Aakash
+ BYJU'S

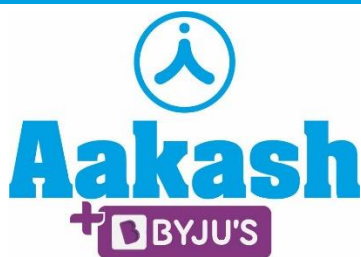*Corporate Office* **: Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456**

Memory Based
Answers & Solutions

Time : 3 hrs.   *for*   **M.M. : 300**

JEE (Main)-2024 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1)   The test is of **3 hours** duration.

(2)   The Test Booklet consists of 90 questions. The maximum marks are 300.

(3)   There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i)   **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii)   **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
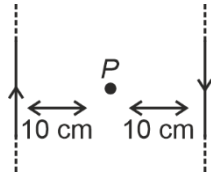
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Two infinite current carrying wires having current I in opposite directions are shown below. Find the magnetic field (in S.I. units) at point P.

(1) $\dfrac{7\mu_0 I}{\pi}$

(2) $\dfrac{10\mu_0 I}{\pi}$

(3) $\dfrac{5\mu_0 I}{\pi}$

(4) $\dfrac{\mu_0 I}{\pi}$

Answer (2)

Sol. $B = 2 \times \dfrac{\mu_0 i}{2\pi d} = 2 \times \dfrac{\mu_0 I}{2\pi \times 0.1} = \dfrac{10\mu_0 I}{\pi}$

2. If the diameter of earth becomes half keeping mass to be constant, then the acceleration due to gravity at surface of earth becomes

(1) Half

(2) Four times

(3) Twice

(4) Three times

Answer (2)

Sol. $g = \dfrac{GM}{R^2}$

$\Rightarrow \quad g \propto \dfrac{1}{R^2}$

$\Rightarrow \quad \dfrac{g'}{g} = \left(\dfrac{R}{\dfrac{R}{2}}\right)^2 = 4$

3. Two masses $m_1 = 4$ gm and $m_2 = 25$ gm are having same kinetic energy, find the ratio of magnitude of their linear momentum.

(1) 1 : 5

(2) 2 : 5

(3) 1 : 1

(4) 1 : 6

Answer (2)

Sol. $k = \dfrac{p^2}{2m}$

$\dfrac{p_1}{p_2} = \dfrac{\sqrt{m_1 k_1}}{\sqrt{m_2 k_2}}$ $\qquad (\because k_1 = k_2)$

$\dfrac{p_1}{p_2} = \sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$

$\therefore \quad p_1 : p_2 = 2 : 5$

4. A charge $Q = 10^{-6}$ C is placed at origin. Find the potential difference between two points A and B whose position vectors are $\left(\sqrt{3}\hat{i} + \sqrt{3}\hat{j}\right)$ m and $\left(\sqrt{6}\hat{j}\right)$ m respectively.

(1) Zero

(2) 1000 volts

(3) 2000 volts

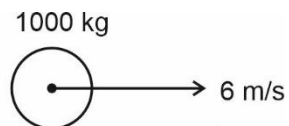(4) 500 volts

Answer (1)

Sol. $V = \dfrac{kQ}{r}$

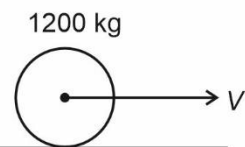Since $r_A = r_B$

$\Rightarrow \Delta V = 0$

5. A body of mass 1000 kg is moving horizontally with velocity 6 m/s. Another body of mass 200 kg is added gently. Then what will be its new velocity?

