30/08/2022

Slot-1

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Answers & Solutions

Time : 45 min. *for* M.M. : 200

CUET UG-2022

(Mathematics)

IMPORTANT INSTRUCTIONS:

1. The test is of 45 Minutes duration.
2. The test contains is divided into two sections.
   a. Section A contains 15 questions which will be compulsory for all candidates.
   b. Section B will have 35 questions out of which 25 questions need to be attempted.
3. Marking Scheme of the test:
   a. Correct answer or the most appropriate answer: Five marks (+5)
   b. Any incorrect option marked will be given minus one mark (-1).

Choose the correct answer :

Question ID: 9320101

Assume P, Q, R and S are matrices of order $2 \times m$, $k \times n$, $m \times 2$ and 2×3 respectively. The restrictions on k, m and n, so that $PQ + RS$ is defined are

(1) $m = 3, n = 2$ (2) $m = n$, k is arbitrary

(3) $m = k$, n is arbitrary (4) $m = k = 2, n = 3$

Answer (4)

Sol. Order of $P = 2 \times m$

Order of $Q = k \times n$

Order of $R = m \times 2$

Order of $S = 2 \times 3$

Now for $PQ + RS$ to be defined.

PQ and RS is to be defined and PQ and RS should be of same order.

For PQ to be defined $m = k$

$\Rightarrow$ Order of $PQ = 2 \times n$

For RS to be defined $2 = 2$

$\Rightarrow$ Order of $RS = m \times 3$

If order of PQ = Order of RS

$\Rightarrow$ $2 \times n \equiv m \times 3$

$\Rightarrow$ $m = 2 \quad n = 3 \quad k = 2$

Question ID: 9320102

The system of equations $3x + 4y = 5$, $6x + 7y = -8$ is written in matrix from as

(1) $\begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 5 & -8 \end{bmatrix}$

(2) $\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

(3) $\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

(4) $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -8 \end{bmatrix}$

Answer (4)

Sol. $3x + 4y = 5$

$6x + 7y = -8$

In matrix form

$$\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -8 \end{bmatrix}$$

Question ID: 9320103

If $2\begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, then the value of $|a + b - c - d|$ is

(1) 3 (2) 24

(3) 6 (4) 16

Answer (3)

Sol. $2\begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 2a+3 & 2d-3 \\ 2b & 2c+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\Rightarrow$ $2a + 3 = 9 \quad \Rightarrow \quad a = 3$

$2d - 3 = 15 \quad \Rightarrow \quad d = 9$

$2b = 12 \quad \Rightarrow \quad b = 6$

$2c + 6 = 18 \quad \Rightarrow \quad c = 6$

$|3 + 6 - 9 - 6|$

$= 6$

Question ID: 9320104

Consider the function $f(x) = x^{\frac{1}{x}}$. Its

(1) minimum value is $e^{\frac{1}{e}}$

(2) maximum value is $e^{\frac{1}{e}}$

(3) minimum value is e^e

(4) maximum value is $\left(\frac{1}{e}\right)^e$

Answer (2)

Sol. $f(x) = x^{\frac{1}{x}}$

$\log f(x) = \frac{1}{x}\log x$

$\frac{f'(x)}{f(x)} = \frac{1}{x^2} + (\log x)\left(-\frac{1}{x^2}\right)$

$f'(x) = \frac{x^{\frac{1}{x}}[1 - \log x]}{x^2}$

$f'(x) = 0$

$\Rightarrow$ $1 - \log x = 0$

$\therefore$ $x = e$

$\therefore$ $f(x)_{\max} = e^{\frac{1}{e}}$

Question ID: 9320105

The given function $f(x) = x^5 - 5x^4 + 5x^3 - 1$; has/have

(a) local maxima at $x = 1$

(b) local maximum value is 0

(c) local minimum at $x = 3$

(d) local minimum value is –28

(e) The point of inflexion is $x = 1$

Choose the **correct** answer from the options given below

(1) (a), (b) only (2) (a), (b), (c) only

(3) (a), (b), (c), (d) only (4) (a), (c), (e) only

Answer (3)

Sol. $f(x) = x^5 - 5x^4 + 5x^3 - 1$

$f'(x) = 5x^4 - 20x^3 + 15x^2$

$= 5x^2(x^2 - 4x + 3) = 0$

$= 5x^2(x - 3)(x - 1) = 0$

$x = 0, 3, 1$

Now,

$f''(x) = 20x^3 - 60x^2 + 30x$

$= 10x(2x^2 - 6x + 3)$

$f''(x) = 0 \Rightarrow x = 0$ $\therefore$ $x = 0$ is point of inflexion.

$f''(1) < 0 \Rightarrow x = 1$ point of maxima.

$f''(3) > 0 \Rightarrow x = 3$ point of minima.

