06/04/2023
Evening

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Memory Based
Answers & Solutions
for
JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. An object starts moving with an initial speed 10 m/s and acceleration 2 m/s^2 along positive x-direction. The time taken to attain 60 m/s speed is

(1) 25 s (2) 20 s

(3) 30 s (4) 15 s

Answer (1)

Sol. $v = u + at$

$60 = 10 + (2)t$

$t = 25$ s

2. Potential energy of an electron is defined as $U = \frac{1}{2} mw^2x^2$ and follows Bohr's law. Radius of orbital as function of n depends on (w is some constant)

(1) n^2 (2) $\frac{1}{\sqrt{n}}$

(3) $\sqrt{n}$ (4) $n^{2/3}$

Answer (3)

Sol. $U = \frac{1}{2}mw^2x^2$

$mvx = \frac{nh}{2\pi}$

$\frac{mv^2}{x} = mw^2x$

$\Rightarrow v = wx$

$\Rightarrow x^2 \propto n$

or $x \propto \sqrt{n}$

3. If W is the weight on the surface of earth then weight of same body at a height $\frac{R_e}{4}$ above the surface of earth is equal to (R_e :- Radius of earth)

(1) $\frac{4}{5}W$ (2) $\frac{16}{25}W$

(3) $\frac{25}{16}W$ (4) $\frac{5}{4}W$

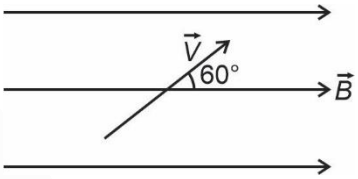
Answer (2)

Sol. $g' = \frac{g \times R^2}{\frac{25}{16}R^2} = \frac{16g}{25}$

$W' = \frac{16W}{25}$

4. A proton is projected with speed v in magnetic field B of magnitude 1 T if angle between velocity and magnetic field is 60° as shown below. Kinetic energy of proton is 2 eV (Mass of proton = 1.67 × 10^{-27} kg, e = 1.6 × 10^{-19} C).

The pitch of the path of proton is approximately



(1) 6.28 × 10^{-2} m (2) 6.28 × 10^{-4} m

(3) 3.14 × 10^{-2} m (4) 3.14 × 10^{-4} m

Answer (2)

Sol. $R = \left(\frac{mv\sin 60°}{qB}\right)$

$T = \left(\frac{2\pi m}{qB}\right)$ K.E $= \frac{1}{2}mv^2$

Pitch $= v\cos\theta \times \left(\frac{2\pi m}{qB}\right)$ $\sqrt{\frac{2K}{m}} = v$

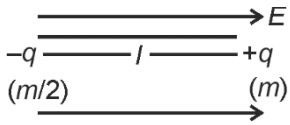
$= \sqrt{\frac{2K}{m}} \times \frac{1}{2} \times \frac{2\pi m}{qB}$

$= \frac{\pi}{eB} \cdot \sqrt{2Km} = \frac{3.14}{1.6\times10^{-19}\times1} \times \sqrt{2\times2\times(1.6)^2\times10^{-46}}$

$= \frac{3.14}{1.6\times10^{-19}} \times 2 \times 1.6 \times 10^{-23}$

= 6.28 × 10^{-4} m

5. An electric dipole is shown in the figure. If it is displaced angularly by a small angle with respect to electric field, then angular frequency of oscillation is given by



(1) $\sqrt{\frac{6qE}{ml}}$ (2) $\sqrt{\frac{3qE}{ml}}$

(3) $\sqrt{\frac{2qE}{ml}}$ (4) $\sqrt{\frac{qE}{ml}}$

Answer (2)

Sol.

$\tau_{com} = qEl\sin\theta$

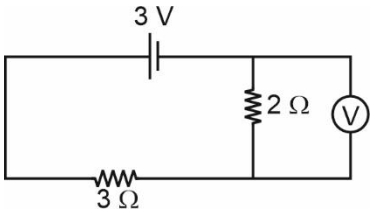
$\because \theta$ is very small

$\tau_{com} = qEl\theta$

$\left(m \times \frac{l^2}{9} + \frac{m}{2}\frac{4l^2}{9}\right)\alpha = qEl\theta$

$\alpha = \frac{3qE}{ml}\theta$

6.



In the circuit shown reading of the ideal voltmeter used is equal to _____ volts

(1) 3 V (2) 1.8 V

(3) 1.2 V (4) Zero

Answer (3)

Sol. Current through 2 Ω resistance

$= \frac{3}{5}\text{A} = 0.6\text{ A}$

$\Rightarrow V_{2\Omega} = 0.6 \times 2 = 1.2\text{ V}$

$\Rightarrow$ Reading of voltmeter = 1.2 V

7. Find the ratio of root mean square speed of oxygen gas molecules to that of hydrogen gas molecules, if temperature of both the gases are same.

