10/08/2022

Slot-1

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Answers & Solutions

Time : 45 min. *for* M.M. : 200

CUET UG-2022

(Mathematics)

IMPORTANT INSTRUCTIONS:

1. The test is of 45 Minutes duration.
2. The test contains is divided into two sections.
   a. Section A contains 15 questions which will be compulsory for all candidates.
   b. Section B will have 35 questions out of which 25 questions need to be attempted.
3. Marking Scheme of the test:
   a. Correct answer or the most appropriate answer: Five marks (+5)
   b. Any incorrect option marked will be given minus one mark (-1).

Choose the correct answer :

Question ID: 481281

If A is square matrix of order 3 and A.(Adj.(A)) = 10I, then the value of $\frac{1}{25}|\text{Adj.}(A)|$ is

(A) 100 (B) 25

(C) 10 (D) 4

Answer (D)

Sol. $A(\text{Adj.}(A)) = 10I$

$|A|\, I = 10I$

$\Rightarrow\ |A| = 10$

$$\frac{1}{25}|\text{Adj}(A)| = \frac{|A|^2}{25} = \frac{100}{25}$$

$= 4$

Question ID: 481282

Let A and B be two non-singular, square matrices of same order, and

A. $(AB)^{-1} = B^{-1}.A^{-1}$

B. $(A + B)^{-1} = B^{-1} + A^{-1}$

C. adj. $A = |A|.A^{-1}$

D. $\det(A^{-1}) = [\det A]^{-1}$

Choose the correct answer from the options given below

(A) A and B only (B) B and C only

(C) B and D only (D) A, C and D only

Answer (D)

Sol. A. $(AB)^{-1} = B^{-1}A^{-1}$

B. $(A + B)^{-1} \neq B^{-1} + A^{-1}$

C. $A\ \text{adj}A = |A|I$

$$\text{adj}(A) = |A|\frac{I}{A}$$

$= |A|\, A^{-1}$

D. $\det(A^{-1}) = (\det A)^{-1}$

Question ID: 481283

If $\begin{vmatrix} -1 & a & a^2 \\ -1 & b & b^2 \\ -1 & c & c^2 \end{vmatrix}^2 = \lambda$ and $a - b = 1$, $b - c = 2$ and $c - a = 3$, then the value of λ is

(A) 6 (B) 36

(C) 72 (D) 144

Answer (B)

Sol. $$\begin{vmatrix} -1 & a & a^2 \\ -1 & b & b^2 \\ -1 & c & c^2 \end{vmatrix}^2 = (-1)^2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}^2 = \lambda$$

$= (a - b)^2 (b - c)^2 (c - a)^2 = \lambda$

$= (1)^2 (2)^2 (3)^2 = \lambda$

$= 36$

Question ID: 481284

If $\begin{bmatrix} 2x-1 & -3 & 6 \\ 3 & 3y-2 & 4 \\ -6 & -4 & 4z-3 \end{bmatrix}$ is skew symmetric matrix, then xyz is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Answer (C)

Sol. $$\begin{bmatrix} 2x-1 & -3 & 6 \\ 3 & 3y-2 & 4 \\ -6 & -4 & 4z-3 \end{bmatrix} = A$$

If A is skew symmetric

$\Rightarrow a_{ij} = 0$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$3y - 2 = 0 \Rightarrow y = \frac{2}{3}$

$4z - 3 = 0 \Rightarrow z = \frac{3}{4}$

Now, $xyz \equiv \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{4}$

Question ID: 481285

The solution of differential equation $\sqrt{x+1} - \sqrt{x-1}\frac{dy}{dx} = 0$ is

(A) $y = \sqrt{x^2-1} + \log\left|x + \sqrt{x^2-1}\right| + C$

(B) $y = \sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right| + C$

(C) $y = \sqrt{x^2-1} + \log\left|x + \sqrt{1-x^2}\right| + C$

(D) $y = \sqrt{1-x^2} + \log\left|x + \sqrt{1-x^2}\right| + C$

Answer (A)

Sol. $\sqrt{x+1}-\sqrt{x-1}\dfrac{dy}{dx}=0$

$$\int\frac{\sqrt{x+1}}{\sqrt{x-1}}dx=\int dy$$

$$\int\frac{x+1}{\sqrt{x^2-1}}dx=y$$

$$\int\frac{x}{\sqrt{x^2-1}}dx+\int\frac{dx}{\sqrt{x^2-1}}=y$$

$$y=\sqrt{x^2-1}+\log\left|x+\sqrt{x^2-1}\right|+C$$

Question ID: 481286

The point(s) on the curve $\dfrac{x^2}{9}+\dfrac{y^2}{64}=1$, at which the tangents are parallel to x-axis are

(A) $(0, \pm 3)$ (B) $(\pm 8, 0)$

(C) $(0, \pm 8)$ (D) $(\pm 3, 0)$

Answer (C)