(1) 5 m/s

(2) 4 m/s

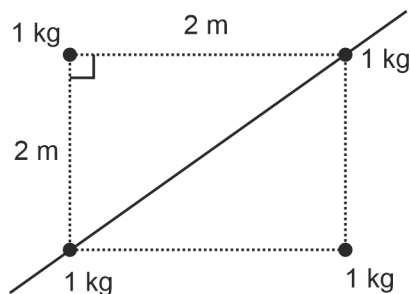(3) 2 m/s

(4) 3 m/s

Answer (1)

Sol. Initial =

Final =

From momentum conservation $\rightarrow P_i = P_f$,

$6 \times 1000 = 1200 V$

$V = 5$ m/s

6. Consider the system shown. Find the moment of inertia about the diagonal shown.

(1) 1 kg·m² (2) 2 kg·m²

(3) 4 kg·m² (4) 6 kg·m²

Answer (3)

Sol. $I = \sum m_i \cdot r_i^2$

$= 0 + 0 + 1(2\sin 45°)^2 \times 2$ kg·m²

$= 4$ kg·m²

7. A rod of length l having resistance R, is cut into two equal parts. These parts are connected in parallel then new resistance shall be

(1) R

(2) $\dfrac{R}{2}$

(3) $\dfrac{R}{4}$

(4) $2R$

Answer (3)

Sol. $R = f\dfrac{l}{A}$ $R \propto l$
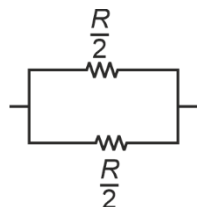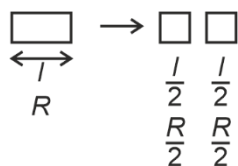
$\therefore$ Req $= \dfrac{R}{4}$

8. **Statement-I :** Linear momentum and moment of force have same dimensions.

Statement-II : Planck's constant and angular momentum have same dimension.

(1) Statement-I is correct while statement-II is false

(2) Statement-I is false while statement-II is correct

(3) Both statements are correct

(4) Both statements are false

Answer (2)

Sol. Linear momentum (p) $\Rightarrow$ [MLT⁻¹]

Angular momentum (L) $\Rightarrow$ [ML²T⁻¹]

Torque $\Rightarrow$ [ML²T⁻²]

Planck's constant $\Rightarrow$ [ML²T⁻¹]

9. In which of the following circuits the diode is reverse biased?
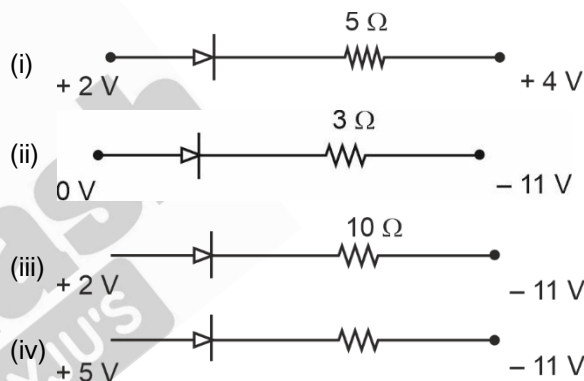
(1) (ii) (2) (i) and (iv)

(3) (iv) (4) (i)

Answer (4)

Sol. For reverse bias $V_P < V_N$

10. A prism has a refractive index $\cot\left(\dfrac{A}{2}\right)$, where A is the refracting angle of the prism. The minimum deviation due to this prism is

(1) $\pi - 3A$ (2) $\pi - 2A$

(3) A (4) $\dfrac{A}{2}$

Answer (2)

Sol. $\delta_{min.} = 2\sin^{-1}\left[\mu \sin\dfrac{A}{2}\right] - A$

$= 2\sin^{-1}\left[\cot\left(\dfrac{A}{2}\right)\sin\dfrac{A}{2}\right] - A$

$= \pi - 2A$

11. A particle performing simple harmonic motion in such that it's amplitude is 4 m and speed of particle at mean position is 10 m/s. Find the distance of particle from mean position where velocity became 5 m/s.