$f(3) = (3)^5 - 5(3)^4 + 5(3)^3 - 1$

$= -28$

$f(1) = 1 - 5 + 5 - 1 = 0$

Question ID: 9320106

Match **List-I** with **List-II**

List-I	**List-II**
(a) If $x = t^2$ and $y = t^3$ then $\dfrac{d^2y}{dx^2}$ at $t = 1$	(i) –2
(b) If $f(x) = \sqrt{x} + 1$, then $f''(1)$	(ii) –1
(c) The minimum value of $f(x) = 9x^2 + 12x + 2$ is	(iii) $\dfrac{3}{4}$
(d) The point of inflexion of the function $f(x) = (x - 2)^4 (x + 1)^3$ is	(iv) $-\dfrac{1}{4}$

Choose the **correct** answer from the options given below

(1) (a) - (i), (b) - (iii), (c) - (ii), (d) - (iv)

(2) (a) - (ii), (b) - (iii), (c) - (i), (d) - (iv)

(3) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(4) (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)

Answer (3)

Sol. (a) $x = t^2$, $y = t^3$

$$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 3t^2$$

$$\frac{dy}{dx} = \frac{3}{2}t$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} \times \frac{dt}{dx}$$

$$= \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

(b) $f(x) = \sqrt{x} + 1$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f''(x) = -\frac{1}{4x^{\frac{3}{2}}} \quad f''(1) = -\frac{1}{4}$$

(c) $f(x) = 9x^2 + 12x + 2$

$f'(x) = 18x + 12 = 0$

$$x = -\frac{2}{3}$$

$$f(x)_{\min} = 9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) + 2$$

$$= 9 \times \frac{4}{9} + (-8) + 2$$

$$= 4 - 8 + 2 = -2$$

(d) $f(x) = (x - 2)^4 (x + 1)^3$

$f'(x) = 3(x + 1)^2(x - 2)^4 + 4(x - 2)^3(x + 1)^3$

$f''(x) = 6(x + 1)(x - 2)^4 + 12(x - 2)^3(x + 1)^2 + 12(x + 1)^2(x - 2)^3 + 12(x - 2)^2(x + 1)^3$

$= (x + 1)[6(x - 2)^4 + 12(x - 2)^3(x + 1) + 12(x + 1)(x - 2)^3 + 12(x - 2)^2(x + 1)^2] = 0$

$\Rightarrow$ $x = -1$ is point of inflexion.

$\therefore$ a $\to$ (iii), b $\to$ (iv), c $\to$ (i), d $\to$ (ii)

Question ID: 9320107

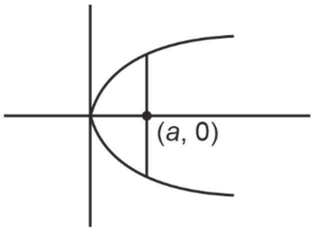
The area enclosed by the curve $y^2 = 4ax$ and its latus - rectum is

(1) $\frac{8}{3}a^2$ (2) $\frac{4}{3}a^2$

(3) $\frac{1}{3}a^2$ (4) $\frac{1}{12}a^2$

Answer (1)

Sol.



$$\text{Area } = 2\int_0^a \sqrt{4ax}\,dx$$

$$= 2.2\sqrt{a}\int_0^a \sqrt{x}\,dx$$

$$= 4\sqrt{a}\left.\frac{x^{\frac{3}{2}}}{3}\times 2\right]_0^a$$

$$= \frac{4a^{\frac{1}{2}}a^{\frac{3}{2}}}{3}\times 2 = \frac{8}{3}a^2$$

Question ID: 9320108

$$\int \frac{xe^x}{(x+1)^2}dx =$$

(1) $\frac{e^x}{x+1}+c$ (2) $\frac{e^x}{x-1}+c$

(3) $\frac{x}{x+1}+c$ (4) $\frac{x}{x-1}+c$

Answer (1)

Sol. $\int \frac{xe^x}{(x+1)^2}dx$

$$\int e^x\left[\frac{x+1-1}{(x+1)^2}\right]dx$$

$$= \int e^x\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx$$

$$\because \quad \int e^x\left[f(x)+f\left(\frac{1}{x}\right)\right]dx = e^x f(x)$$

$$= \frac{e^x}{x+1}+c$$

Question ID: 9320109

The solution of the differential equation $(x+1)\frac{dy}{dx} = 1+y$ is

(1) $\frac{1+y+y^2}{1+x^2} = C$

(2) $\log(x+1)-\log\left(y+\frac{1}{2}\right) = C$

(3) $\frac{x+1}{y+1} = C$

(4) $\log(1+y)-\frac{\sqrt{3}}{2}\log(x+1) = C$

Answer (3)

Sol. $(x+1)\frac{dy}{dx} = 1+y$

$$\frac{dy}{dx}-\frac{y}{x+1} = \frac{1}{x+1}$$

$$\text{IF } = e^{-\int\frac{dx}{x+1}} = e^{-\log|x+1|} = \frac{1}{x+1}$$

$$\therefore \quad \frac{y}{x+1} = \int \frac{1}{(x+1)^2}dx$$

$$\frac{y}{x+1} = -\frac{1}{x+1}+C$$

$$\frac{y+1}{x+1}+C$$

Question ID: 9320110

Order and degree of the differential equation $y\frac{dy}{dx}+\frac{4}{\frac{dy}{dx}} = 5$ are