Sol. $\dfrac{x^2}{9}+\dfrac{y^2}{64}=1$

$$\frac{2x}{9}+\frac{2yy'}{64}=0$$

$$y'=-\frac{64}{9}\frac{x}{y}=0$$

$\Rightarrow$ $\boxed{x=0}$

Now, $\dfrac{0}{9}+\dfrac{y^2}{64}=1$

$\boxed{y=\pm 8}$

Question ID: 481287

A die is tossed four times. The probability of getting an odd number at least once, is

(A) $\dfrac{1}{16}$ (B) $\dfrac{10}{16}$

(C) $\dfrac{4}{16}$ (D) $\dfrac{15}{16}$

Answer (D)

Sol. P(getting odd at least one) = 1 – P(even)

$$=1-\frac{3^4}{6^4}$$

$$=1-\frac{1}{2^4}$$

$$=\frac{15}{16}$$

Question ID: 481288

A manufacturer of electronic circuit has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is ₹50 and that on type B circuit is ₹60, identify the constraints for this LPP, if it was assumed that x circuit B of type A and y circuits of type B was produced by the manufacturer.

A. $x + 2y \geq 15$

B. $2x + y \leq 20$

C. $x + 2y \leq 12$

D. $x, y \leq 0$

Choose the correct answer from the options given below

(A) A & B only (B) B & C only

(C) C & D only (D) A & D only

Answer (B)

Sol. x circuits of type-A

y circuits of type-B

$E = 50x + 60y$

Now A need 20 R, 10 T, 10 C

B need 10 R, 20 T, 30 C

Stock 200 R, 120 T, 150 C

$20x + 10y \leq 200 \Rightarrow 2x + y \leq 20$

$10x + 20y \leq 120$

$\Rightarrow x + 2y \leq 12$

$10x + 30y \leq 150 \Rightarrow x + 3y \leq 15$

& $x \geq 0$, $y \geq 0$

Question ID: 481289

An energy DRONE is flying along the curve $y = x^2 + 7$. A soldier is placed at (3, 7). The nearest distance of the DRONE from soldier's position is

(A) 2

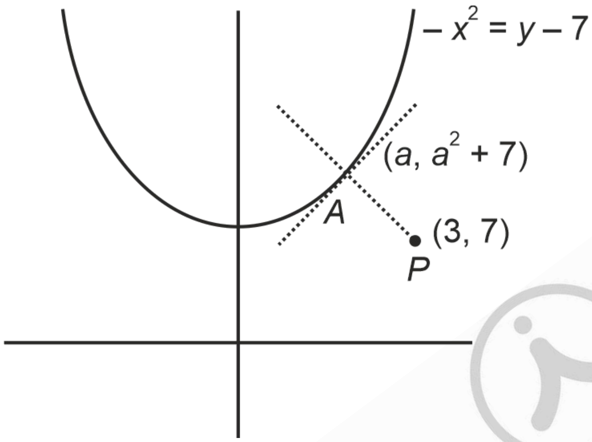
(B) 3

(C) $\sqrt{5}$

(D) $\sqrt{7}$

Answer (C)

Sol.



Nearest distance will lie along comes nearest

Slope at $A = 2x = y'$

$2a = y'$

Slope of normal $= \frac{-1}{2a}$

$m_{PA} = \frac{a^2 + 7 - 7}{a - 3}$

$\therefore \quad \frac{a^2}{a-3} = \frac{-1}{2a}$

$2a^3 + a - 3 = 0$

$\Rightarrow \quad a = 1$

Question ID: 4812810

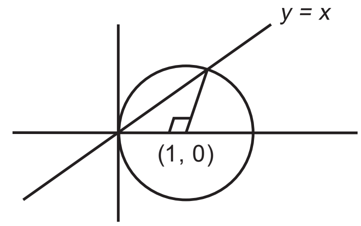
The line $y = x$, partition the area of the circle $(x - 1)^2 + y^2 = 1$, into two segments. The area of the major segment is

(A) $\frac{\pi}{2} + \frac{1}{4}$ (B) $\frac{\pi}{4} + \frac{1}{2}$

(C) $\frac{3\pi}{4} + \frac{1}{6}$ (D) $\frac{3\pi}{4} + \frac{1}{2}$

Answer (D)

Sol.



$$\text{Area} = \pi(1)^2 - \left[\frac{90}{360}\pi(1)^2 - \frac{1}{2}\times 1\times 1\right]$$

$$= \frac{3\pi}{4} + \frac{1}{2}$$

Question ID: 4812811

The maximum value of x^{-x} is

(A) e^{e-1} (B) $e^{\frac{1}{e}}$

(C) $e^{\frac{-1}{e}}$ (D) $e^{\frac{-4}{e}}$

Answer (B)

