Option ID = 47097]

10)
For a solid stationary sphere of radius a placed in an incompressible fluid of uniform stream with velocity $-Ui$:
(I) velocity potential $\phi(r, \theta) = U\cos\theta(r + \frac{a^3}{2r^2})$.
(II) there exist two stagnation points $(a, 0)$, (a, π).
(III) stagnation pressure $p_\infty + \frac{1}{2}\rho U^2$, p_∞ is a pressure at ∞.
(IV) velocity at any point of surface of sphere is $(0, U\sin\theta, 0)$.
Then

[Question ID = 19296]

1. only I, II, IV are true. [Option ID = 47175]
2. only I, III, IV are true. [Option ID = 47177]
3. only I, II, III are true. [Option ID = 47176]
4. only II, III, IV are true. [Option ID = 47178]

Correct Answer :-
- only I, II, III are true. [Option ID = 47176]

11) Let $R = \{a + ib : a, b \in \mathbb{Z}$. Then R is a Euclidean domain with

[Question ID = 19277]

1. exactly two units. [Option ID = 47099]
2. exactly eight units. [Option ID = 47101]
3. exactly four units. [Option ID = 47100]
4. infinitely many units. [Option ID = 47102]

Correct Answer :-

- exactly four units. [Option ID = 47100]

12) Consider the sequence of Lebesgue measurable functions (f_n) on $\mathbb{R}$

$$f_n(x) = \begin{cases} 5, & x \geq 2^n \\ 0, & x < 2^n. \end{cases}$$

Then $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)\,dx$

[Question ID = 19263]

1. does not exist [Option ID = 47046]
2. equals 0. [Option ID = 47043]
3. equals 5. [Option ID = 47044]
4. equals ∞. [Option ID = 47045]

Correct Answer :-

- equals ∞. [Option ID = 47045]

13) Let $f(x) = \sin x + \cos x$ on $[0, \pi]$. Then $||f||_\infty$ is equal to

[Question ID = 19269]

1. 1 [Option ID = 47067]
2. $2\sqrt{2}$ [Option ID = 47070]
3. $\sqrt{2}$ [Option ID = 47068]
4. $1/\sqrt{2}$ [Option ID = 47069]

Correct Answer :-

- $\sqrt{2}$ [Option ID = 47068]

14) Let f be a continuous function on a finite interval $[a, b]$. Then

$$\lim_{t\to\infty} \int_a^b f(x) \sin tx\,dx$$

[Question ID = 19260]

1. equals 0 [Option ID = 47033]
2. equals $\sup_{x\in[a,\,b]} f(x)$ [Option ID = 47034]
3. does not exist [Option ID = 47032]
4. equals $\int_a^b f(x)\,dx$. [Option ID = 47031]

Correct Answer :-

- equals 0 [Option ID = 47033]

15)

Let (X, d) be a metric space and $A \subseteq X$, $B \subseteq X$. Consider the following statements:

I If $x \notin A$ then $d(x, A) > 0$.

II If $A \cap B = \phi$, then $d(A, B) \geq 0$.

III If A is closed and $x \notin A$ then $d(x, A) > 0$.

IV If A and B are closed and $A \cap B = \phi$ then $d(A, B) \geq 0$.

Then,

[Question ID = 19259]

1. all statements are correct. [Option ID = 47030]
2. only III is correct. [Option ID = 47028]
3. only II, III, IV are correct. [Option ID = 47027]
4. only III and IV are correct. [Option ID = 47029]

Correct Answer :-

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16) The set $A = \{x \in \mathbb{Q} \mid -\sqrt{7} \leq x \leq \sqrt{7}\}$ in the subspace $\mathbb{Q}$ of the real line $\mathbb{R}$ is

[Question ID = 19271]

1. neither open nor closed [Option ID = 47078]
2. open but not closed [Option ID = 47075]
3. both open and closed [Option ID = 47077]
4. closed but not open [Option ID = 47076]

Correct Answer :-

- both open and closed [Option ID = 47077]

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17)

A Lipschitz's constant associated with the function $f(x, y) = y^{2/3}$ on $R : |x| \leq 1, |y| \leq 1$

[Question ID = 19288]