(1) 1, 2 respectively (2) 1, 1 respectively

(3) 1, 0 respectively (4) 2, 1 respectively

Answer (1)

Sol. $y\frac{dy}{dx}+\frac{4}{\frac{dy}{dx}} = 5$

$$y\left(\frac{dy}{dx}\right)^2+4 = 5\left(\frac{dy}{dx}\right)$$

$\therefore$ Order = 1

Degree = 2

Question ID: 9320111

Derivative of $x^3 + 1$ with respect to $x^2 + 1$ is

(1) $\frac{2x}{3}$ (2) $\frac{x}{3}$

(3) $\frac{x}{2}$ (4) $\frac{3x}{2}$

Answer (4)

Sol. Derivative of $x^3 + 1$ w.r.t. $x^2 + 1$

$f(x) = x^3 + 1$

$g(x) = x^2 + 1$

$$\frac{f'(x)}{g'(x)} = \frac{3x^2}{2x} = \frac{3}{2}x$$

Question ID: 9320112

Solution of the differential equation $(x + xy)dy - y(1 - x^2)dx = 0$ is

(1) $y = \log\frac{x}{y} - \frac{x^2}{2} + C$ (2) $y = \log\frac{x}{y} + \frac{x^2}{2} + C$

(3) $y = \log xy - \frac{x^2}{2} + C$ (4) $y = \log xy + \frac{x^2}{2} + C$

Answer (1)

Sol. $(x + xy)dy - y(1 - x^2)dx = 0$

$x(1 + y)dy - y(1 - x^2)dx = 0$

$$\int \frac{(1+y)}{y}dy = \int \frac{(1-x^2)}{x}dx$$

$$\log y + y = \log x - \frac{x^2}{2} + C$$

$$y = \log\frac{x}{y} - \frac{x^2}{2} + C$$

Question ID: 9320113

Two numbers are selected at random (without replacement) from the first three positive integers. Let X denotes the larger of the two integers, then the probability distribution of X is

(1)

x	2	3
$P(X = x)$	1/3	2/3

(2)

x	2	3
$P(X = x)$	1/2	1/2

(3)

x	2	3
$P(X = x)$	2/3	1/3

(4)

x	2	3
$P(X = x)$	1/5	4/5

Answer (1)

Sol. S : {1, 2, 3}

Two numbers can be select as (1, 2) (2, 3) (1, 3)

Now

x	2	3
$P(X = x)$	1/3	2/3

where X denotes larger of two integers.

Question ID: 9320114

The probability distribution of number of doublets in three throws of a pair of dice is

(1)

x	0	1	2	3
$P(X = x)$	125/216	75/216	15/216	1/216

(2)

x	0	1	2	3
$P(X = x)$	75/216	125/216	1/216	15/216

(3)

x	0	1	2	3
$P(X = x)$	1/216	75/216	15/216	125/216

(4)

x	0	1	2	3
$P(X = x)$	1/216	15/216	75/216	125/216

Answer (1)

Sol. X = getting number of doublets *i.e.* success

Probability of getting a doublet in a throw

$$P = \frac{6}{36} = \frac{1}{6}$$

$$P(X = 0) = {}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X = 1) = {}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X = 2) = {}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = \frac{15}{216}$$

$$P(X = 3) = {}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{1}{216}$$

Question ID: 9320115

In linear programming, the optimal value of the objective function is attained at the points given by

(1) intersection of the inequalities with the x-axis only

(2) intersection of the inequalities with the axes only

(3) corner points of the feasible region

(4) intersection of the inequalities with the y-axis only

Answer (3)

Sol. The optimal values of the objective function is attained at the corner points of the feasible region.

Question ID: 9320116

If R is a relation on Z (set of all integers) defined by xRy, iff $|x - y| \leq 1$, then

(a) R is reflexive

(b) R is symmetric

(c) R is transitive

(d) R is not symmetric

(e) R is not transitive

Choose the **most appropriate** answer from the options given below

(1) (a) and (d) only

(2) (a), (b) and (c) only

(3) (b) and (c) only

(4) (a), (b) and (e) only

Answer (4)

Sol. xRy, $|x - y| \leq 1$

For reflexive $(a, a) \Rightarrow |a - a| = 0 \leq 1$.

$\therefore$ Relation is reflexive.

For symmetric

$(a, b) \Rightarrow |a - b| \leq 1$.

$(b, a) \Rightarrow |b - a| \leq 1$. [True]

$\therefore$ Relation is symmetric.