Sol. $f(x) = x^{-x}$

$\log f(x) = -x\log x$

$\frac{f'(x)}{f(x)} = -[1 + \log x]$

$f'(x) = -x^{-x}[1 + \log x] = 0$

$x = \frac{1}{e}$

$\therefore \quad f(x) = \left(\frac{1}{e}\right)^{-\frac{1}{e}}$

$f(x)_{\max} = e^{\frac{1}{e}}$

Question ID: 4812812

If $\int (x + \sqrt{x^2 - 1})^2 dx = \alpha \cdot x + \beta x^3 + \gamma(x^2 - 1)^{\frac{3}{2}} + C$, where C is arbitrary constant, then the value of $3(\alpha + \beta + \gamma)$ is

(A) 44

(B) 23

(C) 11

(D) 1

Answer (D)

Sol. $\int (x+\sqrt{x^2-1})^2 dx = \alpha \cdot x + \beta x^3 + \gamma (x^2-1)^{\frac{3}{2}} + C$

$$= \int \left(x^2 + x^2 - 1 + 2x\sqrt{x^2-1} \right) dx$$

$$= \int 2x^2 dx - \int dx + 2\int x\sqrt{x^2-1}\, dx$$

$$= \frac{2x^3}{3} - x + 2I$$

Now,

$$I = \int x\sqrt{x^2-1}\, dx$$

Let $x^2 - 1 = t^2$

$2x\, dx = 2t\, dt$

$$= \int t \cdot t\, dt = \int t^2 dt = \frac{t^3}{3} + C'$$

$$= \frac{\left(\sqrt{(x^2-1)}\right)^3}{3}$$

Question ID: 4812813

The probability distribution of a random variable X is

x	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

The variance of X is

(A) $\frac{31}{64}$ (B) $\frac{15}{64}$

(C) $\frac{103}{64}$ (D) 1

Answer (C)

Sol. $\sum P_i x_i^2 - (\text{Mean})^2$

$$\left(0 \times \frac{1}{4} + 1^2 \times \left(\frac{1}{8}\right) + 2^2 \times \frac{1}{8} + 3^2 \left(\frac{1}{2}\right)\right) - \left(0 \times \frac{1}{4} + 1 \times \frac{1}{8} + 2 \times \frac{1}{8} + 3 \times \frac{1}{2}\right)^2$$

$$= \frac{41}{8} - \left(\frac{15}{8}\right)^2$$

$$= \frac{41}{8} - \frac{225}{64} = \frac{103}{64}$$

Question ID: 4812814

$$\int_0^1 \frac{dx}{x^2 + x + 1}$$

(A) $\frac{\pi}{3\sqrt{3}}$ (B) $\frac{\pi}{2\sqrt{3}}$

(C) $\frac{\pi}{6\sqrt{3}}$ (D) $\frac{\pi}{\sqrt{3}}$

Answer (A)

Sol. $\int_0^1 \frac{dx}{x^2 + x + 1} =$

$$= \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{4}}$$

$$= \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left[\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$$

$$= \frac{2}{\sqrt{3}}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{2}{\sqrt{3}} \times \frac{\pi}{6} = \frac{\pi}{3\sqrt{3}}$$

Question ID: 4812815

If the order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \left(1 + \frac{dy}{dx}\right)^{\frac{1}{3}}$ are a and b respectively, then the value of $a^2 + b^2$ is

(A) 2 (B) 3

(C) 5 (D) 13

Answer (D)

Sol. $\sqrt{\frac{d^2y}{dx^2}} = \left(1 + \frac{dy}{dx}\right)^{\frac{1}{3}}$

Taking power 6 on both sides

$$\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \frac{dy}{dx}\right)^2$$

$\therefore$ Order = 2 = a

Degree = 3 = b

$\therefore$ $a^2 + b^2 = 4 + 9$

$= 13$

Question ID: 4812851

The integrating factor of differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is

(A) $\frac{1}{x}$ (B) x

(C) $\frac{1}{x^2}$ (D) x^2

Answer (D)

Sol. $x\frac{dy}{dx} + 2y = x^2 \log x$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x \log x$

IF = $e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Question ID: 4812852

General solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$ is

A. $\tan^{-1}x + \tan^{-1}y = C$

B. $\sin^{-1}x - \cos^{-1}y = C$

C. $x\sqrt{1-y^2} - y\sqrt{1-x^2} = C$

D. $\sin^{-1}x + \sin^{-1}y = C$

E. $\cos^{-1}x + \cos^{-1}y = C$

(where C is arbitrary constant)

Choose the correct answer from the options given below

(A) B and C only (B) B, D and E only

(C) A and D only (D) A and B only

Answer (B)