(1) $\sqrt{3}$ m

(2) $2\sqrt{3}$ m

(3) $\dfrac{\sqrt{3}}{2}$ m

(4) $\dfrac{1}{\sqrt{2}}$ m

Answer (2)

Sol. $v = \omega\sqrt{A^2 - x^2}$

In 1st case : at $x = 0$, $v = 10$ m/s

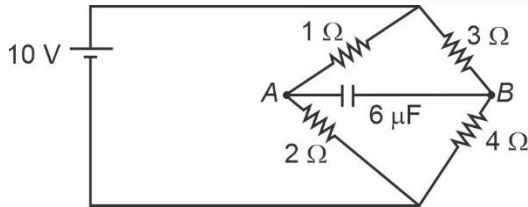then $10 = \omega\sqrt{(4)^2 - 0^2}$

$\omega = \dfrac{10}{4} = \dfrac{5}{2}$ rad/s

In 2nd case :

$5 = \dfrac{5}{2}\sqrt{(4)^2 - x^2}$

$x = 2\sqrt{3}$ m

12. Find charge on capacitor in the given circuit at steady state.

(1) $\dfrac{40}{7}\mu C$

(2) $\dfrac{20}{7}\mu C$

(3) $\dfrac{60}{7}\mu C$

(4) $\dfrac{10}{7}\mu C$

Answer (1)

Sol. $V_A = 10 - \dfrac{10}{3} = \dfrac{20}{3}$ V

$V_B = 10 - \dfrac{3}{7} \times 10 = \dfrac{40}{7}$ V

$V_A - V_B = 20\left[\dfrac{1}{3} - \dfrac{2}{7}\right]$

$= \dfrac{20}{21}$ V

$Q = CV_{AB}$

$= 6 \times \dfrac{20}{21}\mu C$

$= \dfrac{40}{7}\mu C$

13. A proton having velocity $\vec{v}_0$ passes through a region having electric field E and magnetic field B. If the velocity of proton does not change, then which of the following may be true?

(a) $E = 0$, $B = 0$

(b) $E = 0$, $B \neq 0$

(c) $E \neq 0$, $B = 0$

(d) $E \neq 0$, $B \neq 0$

(1) a, b, c, d

(2) a

(3) a, b, d

(4) a, b

Answer (3)

Sol. $\vec{F}_E = q\vec{E}$

$\vec{F}_B = q\left(\vec{v} \times \vec{B}\right)$

Case b is correct when $\vec{v} \parallel \vec{B}$

Case d is correct when $\vec{E} \perp \vec{B} \perp \vec{v}$ and $v = \dfrac{E}{B}$

14. A particle has initial ($t = 0$) velocity $\vec{u} = 5\hat{i}$ and is at origin at this instant. Its acceleration is given by $(3\hat{i} + 4\hat{j})$. When particle's x co-ordinate is 16 units, then its speed is

(1) 13 units

(2) $\sqrt{161}$ units

(3) 12 units

(4) $\sqrt{185}$ units

Answer (4)

Sol. $S = ut + \dfrac{1}{2}at^2$

$\Rightarrow 16 = 5t + \dfrac{3}{2}t^2$

$\Rightarrow t = 2$

$\Rightarrow \vec{v} = \vec{u} + \vec{a}t = 11\hat{i} + 8\hat{j}$

15. A spherometer is used to measure

(1) Radius of curvature of a lens

(2) Length of rod

(3) Density of a solid

(4) Viscosity of a liquid

Answer (1)

Sol. A spherometer is an instrument used for precise measurement of the radius of curvature of curved surface.

16. A particle performing simple harmonic motion according to $y = A\sin\omega t$. Then its kinetic energy (K.E.), potential energy (P.E.) and speed (V) at position $y = \dfrac{A}{2}$ are
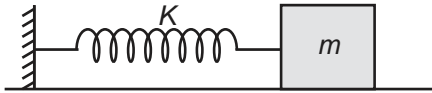
(1) K.E. $= \dfrac{kA^2}{8}$