(1) $\frac{1}{4}$ (2) $\frac{1}{16}$

(3) $\frac{1}{32}$ (4) $\frac{1}{8}$

Answer (1)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{rms} \propto \frac{1}{\sqrt{M}}$

$\frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$

8. In amplitude modulation with carrier frequency (A_c) and modulant frequency (A_m), modulation index is 60%. If $A_c - A_m = 3$ V then $A_c + A_m$ is equal to _____.

(1) 6 V (2) 12 V

(3) 4 V (4) 15 V

Answer (2)

Sol. $\mu = 0.6$

$\frac{A_m}{A_c} = 0.6$

$\frac{A_c - A_m}{A_c + A_m} = \frac{2}{8} = \frac{1}{4}$

$\Rightarrow A_c + A_m = 12$ V

9. For two different photosensitive material having work function ϕ and 2ϕ respectively, are illuminated with light of sufficient energy to emit electron. If the graph of stopping potential versus frequency is drawn, for these two different photosensitive materials the ratio of slope of graph for these two materials is

(1) 1 : 1 (2) 1 : 2

(3) 1 : 4 (4) 4 : 1

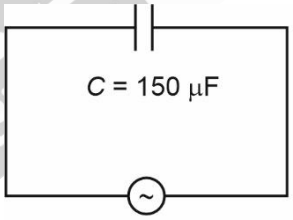
Answer (1)

Sol. $eV = h\nu - \phi$

$V = \left(\frac{h}{e}\right)\nu - \phi$

Slope $= \left(\frac{h}{e}\right)$ is independent of material so ratio is 1 : 1.

10. In the given AC circuit, find maximum current through the capacitor



$E = 36\sin(120\pi t)$ V

(1) 0.65 πA (2) 0.35 πA

(3) 0.2 πA (4) 0.8 πA

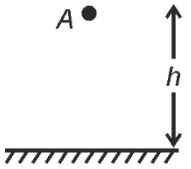
Answer (1)

Sol. $i_0 = \frac{E_0}{X_C} = E_0\,\omega C$

$= 36 \times 120\pi \times 150 \times 10^{-6}$ A

$= 0.65\,\pi$A

11. An object A is released from a height h such that the ratio of its speed before striking the ground and after striking the ground is 4 : 1. If loss of kinetic energy is $\frac{x}{4}\%$ then value of x is



(1) 225 (2) 50

(3) 375 (4) 25

Answer (3)

Sol. $\frac{V_{before}}{V_{after}} = \frac{4}{1}$

$\frac{KE_{before}}{KE_{after}} = \frac{16}{1}$

$\frac{\Delta KE}{KE_{before}} = \frac{15}{16}$

$= \frac{15}{16} \times 100\%$

$= \frac{375}{4}\%$

12. **Assertion (A) :** When tooth paste is pressed, it follows Pascal's principle.

Reason (R) : When pressure is applied on a fluid it is distributed constantly throughout the fluid in all direction and on the wall of the container.

(1) A is correct and R is the correct explanation of A

(2) A is correct and R is wrong explanation of A

(3) A is correct, R is wrong

(4) Both A and R are false

Answer (1)

Sol. Reason (R) is the Pascal's principle and which gives correct explanation of A.

13. **Assertion (A) :** In forward biased p-n junction, diffusion current is from p-region to n-region.

Reason (R) : Diffusion takes place due to concentration gradient.

(1) Both (A) and (R) are true, (R) is the correct explanation of (A)

(2) Both (A) and (R) are true, (R) is not the correct explanation of (A)

(3) (A) is true, (R) is false

(4) Both (A) and (R) are false

Answer (1)

Sol. Diffusion takes place due to concentration gradient.

14. Radius of first orbit in H-atom is a_0. Then, de Broglie wavelength of electron in the third orbit is

(1) $3\pi a_0$ (2) $6\pi a_0$

(3) $9\pi a_0$ (4) $12\pi a_0$

Answer (2)

Sol. $\lambda = \frac{h}{mv}$

$= \frac{2\pi r}{n}$

$= \frac{2\pi a_0 n^2}{n}$

$= 2\pi a_0 (n)$

$= 6\pi a_0$

15. Choose the incorrect statement from the given statements.

(A) Planets revolve around the Sun with constant linear speed.

(B) Energy of planet in elliptical orbit is constant.

(C) Satellite in circular motion have constant energy.

(D) Body falling towards the Earth results in negligible displacement of the Earth.

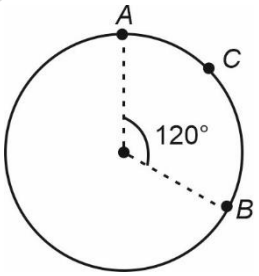
(1) (A) (2) (B)

(3) (C) (4) (D)

Answer (1)

Sol. Planet revolves around the Sun in elliptical orbit with variable speed.