Sol. $\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$

Integrating both sides

$\sin^{-1}y = -\sin^{-1}x + C$

$\sin^{-1}x + \sin^{-1}y = C$

OR $\sin^{-1}x - \cos^{-1}y = C$

OR $\cos^{-1}x + \cos^{-1}y = C$

Question ID: 4812853

The absolute maximum value of $y = x^3 - 3x + 2$, $0 \le x \le 2$, is

(A) 4 (B) 6

(C) 2 (D) 0

Answer (A)

Sol. $\frac{dy}{dx} = 3x^2 - 3 = 0$ at $x = 1$

$y = f(x) = x^3 - 3x + 2$

$f(0) = 2$

$f(2) = 4$

$f(1) = 0$

Question ID: 4812854

Match List I with List II

List - I	List - II
A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$	I. $\frac{\pi}{2}$
B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$	II. $\frac{\pi}{4}$
C. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx$	III. 0
D. $\int_0^{\pi} \lvert\cos x\rvert dx$	IV. 2

(A) A-I, B-III, C-II, D-IV

(B) A-III, B-I, C-II, D-IV

(C) A-II, B-III, C-IV, D-I

(D) A-IV, B-III, C-I, D-II

Answer (B)

Sol. $I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x)^7 dx = 0$ as $(\sin x)^7$ is odd function

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx = 2\int_0^{\frac{\pi}{2}} \sin^2 x\, dx = x - \frac{\sin 2x}{2}\Bigg|_0^{\frac{\pi}{2}}$$

$$I_3 = \int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos y} dx \quad \ldots(1)$$

$$I_3 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \quad \ldots(2)$$

Adding (1) and (2)

$$2I_3 = \int_0^{\frac{\pi}{2}} 1\,dx \Rightarrow I_3 = \frac{\pi}{4}$$

$$I_4 = \int_0^{\pi} |\cos x|\,dx = 2\int_0^{\frac{\pi}{2}} \cos x\,dx = 2\sin x\Big|_0^{\frac{\pi}{2}} = 2$$

Question ID: 4812855

The portion of the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, that lies in the first quadrant is

(A) πab (B) $\frac{\pi ab}{2}$

(C) $\frac{\pi ab}{4}$ (D) $2\pi ab$

Answer (C)

Sol. Area of ellipse = πab

In 1st quadrant only area $= \frac{\pi ab}{4}$

Question ID: 4812856

$$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx =$$

(A) $\frac{e^x}{1+x^2}+C$ (B) $\frac{-e^x}{1+x^2}+C$

(C) $\frac{e^x}{\left(1+x^2\right)^2}+C$ (D) $\frac{-e^x}{\left(1+x^2\right)^2}+C$

Answer (A)

Sol. $\int e^x \left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right)dx = \int e^x\left(\frac{1}{1+x^2}-\frac{2x}{(1+x)^2}\right)dx$

If $f(x)=\frac{1}{1+x^2}$ then $f'(x)=\frac{-2x}{\left(1+x^2\right)^2}dx$

$$\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+C$$

$$=\frac{e^x}{1+x^2}+C$$

Question ID: 4812857

$$\int \frac{\sin\left(\tan^{-1}x\right)}{1+x^2}dx =$$

(A) $-\cos\left(\tan^{-1}x\right)+C$ (B) $\cos\left(\tan^{-1}x\right)+C$

(C) $\tan\left(\cos^{-1}x\right)+C$ (D) $-\tan\left(\cos^{-1}x\right)+C$

Answer (A)

Sol. Let $\tan^{-1}x = t$

$$\Rightarrow \frac{1}{1+x^2}dx = dt$$

$$I = \int \sin t\; dt = -\cos t + C$$

$$= -\cos\left(\tan^{-1}x\right)+C$$

Question ID: 4812858

The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=2$ from the origin is

(A) 1 unit (B) 2 units

(C) 7 units (D) 8 units

Answer (B)

Sol. Equation of plane can be given as

$2x + 3y - 6z = 14$

Distance from (0, 0, 0)

$$=\left|\frac{14}{\sqrt{2^2+3^2+(-6)^2}}\right| = 2\text{ units}$$

Question ID: 4812859

If $|\vec{a}|=8$, $|\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$, then the value $\vec{a}\cdot\vec{b}$ is

(A) $2\sqrt{3}$ (B) $12\sqrt{3}$

(C) $8\sqrt{3}$ (D) $6\sqrt{3}$

Answer (B)

Sol. $|\vec{a}|^2|\vec{b}|^2 = |\vec{a}\times\vec{b}|^2 + \left(\vec{a}\cdot\vec{b}\right)^2$

$576 = 144 + \left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow \vec{a}\cdot\vec{b} = \sqrt{432}$

$= \pm 12\sqrt{3}$

Question ID: 4812860

The direction ratios of the line $\frac{1-x}{3}=\frac{7y-14}{2}=\frac{z-3}{2}$ are

(A) $-3, \frac{1}{7}, 2$

(B) $-3, 2, 2$

(C) $-3, \frac{2}{7}, 1$

(D) $-21, 2, 14$

Answer (D)