   P.E. $= \dfrac{3kA^2}{8}$

   $V = \dfrac{A}{3}\sqrt{\dfrac{k}{m}}$

(2) K.E. $= \dfrac{3kA^2}{8}$

   P.E. $= \dfrac{kA^2}{8}$

   $V = \dfrac{A}{2}\sqrt{\dfrac{3k}{m}}$

(3) K.E. $= \dfrac{3kA^2}{8}$

   P.E. $= \dfrac{kA^2}{4}$

   $V = A\sqrt{\dfrac{3k}{m}}$

(4) K.E. $= \dfrac{kA^2}{4}$

   P.E. $= \dfrac{3kA^2}{8}$

   $V = \dfrac{A}{4}\sqrt{\dfrac{3k}{m}}$

Answer (2)

Sol. $V = \omega\sqrt{A^2 - x^2}$, $k = m\omega^2$, $\omega = \sqrt{\dfrac{k}{m}}$

$V = \sqrt{\dfrac{k}{m}(A^2 - x^2)}$

K.E. $= \dfrac{1}{2}mv^2$

$= \dfrac{1}{2}m.\dfrac{k}{m}(A^2 - x^2)$

$= \dfrac{k}{2}\left(A^2 - \dfrac{A^2}{4}\right)$

$= \dfrac{3kA^2}{8}$

P.E. $= \dfrac{1}{2}kx^2$

$= \dfrac{1}{2}k.\dfrac{A^2}{4}$

$= \dfrac{kA^2}{8}$

Speed (V) $= \sqrt{\dfrac{k}{m}\left(A^2 - \dfrac{A^2}{4}\right)}$

$= \sqrt{\dfrac{k}{m}\left(\dfrac{3A^2}{4}\right)}$

$= \left(\sqrt{\dfrac{3k}{m}}\right).\dfrac{A}{2}$

17. What should be the elevation of outer track of the train to move in a circular path of radius R, width of the track is w ($< < R$) and speed of the train is v ? (Neglect friction)
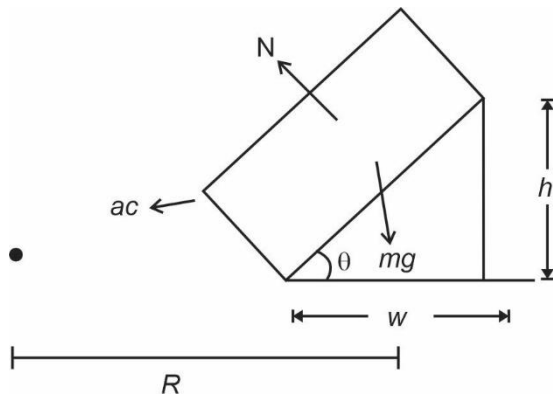
(1) $\dfrac{v^2 w}{Rg}$

(2) $\dfrac{v^2 w}{2Rg}$

(3) $\dfrac{gwv^2}{R}$

(4) $\dfrac{R}{gwv^2}$

Answer (1)

Sol.

$$N\sin\theta = \frac{mv^2}{R}$$

$$N\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{h}{w}$$

$$\therefore \boxed{h = \frac{v^2 w}{Rg}}$$

18. Out of air and liquid, which substance is more viscous?

    (1) Air

    (2) Liquid

    (3) Both have same viscosity

    (4) None of these

Answer (2)

Sol. In general, liquids are more viscous than air because of higher density and intermolecular forces.

19. A metallic frame of given dimension has area vector at 60° with external magnetic field as shown. The frame is taken out from the field in 10 seconds. Find arrange emf induced in the frame.
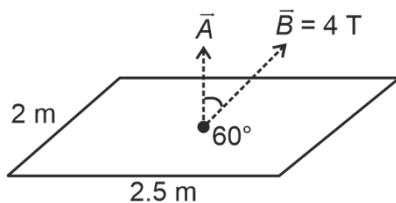
    (1) 1 V

    (2) 2 V

    (3) 3 V

    (4) 4 V

Answer (1)

Sol. $\phi = \vec{B} \cdot \vec{A}$

$$\Rightarrow \quad \varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{10}{10} \text{ V} = 1 \text{ V}$$

20. An electromagnetic wave is given as $E = 200 \sin (1.5x - 4.5 \times 10^8 \, t)$, here E is electric field in N/C. If energy density in electromagnetic field is given as $N \times 10^{-8}$ J/m³. Then N is

    ($\varepsilon_0 = 9 \times 10^{-12}$ SI units.)