16. A particle moves from *A* to *B* via *C* with uniform speed of π m/s. Average velocity during the journey is equal to



(1) $\sqrt{3}$ m/s

(2) $\frac{\sqrt{3}}{2}$ m/s

(3) $\frac{3\sqrt{3}}{2}$ m/s

(4) 2 m/s

Answer (3)

Sol. Displacement

= 2R × sin60° = $\sqrt{3}\,R$

Time = $\frac{2\pi R}{3} \times \frac{1}{\pi} = \frac{2R}{3}$ seconds

$\Rightarrow$ Average velocity

$= \frac{\sqrt{3}R \times 3}{2R} = \frac{3\sqrt{3}}{2}$ m/s

17. The temperature of body drops from 60°C to 40°C in 7 min. The surrounding temperature is 10°C. The temperature of body drops from 40°C to T°C in 7 min. Find the value of T

(1) 16°C (2) 20°C

(3) 28°C (4) 36°C

Answer (3)

Sol. $\frac{60-40}{7} = K(50-10)$

$\frac{40-T}{7} = K\left(\frac{40+T}{2} - 10\right)$

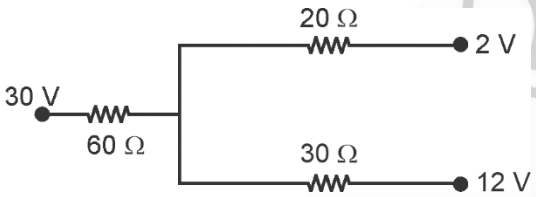
$\Rightarrow \frac{20}{40-T} = \left(\frac{40 \times 2}{T+20}\right)$

$\Rightarrow T + 20 = 160 - 4T$

$\Rightarrow 5T = 140$

$T = \frac{140}{5} = 28℃$

18. In the given circuit, find the current passing through 20 Ω.



(1) 0.45 A (2) 0.23 A

(3) 0.40 A (4) 0.78 A

Answer (3)

Sol. Let x be the potential at the node

$\frac{30-x}{60} + \frac{2-x}{20} + \frac{12-x}{30} = 0$

$\frac{1}{2} + \frac{1}{10} + \frac{4}{10} = \frac{x}{60} + \frac{x}{20} + \frac{x}{30}$

$\frac{10}{10} = \frac{2x + 6x + 4x}{120}$

$120 = 12x$

$x = 10$ V

$i = \frac{10-2}{20} = \frac{8}{20} = 0.4$ A

19. Average energy density of an EM wave with electric field amplitude E_0 and magnetic field amplitude B_0 is equal to

(1) $\frac{1}{2}\varepsilon_0 E_0^2$ (2) $\frac{B_0^2}{\mu_0}$

(3) $\varepsilon_0 E_0^2$ (4) $\frac{1}{2}\mu_0 E_0^2$

Answer (1)

Sol. Total energy = $\frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B^2}{\mu_0}$

and $\varepsilon_0 E_0^2 = \frac{B^2}{\mu_0}$

$\Rightarrow$ total energy = $\frac{1}{2}\varepsilon_0 E_0^2$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A solid sphere and a ring have equal masses and equal radius of gyration. If sphere is rotating about its diameter and ring about an axis passing through and perpendicular to its plane, then the ratio of radius is $\sqrt{\frac{x}{2}}$ then find the value of x.

Answer (5)

Sol. $\frac{2}{5}mR_1^2 = mK_1^2$ and $mR_2^2 = K_2$

$K_1 = \sqrt{\frac{2}{5}}R_1$ $K_2 = R_2$

$K_2 = R_2$ $K_1 = K_2$

$\sqrt{\frac{2}{5}}R_1 = R_2 \Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{5}{2}}$

22.

23.

24.

25.

26.

27.

28.

29.

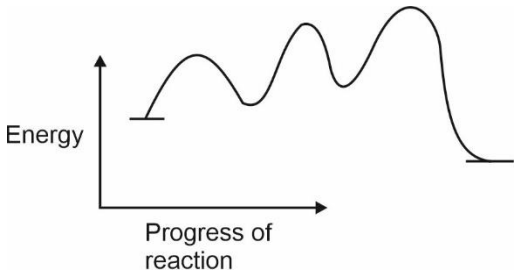
30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. 

(P) Number of intermediates = 2

(Q) Number of transition states = 3

(R) Reaction is endothermic

Correct statement is

(1) P & Q only

(2) P & R only

(3) Q & R only

(4) P, Q, R

Answer (1)

Sol. 3-step reaction

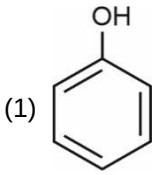
Number of transition states = 3

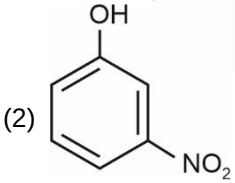
Number of intermediates = 2

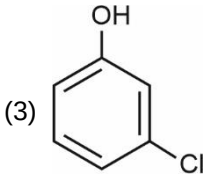
Reaction is exothermic

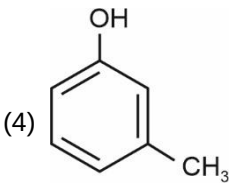
As $\Delta H < 0$

2. Which of the following compound is most acidic?

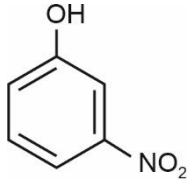
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

is most acidic due to –I effect of $-NO_2$ group.