Sol. $\frac{x-1}{-3}=\frac{y-2}{2/7}=\frac{z-3}{2}$

So direction ratios are $\left(-3, \frac{2}{7}, 2\right)$

OR $(-21, 2, 14)$

Question ID: 4812861

The vertices of a closed convex polygon representing the feasible region of the LPP with, objective function $z = 5x + 3y$ are (0, 0), (3, 1), (1, 3) and (0, 2). The maximum value of z is

(A) 6

(B) 18

(C) 14

(D) 15

Answer (B)

Sol. $z_{(0,0)} = 0$

$z_{(3,1)} = 5.3 + 3.1 = 18$

$Z_{(1,3)} = 5.1 + 3.3 = 14$

$Z_{(0,2)} = 5.0 + 3.2 = 6$

So z_{max} is 18 at (3, 1)

Question ID: 4812862

$$\int \frac{x^2+1}{x^4+1}dx =$$

(A) $\log(x^4 + 1) + C$

(B) $\log\left(\frac{x^2-1}{x^2+1}\right)+C$

(C) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right)+C$

(D) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}\,x}\right)+C$

Answer (C)

Sol. $I=\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx=\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$

Let $x-\frac{1}{x}=t$ then $\left(1+\frac{1}{x^2}\right)dx=dt$

$I=\int \frac{dt}{t^2+2}=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+C$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+C$

Question ID: 4812863

$$\int_0^{4\pi}\frac{x}{1+|\cos x|}dx =$$

(A) 4π

(B) 16π

(C) 32π

(D) 64π

Answer (B)

Sol. $I=\int_0^{4\pi}\frac{x}{1+11\cos x11}dx$ ...(i)

Also $I=\int_0^{4\pi}\frac{4\pi-x}{1+11\cos(4\pi-x)11}dx$ ...(ii)

(i) + (ii), $2I=\int_0^{4\pi}\frac{4\pi}{1+11\cos x\,11}dx$

$\Rightarrow\quad I=\left(\int_0^{4\pi}\frac{1}{1+11\cos x11}dx\right)2\pi$

$=8\pi\int_0^{\pi}\frac{1}{1+11\cos x\,11}dx$

$=8\pi\left(\int_0^{\pi/2}\frac{dx}{1+\cos x}+\int_{\pi/2}^{\pi}\frac{dx}{1-\cos x}\right)$

$=8\pi\left(\tan\left(\frac{x}{2}\right)\Big|_0^{\pi/2}+\left(-\cot\frac{x}{2}\right)\Big|_{\pi/2}^{\pi}\right)$

$= 8\pi \times 2 = 16\pi$

Question ID: 4812864

A man is known to speak truth 4 out of 5 times. He throws a die and reports that five appears. Then the probability that actual five appears on the dice is

(A) $\frac{3}{8}$ (B) $\frac{4}{9}$

(C) $\frac{5}{9}$ (D) $\frac{5}{8}$

Answer (B)

Sol. Given $P(T)=\frac{4}{5}$

So, required probability

$$=\frac{P(T)\cdot\frac{1}{6}}{P(T)\cdot\frac{1}{6}+P(F)\frac{5}{6}}$$

$$=\frac{\frac{4}{5}\cdot\frac{1}{6}}{\frac{4}{5}\cdot\frac{1}{6}+\frac{1}{5}\cdot\frac{5}{6}}=\frac{4}{9}$$

Question ID: 4812865

Five numbers taken out from numbers 1-30 and arrange them in ascending order. The probability that the third number will be 20 is

(A) $\frac{{}^{20}C_2\times{}^{10}C_2}{{}^{30}C_5}$ (B) $\frac{{}^{19}C_2\times{}^{10}C_2}{{}^{30}C_5}$

(C) $\frac{{}^{19}C_2\times{}^{11}C_3}{{}^{30}C_5}$ (D) $\frac{{}^{19}C_2\times{}^{11}C_2}{{}^{30}C_5}$

Answer (B)

Sol. $S=\{1, 2, 3, \ldots\ldots, 30\}$

Required probability $=\frac{{}^{19}C_2\times{}^{10}C_2}{{}^{30}C_5\times 1}$

Question ID: 4812866

Bag I contains 4 red and 5 black balls, while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be black. Then the probability that it was drawn from Bag II, is

(A) $\frac{54}{109}$ (B) $\frac{51}{109}$

(C) $\frac{64}{109}$ (D) $\frac{54}{119}$

Answer (A)

Sol. $P\left(\frac{R}{B_1}\right)=\frac{4}{9}, P\left(\frac{B}{B_1}\right)=\frac{5}{9}$