    (1) 9

    (2) 18

    (3) 36

    (4) 72

Answer (2)

Sol. $\bar{\varepsilon} = \frac{1}{2}\varepsilon_0 E_{ms}^2 + \frac{B_{ms}^2}{2\mu_0}$

$$= \varepsilon_0 E_{ms}^2 = \frac{1}{2}\varepsilon_0 E_0^2$$

$$= \frac{1}{2} \times 9 \times 10^{-12} \times 200 \times 200$$

$$= \frac{36}{2} \times 10^{-8} = 18 \times 10^{-8} \text{ J/m}^3$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In the given meter bridge circuit, null point is found at 60 cm from end A. The unknown resistance S (in Ω) is
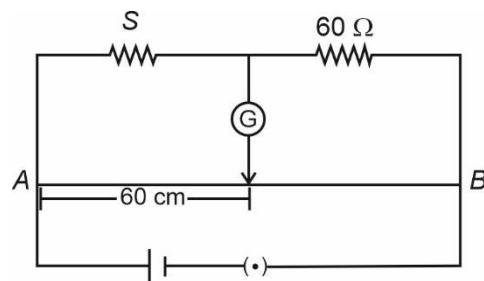
Answer (90.00)

Sol. $\dfrac{S}{60} = \dfrac{60}{(100-60)}$

$S = 90\ \Omega$

22. A particle is moving in one dimension, its displacement – time relation is given as $s = (2t^2 + 5)$ where s is in meters and t is in seconds. Find its velocity (in m/s) at $t = 1$ second.
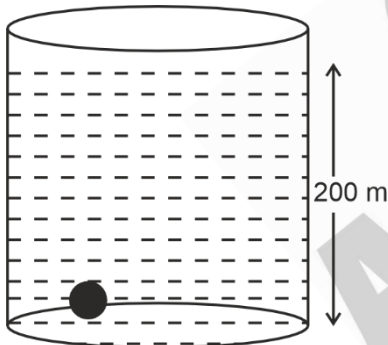
Answer (04.00)

Sol. $s = 2t^2 + 5$

$v = \dfrac{ds}{dt} = 4t$

at $t = 1$, $v = 4$ m/s

23. A sphere of small size is at the bottom of a lake of depth 200 m. Due to pressure its fractional change in volume is $\alpha \times 10^{-7}$. What is value of α, if bulk modulus of sphere is 5×10^{12} Pa? (Use $g = 10$ m/s^2)

Answer (04.00)

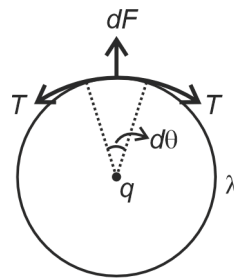**Sol.** $B = \dfrac{|\Delta P|}{\left|\dfrac{\Delta V}{V}\right|}$

$\left|\dfrac{\Delta V}{V}\right| = \dfrac{|\Delta P|}{B} = \dfrac{h\rho g}{B} = \dfrac{200 \times 10^3 \times 10}{5 \times 10^{12}} = 40 \times 10^{-8}$

$= 4 \times 10^{-7}$

$\Rightarrow \alpha = 4$

24. A ring has a uniformly distributed charge of 2π C and radius of 3 cm. A charge 10^{-6} C is placed at the centre of the ring. Tension developed in the ring is 10^X N. Find x.

Answer (07.00)

Sol.