3. Which of the following is most basic

(1) Tl_2O_3

(2) Tl_2O

(3) Cr_2O_3

(4) B_2O_3

Answer (2)

Sol. Tl^+ oxide is more basic than Tl^{3+} Cr_2O_3 is amphoteric

4. Which of the following element is not present in Nessler's reagent?

(1) K

(2) Hg

(3) N

(4) I

Answer (3)

Sol. Nessler's reagent is alkaline solution of K_2HgI_4

5. Which of the following is not obtained on electrolysis of brine solution

(1) NaOH

(2) H_2 gas

(3) Cl_2 gas

(4) Na

Answer (4)

Sol. **Anode**

$$2Cl^- \longrightarrow Cl_2 + 2e^-$$

Cathode

$$2e^- + 2H_2O \longrightarrow H_2 + 2OH^-$$

Na metal is not obtained on electrolysis of brine.

6. $BeCl_2$ exists as in solid state, vapour phase and at high temperature of the order of 1200 K in that order.

(1) Polymer, Dimer and Monomer

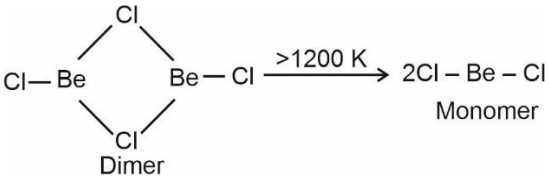
(2) Dimer, Polymer and Monomer

(3) Monomer, Dimer and Polymer

(4) Polymer, Monomer and Dimer

Answer (1)

Sol. $BeCl_2$ has a linear polymeric chain structure with Be-atom undergoing sp^3 hybridisation. In the vapour phase $BeCl_2$ tends to form a chloro-bridged dimer,



which dissociates into the linear monomer at high temperature of the order of 1200 K.

7. Which of the following has highest hydration energy.

(1) Be^{+2}

(2) Mg^{+2}

(3) Ca^{++}

(4) Ba^{+2}

Answer (1)

Sol. Hydration energy decreases down the group in the 2nd group metal cation.

8. Oxidation state of Mn in $KMnO_4$ changes by 3 units in which medium?

(1) Strongly acidic

(2) Strongly basic

(3) Aqueous neutral

(4) Weakly acidic

Answer (3)

Sol. $KMnO_4$ in aqueous neutral medium reduces to MnO_2.

$$2KMnO_4 + H_2O \rightarrow 2MnO_2 + 2KOH + \frac{3}{2}O_2$$

$\therefore$ Oxidation state of Mn in $KMnO_4$ changes from +7 to +4 i.e., by 3 units.

9. IUPAC name of the compound $K_3[Co(C_2O_4)_3]$ is

(1) Potassium trioxalatocobalt (III)

(2) Potassium trioxalatocobaltate (III)

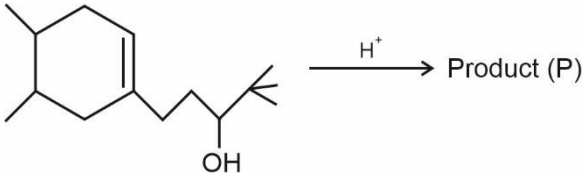
(3) Potassium cobalttrioxalate (II)

(4) Potassium oxalatocobaltate (III)

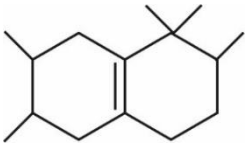
Answer (2)

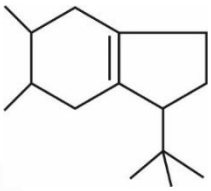
Sol. IUPAC name of $K_3[Co(C_2O_4)_3]$ is Potassium trioxalatocobaltate (III).

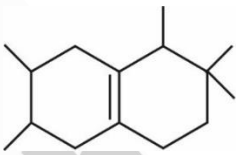
10. Consider the following reaction

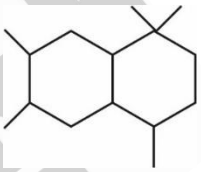


Select the P.

(1) 

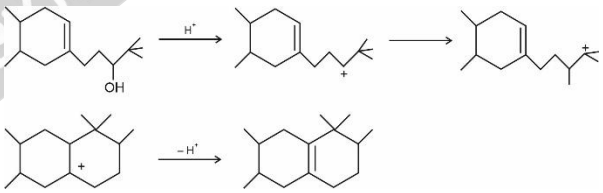
(2) 

(3) 

(4) 

Answer (1)

Sol.



11. During detection of Lead.

Formation of which of following compound is not used as confirmatory test.

(1) $PbSO_4$

(2) $Pb(NO_3)_2$

(3) $PbCrO_4$

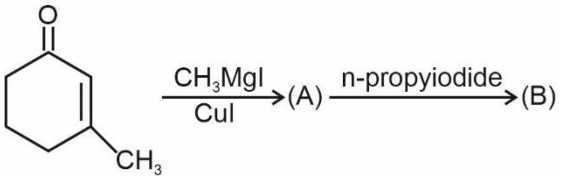
(4) PbI_2

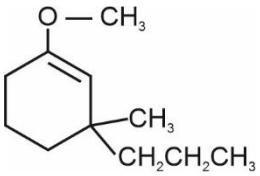
Answer (2)

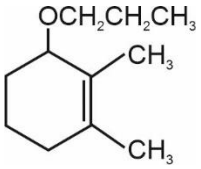
Sol.