$P\left(\frac{R}{B_2}\right)=\frac{5}{11}, P\left(\frac{B}{B_2}\right)=\frac{6}{11}$

$$P\left(\frac{B}{B_2}\right)=\frac{P(B_2)\cdot P\left(\frac{B}{B_2}\right)}{P(B_1)\cdot\left(\frac{B}{B_1}\right)+P(B_2)\cdot P\left(\frac{B}{B_2}\right)}$$

$$=\frac{\frac{1}{2}\cdot\frac{6}{11}}{\frac{1}{2}\cdot\frac{5}{9}+\frac{1}{2}\cdot\frac{6}{11}}$$

$$=\frac{\frac{6}{11}}{\frac{109}{99}}=\frac{54}{109}$$

Question ID: 4812867

A card is picked at random from a pack of 52 playing cards. If the picked card is a queen, then probability of card to be of spade type also, is

(A) $\frac{1}{3}$ (B) $\frac{4}{13}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Answer (C)

Sol. Let Q : card is Queen

S : card is Spade

then $P\left(\frac{S}{Q}\right)=\frac{P(S\cap Q)}{P(Q)}=\frac{\frac{1}{52}}{\frac{4}{52}}=\frac{1}{4}$

Question ID: 4812868

The differential equation representing family of curves $y = ae^{mx} + be^{nx}$, where a and b are arbitrary constants, is

(A) $\frac{d^2y}{dx^2}+(m+n)\frac{dy}{dx}+y=0$

(B) $\frac{d^2y}{dx^2}+mn\frac{dy}{dx}+(m+n)y=0$

(C) $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$

(D) $\frac{d^2y}{dx^2}+(m+n)\frac{dy}{dx}-mny=0$

Answer (C)

Sol. $y = ae^{mx} + be^{nx}$

$$\Rightarrow \quad \frac{dy}{dx} = ame^{mx} + nbe^{nx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = am^2e^{mx} + n^2be^{nx}$$

$$\therefore \quad \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = am^2e^{mx} + bn^2e^{nx}$$

$$-(m+n)\left[ame^{mx} + nbe^{nx}\right] + mn\left[ae^{mx} + be^{nx}\right]$$

$$= 0$$

Question ID: 4812869

The interval in which the function, $f(x) = 7 - 4x - x^2$ is strictly increasing is

(A) $(-\infty, \infty)$ (B) $(-2, \infty)$

(C) $(-\infty, -2)$ (D) $(-\infty, -2]$

Answer (C)

Sol. $f(x) = 7 - 4x - x^2$

then $f'(x) = -4 - 2x$

for strictly increasing

$f'(x) > 0$

$\Rightarrow \quad -(4 + 2x) > 0$

$\Rightarrow \quad x < -2$

$\Rightarrow \quad x \in (-\infty, -2)$

Question ID: 4812870

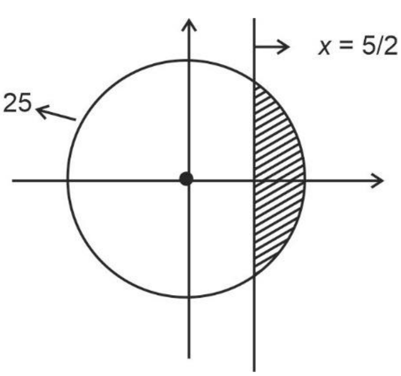
The area (in square units) of minor segment of the circle $x^2 + y^2 = 25$ cut off by the line $x = \frac{5}{2}$ is

(A) $25\left(\frac{\pi}{4} - \frac{\sqrt{3}}{2}\right)$ (B) $\frac{25}{12}\left(4\pi - 3\sqrt{3}\right)$

(C) $\frac{25}{12}\left(3\pi - 4\sqrt{3}\right)$ (D) $25\left(\frac{\sqrt{3}}{2} + \frac{\pi}{4}\right)$

Answer (B)

Sol.



Required area

$$= 2\int_{\frac{5}{2}}^{5} \sqrt{25 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{\frac{5}{2}}^{5}$$

$$= \left\{5 \cdot 0 + 25 \cdot \frac{\pi}{2}\right\} - \left\{\frac{5}{2}\sqrt{\frac{75}{4}} + 25 \cdot \frac{\pi}{6}\right\}$$

$$= \frac{25\pi}{2} - \frac{25}{4}\sqrt{3} - \frac{25\pi}{6} = \frac{25\pi}{3} - \frac{25}{4}\sqrt{3}$$

Question ID: 4812871

If $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\,dx = \frac{\lambda}{2}$, then the value of λ is

(A) π (B) -1

(C) $\frac{\pi}{2}$ (D) $\frac{3}{\sqrt{2}}$

Answer (A)

Sol. Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}\,dx$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\cot x}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\,dx$$

Let sinx – cotx – t

$\Rightarrow \quad (\cot x + \sin x)dx = dt$

$$\therefore \quad 2I = \int_{-1}^{1} \frac{\sqrt{2}dt}{\sqrt{1 - t^2}}$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{2}} \cdot 2\int_0^1 \frac{1}{\sqrt{1 - t^2}}\,dt$$

$$\Rightarrow \quad = \sqrt{2}\left(\sin^{-1} t\right)_0^1 = \frac{\pi}{\sqrt{2}}$$

Question ID: 4812872

A. A relation R on a set A is called an equivalence relation, if it is reflexive, symmetric and transitive.

B. The function $f: R \to R$ defined by $f(x) = e^x$ is not one-one.

C. The one-one function is also known as injective function.

D. The onto function is also known as subjective function.

E. A function $f: X \to Y$ is said to be many-one, if two or more than two elements in set X have the different image in set Y.