For transitive

$(a, b) \Rightarrow |a - b| \leq 1$ [Ex. $1R2 \Rightarrow |2 - 1| \leq 1$

$(b, c) \Rightarrow |b - c| \leq 1$ $2R3 \Rightarrow |2 - 3| \leq 1$

$\not\Rightarrow |a - c| \leq 1$ $1R3$ $\not\Rightarrow |1 - 3| = 2 \not\leq 1$]

$\therefore$ Only reflexive and symmetric and not transitive.

Question ID: 9320117

If the vertices of a triangle ABC are $A(1, 2, 1)$, $B(4, 2, 3)$ and $C(2, 3, 1)$, then the equation of the median passing through the vertex A, is

(1) $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{2}$

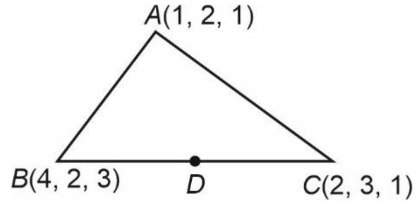
(2) $x - 2 = \frac{y-2}{1} = z - 1$

(3) $x - 1 = 2y - 4 = z - 1$

(4) $\frac{x-1}{2} = 2y - 4 = z - 1$

Answer (4)

Sol.



$$D \equiv \left(\frac{4+2}{2}, \frac{2+3}{2}, \frac{3+1}{2}\right)$$

$$\equiv \left(3, 5/2, 2\right)$$

Equation of median through A

$$\frac{x-1}{2} = \frac{y-2}{1/2} = \frac{z-1}{1}$$

Question ID: 9320118 *(Options (1) and (4) are Same)

A line makes the angle θ with each of the x and z axes. If the angle β which it makes with y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then the value of $\cos^2\theta$ is

(1) $2/5$

(2) $1/5$

(3) $3/5$

(4) $2/5$

Answer (3)

Sol. $\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

or $2\cos^2\theta + 1 - 3\sin^2\theta = 1$

or $2\cos^2\theta = 3\sin^2\theta$

or $2\cos^2\theta = 3 - 3\cos^2\theta$

$\Rightarrow \cos^2\theta = 3/5$

Question ID: 9320119

If $x = 2\sin\theta$ and $y = 2\cos\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = 0$ is

(1) $-\frac{1}{2}$

(2) -1

(3) 0

(4) 1

Answer (1)

Sol. $\frac{dy}{dx} = \frac{-2\sin\theta}{2\cos\theta} = -\tan\theta$

$$\frac{d^2y}{dx^2} = -\sec^2\theta \cdot \frac{d\theta}{dx}$$

$$= \frac{-\sec^2\theta}{2\cos\theta} = -\frac{1}{2}\sec^3\theta$$

at $\theta = 0$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}$$

Question ID: 9320120

If $x = e^{y+e^{y+e^{y+\ldots\infty}}}$, $x > 0$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{x}{1+x}$ (2) $\frac{1}{x}$

(3) $\frac{1-x}{x}$ (4) $\frac{1+x}{x}$

Answer (3)

Sol. $x = e^{y+x}$

Differentiating w.r.t. x

$$1 = e^{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{1}{x} = 1+\frac{dy}{dx}$$

or $\frac{dy}{dx} = \frac{1-x}{x}$

Question ID: 9320121

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, than x is equal to

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) 2

Choose the **most appropriate** answer from the options given below:

(1) (a) and (b) only (2) (a) and (c) only

(3) (a) only (4) (c) only

Answer (3)

Sol. Domain for x is [0, 1]

$$\sin^{-1}(1-x)\Big|_{\max} = \frac{\pi}{2}$$

$$\sin^{-1}(x)\Big|_{\min} = 0$$

So, only possible condition is when

$x = 0$

Question ID: 9320122

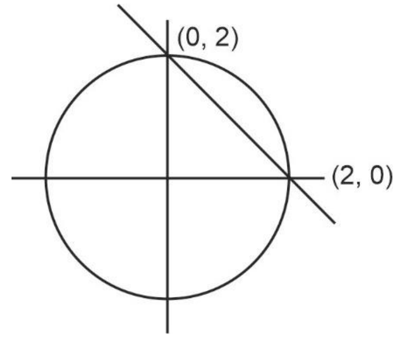
The smaller of the areas enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(1) $2(\pi - 2)$ (2) $\pi - 2$

(3) $2\pi - 1$ (4) $2\pi + 2$

Answer (2)

Sol.