1. does not exist. [Option ID = 47146]
2. equals 1/2. [Option ID = 47145]
3. equals 0. [Option ID = 47143]
4. equals 1. [Option ID = 47144]

Correct Answer :-

- does not exist. [Option ID = 47146]

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18)

Let $I = \int_C y\,dx + (x + 2y)\,dy$, where $C = C_1 + C_2$, C_1 being the line joining $(0, 1)$ to $(1, 1)$ and C_2 is the line joining $(1, 1)$ to $(1, 0)$. The value of I is

[Question ID = 19256]

1. 2 [Option ID = 47017]
2. −1 [Option ID = 47018]
3. 1 [Option ID = 47015]
4. 0 [Option ID = 47016]

Correct Answer :-

- 1 [Option ID = 47015]

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19) Let $F(x) = \int_0^x \frac{\sin t}{t^{3/2}}\,dt$, $0 < x < \infty$. The local maximum value is at the point

[Question ID = 19255]

1. $x = \pi/2$ [Option ID = 47013]
2. $x = 4\pi$ [Option ID = 47014]

3. $x = \pi$ [Option ID = 47011]
4. $x = 2\pi$ [Option ID = 47012]

Correct Answer :-

- $x = \pi$ [Option ID = 47011]

20)

The general integral of the partial differential equation $yzp + xzq = xy$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ (G being an arbitrary function) is

[Question ID = 19289]

1. $z^2 = x^2 - G(x^2 + y^2)$. [Option ID = 47150]
2. $z^2 = y^2 + G(x^2 + y^2)$. [Option ID = 47147]
3. $z^2 = y^2 + G(x^2 - y^2)$. [Option ID = 47149]
4. $z^2 = x - G(x^2 - y^2)$. [Option ID = 47148]

Correct Answer :-

- $z^2 = y^2 + G(x^2 - y^2)$. [Option ID = 47149]

21)

Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then

[Question ID = 19257]

1. For any $\delta > 0$, f is not monotonic on $[0, \delta)$ [Option ID = 47020]
2. f has a local extremum at $x = 0$ [Option ID = 47021]
3. For any $\delta > 0$, f is convex on $[0, \delta)$ [Option ID = 47022]
4. f' is continuous at $x = 0$ [Option ID = 47019]

Correct Answer :-

- For any $\delta > 0$, f is not monotonic on $[0, \delta)$ [Option ID = 47020]

22)

Let $F = \mathbb{Q}((\sqrt{2}, \sqrt{3}))$. Then F is minimal splitting field of the polynomial $(x^2 - 2)(x^2 - 3)$ over $\mathbb{Q}$. The field F is not the minimal splitting field of which of the following polynomials over $\mathbb{Q}$

[Question ID = 19286]

1. $x^4 - 10x^2 + 1$. [Option ID = 47135]
2. $x^{-4} - x^2 + 6$. [Option ID = 47137]
3. $x^4 + x^2 + 1$. [Option ID = 47136]
4. $x^4 + x^2 + 25$.. [Option ID = 47138]

Correct Answer :-

23)

An elementary solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is of the form $(\bar{r} = xi + yj,\ \bar{r'} = x'i + y'j)$

[Question ID = 19290]

1. $u = \log|\bar{r}\bar{r'}|.$ [Option ID = 47154]
2. $u = \log\frac{1}{|\bar{r}+\bar{r'}|}.$ [Option ID = 47151]
3. $u = \log\frac{1}{|\bar{r}\bar{r'}|}.$ [Option ID = 47153]
4. $u = \log\frac{1}{|\bar{r}-\bar{r'}|}.$ [Option ID = 47152]

Correct Answer :-

- $u = \log\frac{1}{|\bar{r}-\bar{r'}|}.$ [Option ID = 47152]

24)

Let $E = \{x \in (0, \sqrt{2}] : x \text{ is a rational number}\} \cup \{y \in [2, 3] : y \text{ is an irrational number}\}$
Then the Lebesgue measure of E is

[Question ID = 19264]

1. 1 [Option ID = 47048]
2. $\sqrt{2}$ [Option ID = 47049]
3. $1/2$ [Option ID = 47050]
4. $\sqrt{2}+1$ [Option ID = 47047]

Correct Answer :-

- 1 [Option ID = 47048]

25)