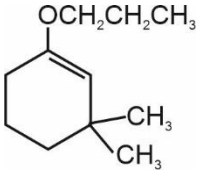
$PbSO_4$	-	White ppt
$PbCrO_4$	-	Yellow ppt
PbI_2	-	Yellow ppt
$Pb(NO_3)_2$	-	Soluble

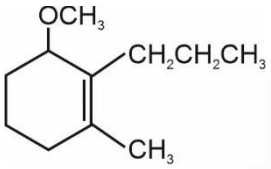
12. Identify the final product (B) formed in the following sequence of reactions.



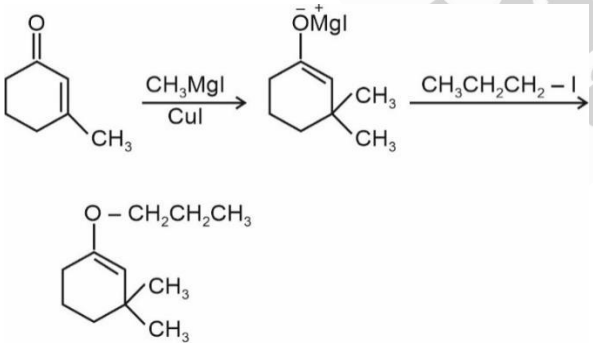
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

13. Consider the following:

(i) D.D.T.

(ii) Aldrin

(iii) Sodium arsenite

(iv) Sodium chlorate

How many of these are pesticides?

(1) 1

(2) 2

(3) 3

(4) 4

Answer (2)

Sol. D.D.T. and Aldrin are pesticides while sodium arsenite and sodium chlorate are herbicides.

14.

	Amino Acid	Letter code
A.	Alanine	P. N
B.	Asparagine	Q. A
C.	Aspartic acid	R. R
D.	Arginine	S. D

(1) A - Q; B - S; C - P; D - R

(2) A - Q; B - S; C - R; D - P

(3) A - S; B - P; C - R; D - Q

(4) A - S; B - P; C - P; D - R

Answer (1)

Sol. Alanine - A

Arginine - R

Aspartic acid - D

Asparagine - N

15.

16.

17.

18.

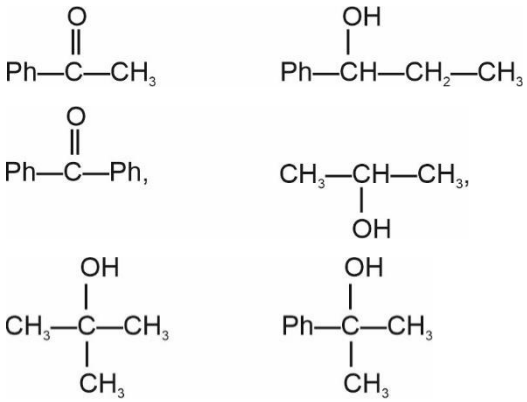
19.

20.

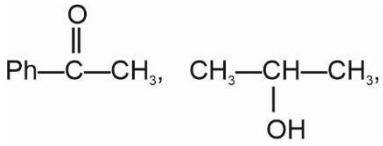
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The number of compounds that give iodoform test



Answer (02.00)

Sol. 

give iodoform test.

22. If a_0 is the radius of H-atom de-Broglie wavelength of e^- in 3rd orbit of Li^{2+} ion is $x\pi a_0$. Find out x.

Answer (02.00)

Sol. $r_3 = \frac{a_0 \times (3)^2}{(3)} = 3a_0$

$2\pi r = 3\lambda$

$2\pi(3a_0) = 3\lambda$

$\Rightarrow \quad \lambda = 2\pi a_0$

x = 2

23. How many of the following will have same relative lowering in vapour pressure?

(A) 1 M NaCl

(B) 1 M Urea

(C) 1.5 M $AlCl_3$

(D) 2 M Na_2SO_4

Answer (02.00)

Sol. $\frac{\Delta P}{p_{solvent}} = i(x_{solute})$

i.M should be same

(A) 1 × 2 = 2

(B) 1 × 1 = 1

(C) 1.5 × 4 = 6

(D) 2 × 3 = 6

(C) & (D) will have same RLVP

24. We are given with 7 type of lattice.

A. Cubic

B. tetragonal

C. Orthorhombic

D. Hexagonal

E. Rhombohedral

F. Monoclinic

G. Triclinic

How many of them can have BCC unit cell?

Answer (03.00)

Sol. Cubic, tetragonal and orthorhombic can have BCC unit cell.