Area of smaller region

$$= \frac{1}{4}\times 4\pi - \frac{1}{2}\cdot 2\cdot 2$$

= $(\pi - 2)$ sq. units

Question ID: 9320123

If $0 < x < \pi$ and the matrix $\begin{bmatrix} 4\sin x & -1 \\ -3 & \sin x \end{bmatrix}$ is singular, then the values of x are :

(1) $\frac{\pi}{3}, \frac{2\pi}{3}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{\pi}{6}, \frac{\pi}{3}$ (4) $\frac{\pi}{6}, \frac{2\pi}{3}$

Answer (1)

Sol. $4\sin^2 x - 3 = 0$

$$\Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow x = \frac{\pi}{3} \text{ OR } \frac{2\pi}{3}$$

Question ID: 9320124

$$\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx =$$

(1) 3 (2) 4

(3) 6 (4) 0

Answer (3)

Sol. $I = \int_{1/3}^{1} \frac{x\left(\frac{1}{x^2}-1\right)^{1/3}}{x^4}dx = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3}dx$

Let $\frac{1}{x^2} - 1 = t \Rightarrow \frac{-2}{x^3}dx = dt$

So $I = \int_8^0 \frac{-t^{1/3}}{2}dt = \frac{1}{2}\int_0^8 t^{1/3}dt$

$$= \frac{1}{2}\cdot \frac{t^{4/3}}{\frac{4}{3}}\Bigg|_0^8$$

$$= \frac{3}{8}\cdot 8^{4/3} = 6$$

Question ID: 9320125

The function $f(x) = e^{|x|}$ is

(a) continuous everywhere on R

(b) not continuous at $x = 0$

(c) Differentiable everywhere on R

(d) not differentiable at $x = 0$

(e) continuous and differentiable on R

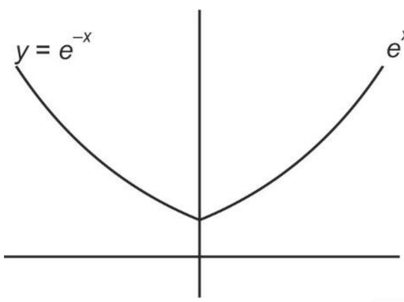
Choose the **most appropriate** answer from the options given below :

(1) (e) only (2) (b) and (c) only

(3) (a) and (d) only (4) (b) and (d) only

Answer (3)

Sol.



$$f(x) = \begin{cases} e^x & x \ge 0 \\ e^{-x} & x < 0 \end{cases}$$

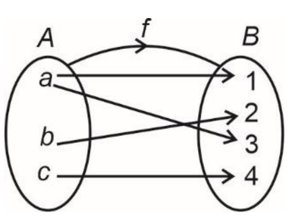
$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x) = f(0) = 1$ so $f(x)$ is continuous.

But RHD at $x = 0$ is 1

LHD at $x = 0$ is –1

So $f(x)$ is not differentiable at $x = 0$.

Question ID: 9320126



Which of the following is **true** on the basis of above diagram?

(1) 'f' is a function from $A \to B$

(2) 'f' is one-one function from $A \to B$

(3) 'f' is onto function from $A \to B$

(4) 'f' is not a function from $A \to B$

Answer (4)

Sol. $f(a) = 1$ and 4, which is not possible for any function.

Question ID: 9320127

If the points (2, –3), (λ, –1) and (0, 4) are collinear, then the value of λ is :

(1) $\frac{7}{10}$ (2) $\frac{3}{10}$

(3) $\frac{7}{3}$ (4) $\frac{10}{7}$

Answer (4)

Sol. $\begin{vmatrix} \lambda & -1 & 1 \\ 0 & 4 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 0$

$\Rightarrow \lambda(7) + 1(-2) + 1(-8) = 0$

$\Rightarrow 7\lambda - 10 = 0$

Question ID: 9320128

The value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ is :

(1) $\frac{120}{169}$ (2) $\frac{-120}{169}$

(3) $\frac{-60}{169}$ (4) $\frac{60}{169}$

Answer (2)

Sol. $\sin\left(2\pi - 2\cot^{-1}\frac{5}{12}\right) = -\sin\left(2\cot^{-1}\frac{5}{12}\right)$

Let $\cot^{-1}\frac{5}{12} = \theta \Rightarrow \cot\theta = \frac{5}{12}$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^3\theta} = \frac{\frac{24}{5}}{1+\frac{144}{25}} = \frac{\frac{24}{5}}{\frac{169}{25}}$$

$$= \frac{120}{169}$$

Question ID: 9320129

Let $y = m \sin rx + n \cos rx$. What is the value of $\frac{d^2y}{dx^2}$?

(1) ry (2) $-ry$

(3) r^2y (4) $-r^2y$

Answer (4)

Sol. $\frac{dy}{dx} = m \cdot r(\cos rx) - nr(\sin rx)$

$\Rightarrow \frac{d^2y}{dx^2} = mr^2(-\sin rx) - nr^2(\cos rx)$

$= -r^2y$

Question ID: 9320130

The integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

(1) $\sec x$
(2) $\cos x$
(3) $\sec x + \tan x$
(4) $\tan x$

Answer (1)

Sol. Dividing by $\cos x$

$$\frac{dy}{dx} + y \tan x = \sec x$$

So I.F. $= e^{\int \tan x\, dx} = e^{\ln(\sec x)} = \sec x$

Question ID: 9320131

The order and degree of the differential equation $\left[\left(\frac{d^2y}{dx^2}\right)^2 - 3\right]^{\frac{1}{3}} = 2\left(\frac{dy}{dx}\right)^{\frac{1}{4}}$ are