Let H be a Sylow p-subgroup and K be a p-subgroup of a finite group G. Which of the following is incorrect is incorrect (H char G means H is characteristic in G)

[Question ID = 19282]

1. $K \triangleleft G \Rightarrow K \subset H.$ [Option ID = 47119]
2. $K \triangleleft G \Rightarrow K \,\text{char}\, H.$ [Option ID = 47121]
3. $K \subset H$ if $K \triangleleft G.$ [Option ID = 47120]
4. $K \triangleleft G \not\Rightarrow H \cap K \triangleleft H$. [Option ID = 47122]

Correct Answer :-

- $K \triangleleft G \not\Rightarrow H \cap K \triangleleft H$. [Option ID = 47122]

26)

A two dimensional motion with complex potential $w = U(z + \frac{a^2}{z}) + ik \log \frac{z}{a}$ has
(I) stream lines as circle $|z| = a$.
(II) circulation zero about circle $|z| = a$.
(III) has two stagnation points in general.
(IV) velocity at infinity equal to (-U).
Then

[Question ID = 19295]

1. only I, II, IV are true. [Option ID = 47172]
2. only I, III, IV are true. [Option ID = 47173]
3. only I, II, III are true. [Option ID = 47171]
4. only II, III, IV are true. [Option ID = 47174]

Correct Answer :-
- only I, III, IV are true. [Option ID = 47173]

27)

Let G be an abelian group of order 15. Define a map $\phi : G \to G$ by $\phi(g) = g^8$ for all $g \in G$. Consider the statements:
I ϕ is a homomorphism.
II ϕ is one-to-one.
III ϕ is onto.
Then

[Question ID = 19281]

1. only I and III are true. [Option ID = 47117]
2. only I and II are true. [Option ID = 47116]
3. only I is true. [Option ID = 47115]
4. all statements are true. [Option ID = 47118]

Correct Answer :-
- all statements are true. [Option ID = 47118]

28)

Let ξ be a primitive n^{th} root of unity where $n \equiv 2 (\mod 4)$. Then $[\mathbb{Q}(\xi) : \mathbb{Q}(\xi^2)]$ is
(Here $[V : F]$ denotes the dimension of the vector space V over F)

[Question ID = 19285]

1. 1 [Option ID = 47131]
2. 2 [Option ID = 47132]
3. $\phi(n)$ [Option ID = 47133]
4. $\phi(n)/2$ [Option ID = 47134]

Correct Answer :-
- 1 [Option ID = 47131]

29)

The closed topologist's sine curve $\{(x, \sin \frac{1}{x}) \,|\, x \in (0, 1]\}$ as subspace of real line $\mathbb{R}$ is

[Question ID = 19272]

1. a path connected space [Option ID = 47081]
2. connected but not locally connected [Option ID = 47079]

3. a locally path connected space [Option ID = 47082]
4. locally connected but not connected [Option ID = 47080]

Correct Answer :-
- connected but not locally connected [Option ID = 47079]

30)

Let $R(T)$ and $N(T)$ denote the range space and null space of the linear transformation $T : P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ which is given by

$$T(f) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}.$$

Then

[Question ID = 19275]

1. $\dim(R(T)) = 2$ and $\dim(N(T)) = 1$ [Option ID = 47094]
2. $\dim(R(T)) = 0$ and $\dim(N(T)) = 2$ [Option ID = 47093]
3. $\dim(R(T)) = 2$ and $\dim(N(T)) = 0$ [Option ID = 47091]
4. $\dim(R(T)) = 1$ and $\dim(N(T)) = 1$ [Option ID = 47092]

Correct Answer :-
- $\dim(R(T)) = 2$ and $\dim(N(T)) = 1$ [Option ID = 47094]

31) The bilinear transformation on $\mathbb{C}$ which maps $z = 0, -i, -1$ into $w = i, 1, 0$ is

[Question ID = 19265]

1. $-i\frac{z+1}{z-1}$ [Option ID = 47053]
2. $\frac{z+1}{z-1}$ [Option ID = 47052]
3. $i\frac{z+1}{z-1}$ [Option ID = 47051]
4. $i\frac{z-1}{z+1}$ [Option ID = 47054]

Correct Answer :-
- $-i\frac{z+1}{z-1}$ [Option ID = 47053]

32) Let $A, B \in M_n(\mathbb{C})$. Consider the following statements

I If A, B and $A + B$ are invertible, then $A^{-1} + B^{-1}$ is invertible.
II If A, B and $A + B$ are invertible, then $A^{-1} - B^{-1}$ is invertible.
III If AB is nilpotent, then BA is nilpotent.
IV Characteristic polynomials of AB and BA are equal if A is invertible.