Choose the correct answer from the option given below:

(A) A, C, D only (B) B, C, D only

(C) C, D, E only (D) B, D, E only

Answer (A)

Sol. (A) Definition

(B) $f(x) = e^x$ is a one-one function

$(\because f'(x) = e^x > 0 \quad \forall x \in R$

(C) and (D) are also correct

Question ID: 4812873

If $|\vec{a}| = 3|\vec{b}|, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then $|\vec{a} - \vec{b}|$ is equal to :

(A) 14 (B) $2\sqrt{7}$

(C) 28 (D) 25

Answer (B)

Sol. Given

$|\vec{a}| = 3|\vec{b}|, |\vec{b}| = 2$

and let θ be the angle between $\vec{a}$ and $\vec{b}$, then θ = 60°

Now, $|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}}$

$= \sqrt{36 + 4 - 2 \cdot 6 \cdot 2 \cdot \frac{1}{2}}$

$= \sqrt{28} = 2\sqrt{7}$

Question ID: 4812874

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and square matrix B satisfy AB = $8I$, then the value of $|B|$ is:

(A) 512

(B) 64

(C) 32

(D) 8

Answer (B)

Sol. $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$|A| = 1(-4) + 1(7) + 1(5)$

$= 8$

and $AB = 8I$

$\Rightarrow |AB| = |8I|$

$\Rightarrow |B| = \frac{8^3}{8} = 64$

Question ID: 4812875

The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is :

(A) $\frac{5\pi}{6}$

(B) $\frac{-\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{3}$

Answer (C)

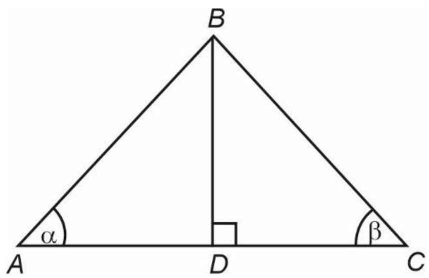
Sol. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}$

and let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{2}\sqrt{2}} = \frac{-1}{2}$

$\therefore \quad \theta = \frac{2\pi}{3}$

Passage : Two men on either side of a pole of 30 m high observe its top at angles of elevation α and β respectively. The distance between the two men is $40\sqrt{3}$ m and the distance between the first man at *A* and the pole is $30\sqrt{3}$ m.



Based on the above information, answer the question:

Question ID: 4812876

The value of α is

(A) $\frac{\pi}{3}$

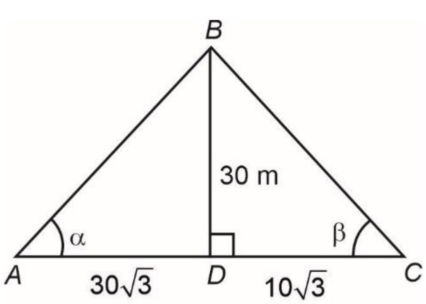
(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{8}$

Answer (C)

Sol.



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \alpha = 30° \text{ or } \frac{\pi}{6}$

Question ID: 4812877

$\tan(\alpha + \beta) =$

(A) $\sqrt{3} + \frac{1}{\sqrt{3}}$

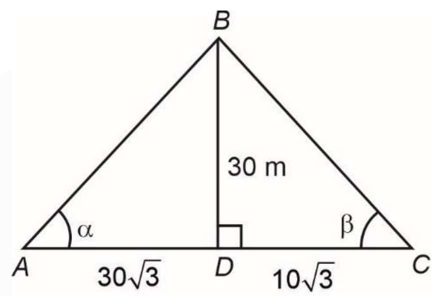
(B) $\sqrt{3} - \frac{1}{\sqrt{3}}$

(C) $2\sqrt{3} - 1$

(D) Infinite (does not exist)

Answer (D)

Sol.