25. How many of the given molecules are square planar in shape?

XeF_4, SF_4, $[Ni(CO)_4]$, $[Ni(CN)_4]^{2-}$, $[NiCl_4]^{2-}$, $[FeCl_4]^{2-}$, $[Cu(NH_3)_4]^{2+}$, $[PdCl_4]^{2-}$

Answer (04.00)

Sol. XeF_4 : square planar

SF_4 : see saw

$[Ni(CO)_4]$: tetrahedral

$[Ni(CN)_4]^{2-}$: square planar

$[NiCl_4]^{2-}$: tetrahedral

$[FeCl_4]^{2-}$: tetrahedral

$[Cu(NH_3)_4]^{2+}$: square planar

$[PdCl_4]^{2-}$: square planar

26. Volume of HBr (0.02 M) (in ml) needed to completely neutralise $Ba(OH)_2$ (0.01 M, 10 ml)

Answer (10)

Sol. mEq of HBr = mEq of $Ba(OH)_2$

$0.02 \times V = 0.01 \times 10 \times 2$

$V = \frac{0.02 \times 10}{0.02} = 10 \text{ ml}$

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $f(x) + f(\pi - x) = \pi^2$ then $\int_0^{\pi} f(x)\sin x\,dx$

(1) π^2 (2) $\frac{\pi^2}{4}$

(3) $2\pi^2$ (4) $\frac{\pi^2}{8}$

Answer (1)

Sol. $I = \int_0^{\pi} f(x)\sin x\,dx$

$I = \int_0^{\pi} f(\pi - x)\sin x\,dx$

$2I = \int_0^{\pi} (\sin x)(f(x) + f(\pi - x))\,dx$

$2I = \pi^2 \int_0^{\pi} \sin x\,dx$

$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x\,dx$

$I = \pi^2 (-\cos x)_0^{\frac{\pi}{2}}$

$= \pi^2$

2. The system of the equations

$x + y + z = 6$

$x + 2y + \alpha z = 5$ has

$x + 2y + 6z = \beta$

(1) Infinitely many solution for $\alpha = 6, \beta = 3$

(2) Infinitely many solution for $\alpha = 6, \beta = 5$

(3) Unique solution for $\alpha = 6, \beta = 5$

(4) No solution for $\alpha = 6, \beta = 5$

Answer (2)

Sol. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 2 & 6 \end{vmatrix} = 6 - \alpha$

$\therefore$ for $\alpha \neq 6$ system has unique solution

Now, when $\alpha = 6$

$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & 6 \\ \beta & 2 & 6 \end{vmatrix} = 0 - (30 - 6\beta) + (10 - 2\beta)$

$= 4(\beta - 5)$

$\Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 0 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$

Clearly at $\beta = 5$, $\Delta_i = 0$ for $i = 1, 2, 3$

$\therefore$ at $\alpha = 6$, $\beta = 5$ system has infinite solutions.

3. Let statement 1 : $(2002)^{2023} - (1919)^{2002}$ is divisible by 8.

Statement 2 : $13.13^n - 12n - 13$ is divisible by 144 $\forall\, n \in N$, then

(1) Statement-1 and statement-2 both are true

(2) Only statement-1 is true

(3) Only statement-2 is true

(4) Neither statement-1 nor statement-2 are true

Answer (3)

Sol. $\because$ $(2002)^{2023} = 8m$

$\therefore$ $(2002)^{2023}$ is divisible by 8

and $(1919)^{2002}$ is not divisible by 8

$\therefore$ $(2002)^{2023} - (1919)^{2002}$ is not divisible by 8.

Now

$13.(13)^n - 12n - 13$

$= 13(1 + 12)^n - 12n - 13$

$= 13\left[1 + 12n + {}^nC_2 12^2 + \cdots\right] - 12n - 13$

$= 144n + 144\,{}^nC_2 + \cdots$

$= 144\left[n + {}^nC_2 + \cdots\right]$

$= 144K$

$\therefore$ Statement-2 is correct

4. If the coefficient of x^7 in $\left(\alpha x^2+\frac{1}{2\beta x}\right)^{11}$ and x^{-7} in $\left(x+\frac{1}{3\beta x^2}\right)^{11}$ are equal then

(1) $\alpha^6\beta=\frac{2^5}{3^6}$ (2) $\alpha^6\beta=\frac{2^6}{3^5}$

(3) $\alpha\beta^6=\frac{2^5}{3^6}$ (4) $\alpha\beta^6=\frac{2^6}{3^5}$

Answer (1)

Sol. Coefficient of x^7 in $\left(\alpha x^2+\frac{1}{2\beta x}\right)^{11}$

$T_{r+1}={}^{11}C_r\left(\alpha x^2\right)^{11-r}\left(\frac{1}{2\beta x}\right)^r$

Now, $22-2r-r=7$

$r=5$

Coeff. $=\ {}^{11}C_5\frac{\alpha^6}{(2\beta)^5}$

Coeff. of x^{-7} in $\left(x+\frac{1}{3\beta x^2}\right)^{11}$ will be, if $r=6$ is $\frac{{}^{11}C_6}{3^6\beta^6}$