Then

[Question ID = 19274]

1. only I, III, and IV are true [Option ID = 47089]
2. all the statements are true.. [Option ID = 47090]
3. only III is true [Option ID = 47088]
4. only I and II are true [Option ID = 47087]

Correct Answer :-
- only I, III, and IV are true [Option ID = 47089]

33)

For the boundary value problem: $L(y) = y'' = 0,\ y(0) = 0,\ \ y'(1) = 0$, the Green's function is

[Question ID = 19291]

1. $G(x,\,\xi) = \begin{cases} \xi, & x \le \xi \\ x, & x > \xi \end{cases}$ [Option ID = 47156]
2. $G(x,\,\xi) = \begin{cases} -x, & x \le \xi \\ -\xi, & x > \xi \end{cases}$ [Option ID = 47157]
3. $G(x,\,\xi) = \begin{cases} -x, & x \le \xi \\ -\xi, & x > \xi \end{cases}$ [Option ID = 47158]
4. $G(x,\,\xi) = \begin{cases} x, & x \le \xi \\ \xi, & x > \xi \end{cases}$ [Option ID = 47155]

Correct Answer :-

- $G(x,\,\xi) = \begin{cases} x, & x \le \xi \\ \xi, & x > \xi \end{cases}$ [Option ID = 47155]

34) Let $E = \{x \in [0,\,\pi) : \sin 4x < 0\}$. Then Lebesgue measure of E is

[Question ID = 19262]

1. $\pi/2$ [Option ID = 47040]
2. $\pi/4$ [Option ID = 47039]
3. $3\pi/4$ [Option ID = 47041]
4. $\pi/3$ [Option ID = 47042]

Correct Answer :-

- $\pi/2$ [Option ID = 47040]

35)

Let $x_1, x_2, \cdots, x_n$ be non-zero real numbers. With $x_{ij} = x_i x_j$, let X be the $n \times n$ matrix (x_{ij}). Then

[Question ID = 19273]

1. the matrix X is positive definite if $(x_1, x_2, \cdots, x_n)$ is a non-zero vector [Option ID = 47084]
2. the matrix X is positive semi definite for all $(x_1, x_2, \cdots, x_n)$ [Option ID = 47085]
3. for all $(x_1, x_2, \cdots, x_n)$, zero is an eigenvalue of X. [Option ID = 47086]
4. it is possible to chose $x_1, x_2, \cdots, x_n$ so as to make the matrix X non singular [Option ID = 47083]

Correct Answer :-

36)

Let $A = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous on } \mathbb{Q} \text{ and discontinuous } \mathbb{Q}'\}$, where $\mathbb{Q}$ is the set of all rational numbers and $\mathbb{Q}'$ is the set of all irrational numbers. Let μ be a counting measure on A. Then

[Question ID = 19258]

1. $\mu(A) = \sum_{q \in \mathbb{Q}} \frac{1}{2^q}$ [Option ID = 47026]
2. $\mu(A)$ is infinite [Option ID = 47023]
3. $\mu(A) = 0$ [Option ID = 47024]
4. $\mu(A) = 2$ [Option ID = 47025]

Correct Answer :-

. $\mu(A) = 0$ [Option ID = 47024]

37) Let $R = \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$. Then the total number of zero divisors in R is

[Question ID = 19278]

1. 15 [Option ID = 47106]
2. 10 [Option ID = 47105]
3. 20 [Option ID = 47104]
4. 22 [Option ID = 47103]

Correct Answer :-

38)

Let $a, b \in \mathbb{C}$ such that $0 < |a| < |b|$. Then the Laurent expression of $\frac{1}{(z-a)(z-b)}$ in the annulus $|a| < |z| < |b|$ is

[Question ID = 19266]