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \alpha = 30° \text{ or } \frac{\pi}{6}$

$\tan\beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \quad \beta = \frac{\pi}{3}$

So, $\tan(\alpha + \beta) = \tan\left(\frac{\pi}{2}\right) \to \infty$(does not exist)

Question ID: 4812878

Area of ΔABD is

(A) $150\sqrt{3}\ \text{m}^2$

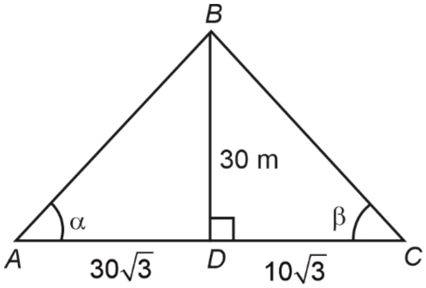
(B) $900\sqrt{3}\ \text{m}^2$

(C) $450\sqrt{3}\ \text{m}^2$

(D) $100\sqrt{3}\ \text{m}^2$

Answer (C)

Sol.



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \alpha = 30° \text{ or } \frac{\pi}{6}$

$\tan\beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \quad \beta = \frac{\pi}{3}$

So, $\tan(\alpha+\beta) = \tan\left(\frac{\pi}{2}\right) \to \infty$(does not exist)

Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3}\ \text{m}^2$

Question ID: 4812879

$\angle ABC$ is equal to:

(A) $\frac{\pi}{4}$

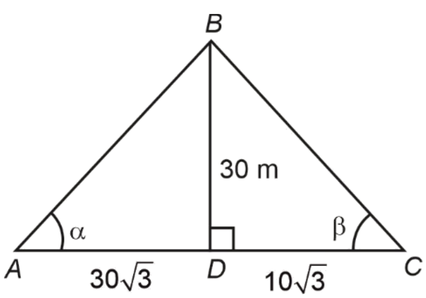
(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$

Answer (C)

Sol.



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \alpha = 30° \text{ or } \frac{\pi}{6}$

$\tan\beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \quad \beta = \frac{\pi}{3}$

So, $\tan(\alpha+\beta) = \tan\left(\frac{\pi}{2}\right) \to \infty$(does not exist)

Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3}\ \text{m}^2$

$\angle ABC = \frac{\pi}{2}$

Question ID: 4812880

The value of $\frac{1}{AB^2} + \frac{1}{BC^2}$ is:

(A) $\frac{1}{BD^2}$

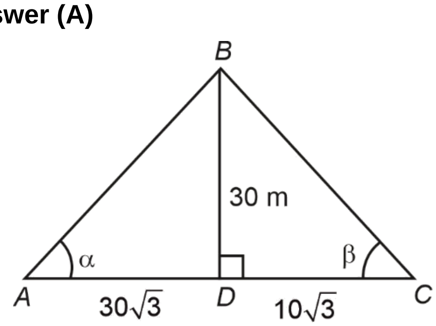
(B) $\frac{1}{AD^2}$

(C) $\frac{1}{CD^2}$

(D) $\frac{1}{AC^2}$

Answer (A)

Sol.



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \alpha = 30° \text{ or } \frac{\pi}{6}$

$\tan\beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \quad \beta = \frac{\pi}{3}$

So, $\tan(\alpha+\beta) = \tan\left(\frac{\pi}{2}\right) \to \infty$(does not exist)

Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3}\ m^2$

$\angle ABC = \frac{\pi}{2} - \alpha$

$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{30^2(4)} + \frac{1}{1200} = \frac{1}{3600} + \frac{1}{1200}$

$= \frac{4}{3600} = \frac{1}{900}$

$= \frac{1}{BD^2}$

Passage:

Manjeet wants to donate a rectangular plot of land for a school in his village. When he has asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. If length and breadth of the plot are x and y respectively, then based on above information answer the question.

Question ID: 4812881

The length (x) and breadth (y) of plot satisfy equations:

(A) $x - y = 50$, $2x - y = 550$

(B) $x - y = 50$, $2x + y = 550$

(C) $x + y = 50$, $2x + y = 550$

(D) $x + y = 50$, $2x + y = 550$

Answer (B)

Sol. $(x - 50)(y + 50) = xy$

$\Rightarrow \quad x - y = 50 \qquad \ldots(i)$

$(x - 10)(y - 20) = xy - 5300$

$\Rightarrow \quad 2x + y = 550 \qquad \ldots(ii)$

From these equations

$y = 200$ m

$y = 150$ m

Question ID: 4812882

The linear equation involving x and y are written in matrix form as:

(A) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

Answer (A)

Sol. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ -550 \end{bmatrix}$

Question ID: 4812883

The length of the plot is:

(A) 150 m

(B) 400 m

(C) 200 m

(D) 320 m

Answer (C)

Sol. l = length of plot = 200 m

Question ID: 4812884

The breadth of plot is:

(A) 150 m

(B) 200 m

(C) 430 m

(D) 350 m

Answer (A)

Sol. Breadth of the plot = b = 150 m

Question ID: 4812885

Area of the rectangular plot is:

(A) 60,000 sq. m

(B) 3,000 sq. m

(C) 25,000 sq. m

(D) 30,000 sq. m

Answer (D)

Sol. Area of plot = 200 × 150 = 30,000 m^2