(1) order = 2, degree = 2
(2) order = 2, degree = 4
(3) order = 2, degree = 8
(4) order = 1, degree = 1

Answer (3)

Sol. Differential equation can be reduced to

$$\left(\left(\frac{d^2y}{dx^2}\right)^2 - 3\right)^4 = 2^{12}\left(\frac{dy}{dx}\right)^3$$

So order = 2, Degree = 8

Question ID: 9320132

$\int \sqrt{1-49x^2}\, dx$ is equal to

(1) $\frac{x}{2}\left(\sqrt{1-49x^2}\right) + \frac{1}{98}\sin^{-1} 7x + C$

(2) $\frac{7x}{2}\sqrt{1+49x^2} + \frac{1}{49}\sin^{-1} x + C$

(3) $\frac{x}{2}\sqrt{1+\frac{1}{7x^2}} - \frac{1}{49}\sin^{-1} 7x + C$

(4) $\frac{x}{2}\sqrt{1-49x^2} + \frac{1}{14}\sin^{-1} 7x + C$

Answer (4)

Sol. $I = 7\int \sqrt{\frac{1}{49} - x^2}\, dx$

$$= 7\left(\frac{x}{2}\sqrt{\frac{1}{49} - x^2} + \frac{1}{98}\sin^{-1}\frac{x}{\frac{1}{7}}\right) + C$$

$$= \frac{x}{2}\sqrt{1-49x^2} + \frac{1}{14}\sin^{-1}(7x) + C$$

Question ID: 9320133

The shortest distances of the point (1, 2, 3) from x, y, z axes respectively are

(1) 1, 2, 3

(2) $\sqrt{5}, \sqrt{13}, \sqrt{10}$

(3) $\sqrt{10}, \sqrt{13}, \sqrt{5}$

(4) $\sqrt{13}, \sqrt{10}, \sqrt{5}$

Answer (4)

Sol. $D_x = \sqrt{2^2 + 3^2} = \sqrt{13}$

$D_y = \sqrt{1^2 + 3^2} = \sqrt{10}$

$D_z = \sqrt{1^2 + 2^2} = \sqrt{5}$

Question ID: 9320134

Distance between two planes $x + 2y - z = 5$ and $2x + 4y - 2z + 2 = 0$ is

(1) $\sqrt{6}$ unit

(2) 7 unit

(3) $\frac{5}{\sqrt{6}}$ unit

(4) $\frac{4}{\sqrt{6}}$ unit

Answer (1)

Sol. $P_1 : 2x + 4y - 2z = 10$

$P_2 : 2x + 4y - 2z = -2$

$$d = \frac{12}{\sqrt{2^2 + 4^2 + 2^2}} = \frac{12}{\sqrt{24}} = \sqrt{6} \text{ units}$$

Question ID: 9320135

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two non zero vectors inclined at an angle θ, then identify the correct option out of the given options.

(a) $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|.|\vec{b}|}$

(b) $\vec{a}$ and $\vec{b}$ are perpendicular, if $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(c) $\vec{a}$ and $\vec{b}$ are perpendicular, if $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} \neq \dfrac{c_1}{c_2}$

(d) for $\theta = \pi$, $\vec{a}\times\vec{b} = 0$

(e) $\cos\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\cdot|\vec{b}|}$

Choose the **most appropriate** answer from the options given below

(1) (a), (b) and (d) only (2) (a), (b) and (e) only

(3) (b), (d) and (e) only (4) (a) and (b) only

Answer (1)

Sol. Given $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Let, angle between $\vec{a}$ and $\vec{b}$ is θ

So, $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

and if $\vec{a}$ is perpendicular to $\vec{b}$ then

$\vec{a}\cdot\vec{b} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$

and if $\theta = \pi$, then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\pi\,\hat{n}$

$$= \vec{0}$$

So, option (1) is correct.

Question ID: 9320136

If $\vec{p} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{q} = 2\hat{i} + \hat{j} - \hat{k}$, then the area of parallelogram having diagonals $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$ is

(1) $4\sqrt{11}$ sq. unit (2) $\sqrt{44}$ sq. unit

(3) $\sqrt{11}$ sq. unit (4) $3\sqrt{11}$ sq. unit

Answer (3)

Sol. Given: $\vec{p} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{q} = 2\hat{i} + \hat{j} - \hat{k}$

So, diagonals $\vec{d_1} = \vec{p} + \vec{q} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$$\vec{d_2} = \vec{p} - \vec{q} = -\hat{i} - \hat{k}$$

$\therefore$ Area of parallelogram $= \dfrac{1}{2}\left|\vec{d_1}\times\vec{d_2}\right|$

Now,

$$\vec{d_1}\times\vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -3 \\ -1 & 0 & -1 \end{vmatrix} = -2\hat{i} + 6\hat{j} + 2\hat{k}$$