1. $\frac{1}{a-b}\left[\sum_{n=0}^{\infty} \frac{z^n}{b^n} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}}\right]$ [Option ID = 47057]
2. $\frac{1}{a-b}\left[\sum_{n=0}^{\infty} \frac{z^n}{b^{n+1}} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}}\right]$ [Option ID = 47055]
3. $\frac{1}{a-b}\left[\sum_{n=0}^{\infty} \frac{z^n}{a^{n+1}} + \sum_{n=0}^{\infty} \frac{b^n}{z^{n+1}}\right]$ [Option ID = 47056]
4. $\frac{1}{a-b}\left[\sum_{n=0}^{\infty} \frac{z^n}{a^n} + \sum_{n=0}^{\infty} \frac{b^{n+1}}{z^n}\right]$ [Option ID = 47058]

Correct Answer :-

. $\frac{1}{a-b}\left[\sum_{n=0}^{\infty} \frac{z^n}{b^{n+1}} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}}\right]$ [Option ID = 47055]

39) Consider the following statements:

I $x^3 - 9$ is not irreducible over $\mathbb{Z}_7$.

II $x^3 - 9$ is not irreducible over $\mathbb{Z}_{11}$.

Then

[Question ID = 19279]

1. II is false but I is true. [Option ID = 47107]
2. both I and II are true. [Option ID = 47109]
3. both I and II are false. [Option ID = 47110]

4. I is false but II is true. [Option ID = 47108]

Correct Answer :-

- I is false but II is true. [Option ID = 47108]

40)
The contour integral $\int_C \frac{e^z}{(z^2+\pi^2)^2}\,dz$, where C is the circle $|z| = 4$ taken anti-clockwise equals

[Question ID = 19267]

1. $\frac{i}{2\pi}$ [Option ID = 47061]
2. $\frac{4}{\pi i}$ [Option ID = 47059]
3. $\frac{4}{\pi}$ [Option ID = 47060]
4. $\frac{i}{\pi}$ [Option ID = 47062]

Correct Answer :-

- $\frac{i}{\pi}$ [Option ID = 47062]

41)
The pressure $p(x, y, z)$ in steady flow of inviscid incompressible fluid of density ρ with velocity $\bar{q} = (kx, -ky, 0)$, k is a constant, under no external force when $p(0,0,0) = p_0$, is

[Question ID = 19341]

1. $p_0 - \rho k^2(y^2 - x^2)/2.$ [Option ID = 47358]
2. $p_0 - \rho k^2(y^2 + x^2).$ [Option ID = 47354]
3. $p_0 - \rho k^2(y^2 - x^2).$ [Option ID = 47352]
4. $p_0 - \rho k^2(y^2 + x^2)/2.$ [Option ID = 47356]

Correct Answer :-

- $p_0 - \rho k^2(y^2 + x^2)/2.$ [Option ID = 47356]

42) let E be a Lebesgue non-measurable subset of $\mathbb{R}$. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2, & x \in E \\ -2, & x \in E^c. \end{cases}$$

Then

[Question ID = 19261]

1. neither f nor $|f|$ is Lebesgue measurable [Option ID = 47038]
2. f is Lebesgue measurable but $|f|$ is not Lebesgue measurable [Option ID = 47036]
3. f is not Lebesgue measurable but $|f|$ is Lebesgue measurable [Option ID = 47037]
4. f and $|f|$ both are Lebesgue measurable. [Option ID = 47035]

Correct Answer :-

• f is not Lebesgue measurable but $|f|$ is Lebesgue measurable [Option ID = 47037]

43) Every non trivial solution of the equation $y'' + (\sinh x)y = 0$ has

[Question ID = 19292]

1. only finitely many zeros in $(0, \infty)$. [Option ID = 47162]
2. infinitely many zeros in $(-\infty, 0)$. [Option ID = 47160]
3. infinitely many zeros in $(0, \infty)$. [Option ID = 47159]
4. at most one zero in $(0, \infty)$. [Option ID = 47161]

Correct Answer :-

• infinitely many zeros in $(0, \infty)$. [Option ID = 47159]

44) Which of the following statements is true [Question ID = 19253]