${}^{11}C_5\frac{\alpha^6}{2^5\beta^5}=\frac{{}^{11}C_5}{3^6\beta^6}$

$\alpha^6\beta=\frac{2^5}{3^6}$

5. If $(21)^{18}+20\cdot(21)^{17}+(20)^2\cdot(21)^{16}+\ldots\ldots\ldots(20)^{18}=k\left(21^{19}-20^{19}\right)$ then $k=$

(1) $\frac{21}{20}$ (2) 1

(3) $\frac{21}{20}$ (4) $\frac{20}{21}$

Answer (2)

Sol. $a=(21)^{18},\ r=\frac{20}{21},\ n=19$

$S=(21)^{18}\dfrac{\left(1-\left(\frac{20}{21}\right)^{19}\right)}{1-\frac{20}{21}}$

$\Rightarrow\ \frac{(21)^{19}}{(21)^{19}}\left((21)^{19}-(20)^{19}\right)$

$=(21)^{19}-(20)^{19}$

6. If $1^2-2^2+3^2-4^2+\ldots.-(2022)^2+(2023)^2=m^2n$, where $m, n\in N$ and $m>19$ then n^2-m is

(1) 615

(2) 562

(3) 812

(4) 264

Answer (1)

Sol. $1^2-2^2+3^2-4^2+\ldots..(2021)^2-(2022)^2+(2023)^2$

$=\underbrace{-3-7-11\ldots\ldots}_{1011\text{ terms}}+(2023)^2$

$=-\frac{1011}{2}\left(6+(1010)4\right)+(2023)^2$

$=-1011(3+2020)+(2023)^2$

$=(2023)(-1011)+(2023)^2$

$=(2023)(2023-1011)$

$=2023(1012)$

$=(17)^2\cdot7\left(2^2\cdot253\right)=(34)^2(1771)$

$=m=34,\ n=1771$

$\therefore\ n-m^2$

$=1771-1156$

$=615$

7. If $a\neq\pm b$ and are purely real, $z\in$ complex number, $\text{Re}(az^2+bz)=a$ and $\text{Re}(bz^2+az)=b$ then number of value of z possible is

(1) 0

(2) 1

(3) 2

(4) 3

Answer (1)

Sol. $a(x^2-y^2)+bx=a$ ...(i)

$b(x^2-y^2)+ax=b$ ...(ii)

(i) – (ii)

$(a-b)(x^2-y^2)+(b-a)x=a-b \qquad (a\neq b)$

$\Rightarrow\ x^2-y^2-x=1$

(i) + (ii)

$(a+b)(x^2-y^2)+x(a+b)=a+b \qquad (a\neq -b)$

$\Rightarrow\ x^2-y^2+x=1$

$\Rightarrow\ x=0$

$\Rightarrow\ y^2=-1$ (not possible $\because\ y\in R$)

$\therefore$ No complex number possible.

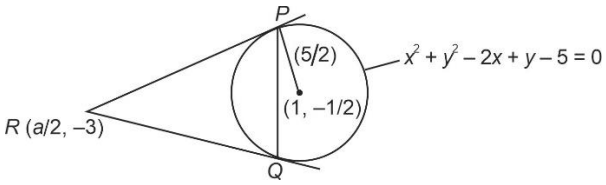
8. Two tangents are drawn from point $R\left(\frac{9}{2}, -3\right)$ intersect the circle $x^2 + y^2 - 2x + y = 5$ at P and Q then the area of ΔPQR is

(1) $\frac{1710}{290}$ (2) $\frac{1715}{296}$

(3) $\frac{296}{1715}$ (4) $\frac{290}{1710}$

Answer (2)

Sol.



$$\text{Area} = \frac{RL^3}{R^2 + L^2}$$

$$L = \frac{7}{2}$$

$$\text{Area} = \frac{\frac{5}{2} \times \left(\frac{7}{2}\right)^3}{\left(\frac{5}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \frac{\frac{1715}{16}}{\frac{25+49}{4}} = \frac{1715 \times 4}{16 \times 74} = \frac{1715}{296}$$

9. Equation of plane passing through intersection of $P_1 : \vec{r}\,(\hat{i} + \hat{j} + \hat{k}) = 6$ and $P_2 : \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and (0, 2, –2) is P. Then square of distance of (12, 12, 18) from P is

(1) 310 (2) 1240

(3) 155 (4) 620

Answer (4)

Sol. $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda\,(2x + 3y + 4z + 5) = 0$

$(1 + 2\lambda)\,x + (1 + 3\lambda)y + (1 + 4\lambda)z + 5\lambda - 6 = 0$

Passing through (0, 2, –2)

$\Rightarrow \quad \lambda = 2$

$\therefore$ Plane : $5x + 7y + 9z = -4$

$$\text{Distance} = \left|\frac{5(12) + 7(12) + 9(18) + 4}{\sqrt{5^2 + 7^2 + 9^2}}\right|^2$$

$$= 620$$

10. If V is volume of parallelopiped whose three coterminous edges are $\vec{a}, \vec{b}, \vec{c}$, then volume of a parallelopiped whose coterminous edges are $\vec{a}, \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}$ is