$\therefore$ Area $= \dfrac{1}{2}\sqrt{4+36+4} = \sqrt{11}$ sq. units

Question ID: 9320137

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

(1) 3

(2) $-\dfrac{3}{2}$

(3) $\dfrac{3}{2}$

(4) –3

Answer (2)

Sol. Given: $\vec{a} + \vec{b} + \vec{c} = 0$

Taking dot product with $(\vec{a} + \vec{b} + \vec{c})$ on both sides

$(\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\therefore (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -\dfrac{3}{2}$

Question ID: 9320138

The corner points of the feasible region for an *L.P.P.* are (0, 10), (5, 5), (15, 15) and (0, 20). If the objective function is $z = px + qy$; $p, q > 0$, then the condition on p and q so that the maximum of z occurs at (15, 15) and (0, 20) is

(1) $p = q$ (2) $p = 2q$

(3) $q = 3p$ (4) $q = 2p$

Answer (3)

Sol. Given the corner points of feasible region for L.P.P. are (0, 10), (5, 5), (15, 15) and (0, 20) and since objective function $z = px + qy$; $p, q > 0$ is maximum for (15, 15) and (0, 20)

$\therefore$ z will be maximum for all $x, y \in$ which belongs to feasible region and lies on the line joining (15, 15) and (0, 20)

$\therefore \quad L : y - 20 = \frac{-5}{15}(x - 0) \quad \Rightarrow \quad 5x + 15y = 300$

$\Rightarrow x + 3y = 600$

So, $q = 3p$

Question ID: 9320139

$\int x\sqrt{x+2}\,dx$ is equal to :

(1) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(2) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(3) $\frac{1}{5}(x+2)^{\frac{5}{2}} - \frac{2}{3}(x+2)^{\frac{3}{2}} + C$

(4) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

Answer (1)

Sol. $I = \int x\sqrt{x+2}\,dx$

Let $\sqrt{x+2} = t$

$\Rightarrow \quad x + 2 = t^2$

$\Rightarrow \quad dx = 2t\,dt$

$\therefore \quad I = \int \left(t^2 - 2\right) t(2t)\,dt$

$= \int \left(2t^4 - 4t^2\right) dt = \frac{2}{5}t^5 - \frac{4}{3}t^3 + C$

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

Question ID: 9320140

Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black marbles respectively. One of the urns is selected at random and a marble is drawn from it. If the marble drawn is red, then the probability that it is drawn from the first urn is

(1) $\frac{6}{10}$ (2) $\frac{4}{10}$

(3) $\frac{5}{10}$ (4) $\frac{2}{5}$

Answer (4)

Sol. $\underbrace{6R, 4B}_{U_1} \quad \underbrace{4R, 6B}_{U_2} \quad \underbrace{5R, 5B}_{U_3}$

Let

E : Drawn marble is red

E_1 : Drawn marble from urn I

E_2 : Drawn marble from urn II

E_3 : Drawn marble from urn III

So, $P\left(E_1\right) = P\left(E_2\right) = P\left(E_3\right) = \frac{1}{3}$

Now,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E \cap E_1)}{P(E)} = \frac{P(E_1)\ P\left(\frac{E}{E_1}\right)}{P(E_1)\,P\left(\frac{E}{E_1}\right) + P(E_2)\,P\left(\frac{E}{E_2}\right) + P(E_3)\,P\left(\frac{E}{E_3}\right)}$$

$$\frac{\frac{6}{10}}{\frac{6}{10} + \frac{4}{10} + \frac{5}{10}} = \frac{6}{15} = \frac{2}{5}$$

Question ID: 9320141

Read the text carefully and answer the questions:

Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that exactly one of them complete the task on time is

(1) $\frac{2}{15}$ (2) $\frac{2}{5}$

(3) $\frac{3}{20}$ (4) $\frac{13}{30}$

Answer (4)

Sol. Given: $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}; P(C) = \frac{1}{5}$

So, probability that exactly one of them complete the task on time.

$P = \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5}$

$= \frac{12 + 8 + 6}{60} = \frac{26}{60} = \frac{13}{30}$

Question ID: 9320142

Read the text carefully and answer the questions:

Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that exactly two of them complete the task on time is

(1) $\frac{3}{20}$ (2) $\frac{13}{30}$

(3) $\frac{1}{5}$ (4) $\frac{2}{15}$

Answer (1)

Sol. Given: $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}, P(C)=\frac{1}{5}$

So, probability that exactly two of them complete the task on time is

$$P=\frac{1}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{1}{5}+\frac{1}{3}\times\frac{3}{4}\times\frac{1}{5}$$

$$=\frac{4+2+3}{60}=\frac{9}{60}=\frac{3}{20}$$

Question ID: 9320143

Read the text carefully and answer the questions:

Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that B alone complete the task on time is:

(1) $\frac{13}{30}$ (2) $\frac{3}{20}$

(3) $\frac{2}{5}$ (4) $\frac{2}{15}$

Answer (4)