1. If $0 \leq a_n \leq b_n$ and $\sum b_n$ diverges then $\sum a_n$ diverges [Option ID = 47005]
2. If $\lim_{n\to\infty} a_n = 0$, then $\sum \frac{a_n}{a_n^2+n^2}$ converges [Option ID = 47004]
3. $\sum_{k=1}^{\infty} \left(\tan^{-1}\frac{1}{k} - \tan^{-1}\frac{1}{k+1}\right) = \frac{\pi}{8}$ [Option ID = 47003]
4. $\sum_{n=1}^{\infty} \frac{1}{n^n} \geq 2$ [Option ID = 47006]

Correct Answer :-

• If $\lim_{n\to\infty} a_n = 0$, then $\sum \frac{a_n}{a_n^2+n^2}$ converges [Option ID = 47004]

45) Which of the following statements is not true [Question ID = 19254]

1. The set of all algebraic numbers is countable. [Option ID = 47010]
2. The set of rational numbers is equivalent to the set of natural numbers [Option ID = 47008]
3. Given a set A, there exists a function $f : A \to P(A)$ that is onto ($P(A)$ denotes power set of A) [Option ID = 47009]
4. There is one-one function taking $(-1, 1)$ onto $\mathbb{R}$. [Option ID = 47007]

Correct Answer :-

• Given a set A, there exists a function $f : A \to P(A)$ that is onto ($P(A)$ denotes power set of A) [Option ID = 47009]

46) Which of the following statements is not true [Question ID = 19270]

1. An uncountable discrete space is not separable. [Option ID = 47072]
2. Every closed subspace of a separable space is separable. [Option ID = 47073]
3. Every compact metric space is Lindelof. [Option ID = 47074]
4. Every second countable space is separable. [Option ID = 47071]

Correct Answer :-

• Every closed subspace of a separable space is separable. [Option ID = 47073]

47) Which of the following is not correct (Here [V : F] denotes the dimension of the vector space V over F) [Question ID = 19284]

1. $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, i, \sqrt{6}) : \mathbb{Q}] = 16.$ [Option ID = 47130]
2. $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, i) : \mathbb{Q}] = 8.$ [Option ID = 47129]
3. $[\mathbb{Q}(\sqrt{3}) : \mathbb{Q}] = 2.$ [Option ID = 47127]

4. $[\mathbb{Q}(\sqrt{3},\, i) : \mathbb{Q}] = 4.$ [Option ID = 47128]

Correct Answer :-

- $[\mathbb{Q}(\sqrt{2},\, \sqrt{3},\, i,\, \sqrt{6}) : \mathbb{Q}] = 16.$ [Option ID = 47130]

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48) Which of the following Banach spaces is not a Hilbert space [Question ID = 19268]

1. $(L^2([0,\, 1]),\, ||.||_2)$ [Option ID = 47064]
2. $\mathbb{R}^n$ with the norm $||x|| = \sqrt{\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2}$, where $x = (\xi_1, \xi_2, \cdots \xi_n)$ [Option ID = 47065]
3. $\mathbb{R}^n$ with the norm $||x|| = \max\{|\xi_1|, |\xi_2|, \cdots, |\xi_n|\}$, where $x = (\xi_1, \xi_2, \cdots \xi_n)$ [Option ID = 47066]
4. $(l^2,\, ||.||_2)$ [Option ID = 47063]

Correct Answer :-

- $\mathbb{R}^n$ with the norm $||x|| = \max\{|\xi_1|, |\xi_2|, \cdots, |\xi_n|\}$, where $x = (\xi_1, \xi_2, \cdots \xi_n)$ [Option ID = 47066]

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49) Which of the following websites is of Mathematical Reviews [Question ID = 19251]

1. https://mathscinet.ams.org [Option ID = 46997]
2. https://mathscinet.ac.in [Option ID = 46995]
3. https://math.ac.au [Option ID = 46996]
4. https://www.mathjournal.org. [Option ID = 46998]

Correct Answer :-

- https://mathscinet.ams.org [Option ID = 46997]

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50) Let G be a cyclic group of order 42. The number of distinct composition series of G is [Question ID = 19283]

1. 8 [Option ID = 47126]
2. 16 [Option ID = 47123]
3. 10 [Option ID = 47125]
4. 6 [Option ID = 47124]

Correct Answer :-

- 6 [Option ID = 47124]