(1) $6V$ (2) V

(3) $2V$ (4) $3V$

Answer (2)

Sol. $\left[\vec{a} \quad \vec{b} + \vec{c} \quad \vec{a} + 2\vec{b} + 3\vec{c}\right]$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$

$$= \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$

$= V$

11. $S_1 : (p \Rightarrow q) \vee (\sim p \wedge q)$ is a tautology

$S_2 : (q \Rightarrow p) \Rightarrow (\sim p \wedge q)$ is a contradiction

(1) Both S_1 and S_2 are true

(2) Neither S_1 Nor S_2 are true

(3) Only S_1 are true

(4) Only S_2 are true

Answer (2)

Sol. $(p \Rightarrow q) \vee (\sim p \wedge q)$

$\equiv (\sim p \vee q) \vee \sim p \wedge q$

$\equiv \sim p \vee q \vee \sim p \wedge q$

$\equiv \sim p \vee q$

Which is not a tautology

$(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$

$\equiv (\sim q \vee p) \Rightarrow (\sim p \wedge q)$

$\equiv \sim(\sim q \vee p) \vee (\sim p \wedge q)$

$\equiv (q \wedge (\sim p) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$

Which is not a contradiction.

12. $(1 + \ln x)\frac{dx}{dy} + x \ln x = e^y$

Solution of this differential equation satisfies (1, 90) and (α, 92) then α^α is

(1) $\frac{e^{90}}{90}$ (2) $\frac{e^{92} - e^{90}}{45}$

(3) $e^{\left(\frac{e^{92} - e^{90}}{92}\right)}$ (4) $e^{92} - e^{90}$

Answer (3)

Sol. $(1+\ln x)dx + x\ln x\,dy = e^y dy$

$d(y.x\ln x) = d(e^y)$

$\Rightarrow \boxed{xy\ln x = e^y + C}$

Through (1, 90) $\Rightarrow C = -e^{90}$

$xy\ln x = e^y - e^{90}$

$\therefore \quad \alpha.92\ln\alpha = e^{92} - e^{90}$

$\ln\alpha^\alpha = \dfrac{e^{92}-e^{90}}{92}$

$\alpha^\alpha = e^{\left(\frac{e^{92}-e^{90}}{92}\right)}$

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The rank of the word "PUBLIC" is

Answer (198)

Sol. $\therefore$

5	6	1	4	3	2
P	*U*	*B*	*L*	*I*	*C*
4	4	0	2	1	0
5!	4!	3!	2!	1!	0!

$\therefore$ Rank = (1 × 1! + 2 × 2! + 4 × 4! + 4 × 5!) + 1

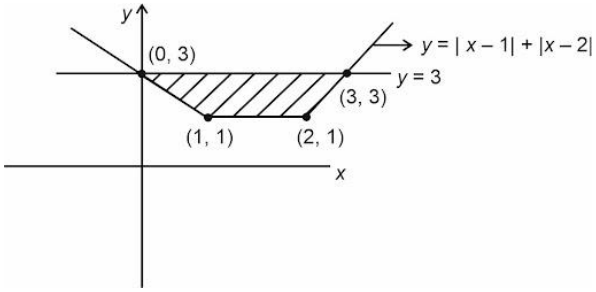
= (1 + 4 + 96 + 480) + 1

= 582

22. The area enclosed by $y = |x-1| + |x-2|$ and $y = 3$ is

Answer (04)

Sol.



$= \dfrac{1}{2}[1+3]\times 2$

= 4

23. If the number of all 4 letter words with 2 vowels and 2 consonants from the word UNIVERSE is *n*, then *n* – 500 is

Answer (4)

Sol. Vowels → I, U, E, E

Consonants → N, V, R, S

(I) 2 Vowels some (II) 2 Vowels different

$4C_2 \times \dfrac{4!}{2!} = 72$ $\qquad$ $3C_2 \times 4C_2 \times 4! = 432$

72 + 432 = 504

24. Three dice are thrown. Then the probability that no outcomes is similar is $\dfrac{p}{q}$ then $q - p$ is (where *p* and *q* are co-prime)

Answer (04)

Sol. $P(E) = \dfrac{6\times5\times4}{6\times6\times6}$

$= \dfrac{20}{36} = \dfrac{5}{9} = \dfrac{p}{q}$

$q - p = 9 - 5 = 4$

25. $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 2qp$

$P^\alpha - P^\beta = \delta I - 13P$

Then find the value of $\alpha + \beta + \gamma - \delta$

Answer (24)

Sol. $P^3 = P - P^2$

$= P - (I - P) = 2P - I$

$P^4 = 2P^2 - P$

$= 2(I - P) - P = 2I - 3P$

Similarly

$P^6 = 5I - 8P$

and $P^8 = 13I - 21P$

$P^8 + P^6 = 18I - 29P$

$P^8 - P^6 = 8I - 13P$

$\alpha = 8, \beta = 6, \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

26.

27.

28.

29.

30.

❑ ❑ ❑