Sol. Given $P(A)=\frac{1}{3};\ P(B)=\frac{1}{4};\ P(C)=\frac{1}{5}$

So, the probability that B alone complete the task on time

$$P=\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}=\frac{2}{15}$$

Question ID: 9320144

Read the text carefully and answer the questions:

Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that the task is completed on time by none of them is

(1) $\frac{3}{20}$ (2) $\frac{2}{5}$

(3) $\frac{13}{30}$ (4) $\frac{2}{15}$

Answer (2)

Sol. Given $P(A)=\frac{1}{3};\ P(B)=\frac{1}{4};\ P(C)=\frac{1}{5}$

So, probability that the task is completed on time by none of them

$$P=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$$

Question ID: 9320145

Read the text carefully and answer the questions:

Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that task is completed on time by at least one of them is:

(1) $\frac{2}{5}$ (2) $\frac{3}{20}$

(3) $\frac{3}{5}$ (4) $\frac{2}{15}$

Answer (3)

Sol. Given $P(A)=\frac{1}{3};\ P(B)=\frac{1}{4};\ P(C)=\frac{1}{5}$

So, probability that the task is completed on time by at least one of them

$$P=1-\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=1-\frac{2}{5}=\frac{3}{5}$$

Question ID: 9320146

Read the text carefully and answer the questions:

Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The equations in terms of x and y are:

(1) $x - y = 50$, $2x + y = 550$

(2) $x + y = 40$, $2x - y = 550$

(3) $x - y = 10$, $2x + y = 50$

(4) $x - y = 30$, $2x + y = 505$

Answer (1)

Sol. Let length of the plot is = x m

and breadth of the plot is = y m

Then, According to question

$(x - 50)(y + 50) = xy$ … (i)

$(x - 10)(y - 20) = xy - 5300$ …(ii)

From (i) $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii) $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

Question ID: 9320147

Read the text carefully and answer the questions:

Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The value x is:

(1) 150 m (2) 100 m

(3) 200 m (4) 300 m

Answer (3)

Sol. Let length of the plot is = x m

and breadth of the plot is = y m

Then, According to question

$(x - 50)(y + 50) = xy$ … (i)

$(x - 10)(y - 20) = xy - 5300$ …(ii)

From (i) $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii) $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

$x - y = 50$ … (iii)

$2x + y = 550$ …(iv)

(iii) + (iv)

$\Rightarrow 3x = 600$

$\Rightarrow x = 200$

Question ID: 9320148

Read the text carefully and answer the questions:

Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The value of y is

(1) 50 m

(2) 100 m

(3) 240 m

(4) 150 m

Answer (4)

Sol. Let length of the plot is = x m

and breadth of the plot is = y m

Then, According to question

$(x - 50)(y + 50) = xy$ … (i)

$(x - 10)(y - 20) = xy - 5300$ …(ii)

From (i) $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii) $-20x - 10y + 200 = -5300$

$\Rightarrow$ $20x + 10y = 5500$

$\Rightarrow$ $2x + y = 550$

$x - y = 50$ … (iii)

$2x + y = 550$ …(iv)

(iii) + (iv)

$\Rightarrow$ $3x = 600$

$\Rightarrow$ $x = 200$

$x - y = 50$ and $x = 200$

then $y = 150$ m

Question ID: 9320149

Read the text carefully and answer the questions:

Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The value of the expression $\frac{x^2+y^2}{x-y}$ is:

(1) 625

(2) 1250

(3) 312.5

(4) 3125

Answer (2)

Sol. Let length of the plot is = x m

and breadth of the plot is = y m

Then, According to question

$(x - 50)(y + 50) = xy$ … (i)

$(x - 10)(y - 20) = xy - 5300$ …(ii)

From (i) $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii) $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

$x - y = 50$ … (iii)

$2x + y = 550$ …(iv)

(iii) + (iv)

$\Rightarrow$ $3x = 600$

$\Rightarrow$ $x = 200$

$x = 200$, $y = 150$

$$\frac{x^2+y^2}{x-y} = \frac{(200)^2+(150)^2}{50} = \frac{62500}{50} = 1250$$

Question ID: 9320150

Read the text carefully and answer the questions:

Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The area of rectangular field is:

(1) 30000 sq. m

(2) 3000 sq. m

(3) 300000 sq. m

(4) 60000 sq. m

Answer (1)

Sol. Let length of the plot is = x m

and breadth of the plot is = y m

Then, According to question

$(x - 50)(y + 50) = xy$ … (i)

$(x - 10)(y - 20) = xy - 5300$ …(ii)

From (i) $50x - 50y - 2500 = 0$

$\Rightarrow$ $x - y = 50$

From (ii) $-20x - 10y + 200 = -5300$

$\Rightarrow$ $20x + 10y = 5500$

$\Rightarrow$ $2x + y = 550$

$x - y = 50$ … (iii)

$2x + y = 550$ …(iv)

(iii) + (iv)

$\Rightarrow$ $3x = 600$

$\Rightarrow$ $x = 200$

$x = 200$, $y = 150$

Area = $200 \times 150 = 30000$ sq. m.