$\Rightarrow dF = Td\theta$

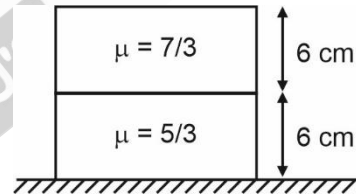$\Rightarrow Td\theta = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q \cdot \lambda R d\theta}{R^2}$

$\Rightarrow T = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q\lambda}{R}$

$= 9 \times 10^9 \times 10^{-6} \times \dfrac{2\pi}{2\pi R^2}$

$= \dfrac{9 \times 10^3}{9} \times 10^4$ N

$= 10^7$ N

25. Two slabs of same thickness of 6 cm each are placed over one other as shown on table.

Apparent depth of table surface is N cm. (N is nearest integer)

Answer (06.00)

Sol. $h_{app} = \dfrac{t_1}{\mu_1} + \dfrac{t_2}{\mu_2} = \dfrac{6}{7} \times 3 + \dfrac{6}{5} \times 3$

$= \dfrac{18}{7} + \dfrac{18}{5}$

$= 2.57 + 3.60 = 6.17$ cm $\Rightarrow N = 06$
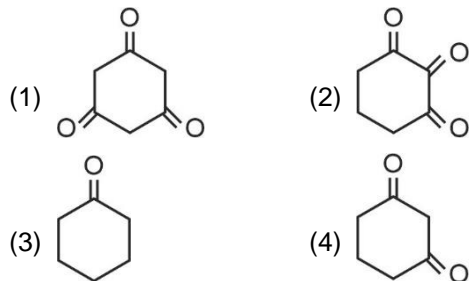
26.

27.

28.

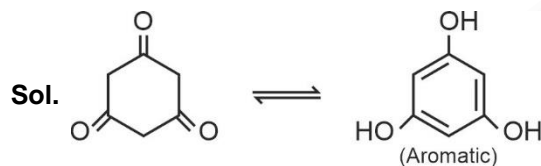29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.
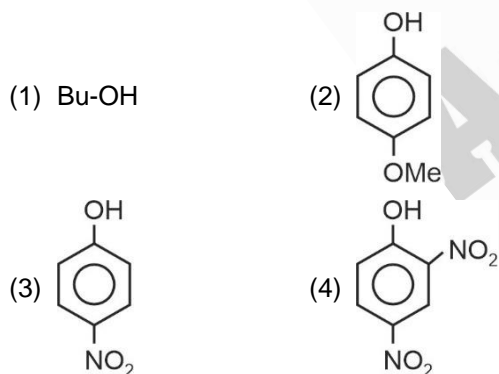
Choose the correct answer :

1. Which of the following has highest enol content?

(1)

(2)

(3)

(4)

Answer (1)

Sol.

(Aromatic)

2. Which of the following is most acidic?
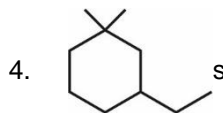
(1) Bu-OH

(2)

(3)

(4)

Answer (4)

Sol. Option (4) has 2 strong withdrawing groups at ortho/para thus conjugate base will be most stabilized.

3. Which of the following cannot show variable oxidation state?

(1) Chlorine   (2) Fluorine

(3) Bromine   (4) Iodine

Answer (2)

Sol. Fluorine has no vacant *d*-orbitals, so no electron excitation is possible and so it does not exhibit variable oxidation state.

4.

IUPAC name is

(1) 1-Ethyl-3,3-dimethyl cyclohexane

(2) 3-Ethyl-1,1-dimethyl cyclohexane

(3) 1-Ethyl-3,3-dimethyl cyclohexane

(4) 3-Ethyl-1,1-dimethyl cyclohexane

Answer (2)

Sol. Naming will be done in alphabetic order.
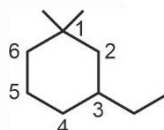
Correct IUPAC name : 3-Ethyl-1,1-dimethyl cyclohexane

5.

The given compound is

(1) Alicyclic   (2) Aromatic

(3) Antiaromatic   (4) Acyclic

Answer (1)

Sol. Given compound has a ring which is not aromatic or antiaromatic.

6. Which of the following is polar molecule?

(1) $CH_2 = CH_2$ (2) $CHCl_3$

(3) CCl_4 (4) CH_4

Answer (2)

Sol. Asymmetrical molecules can be polar.

$CH_2 = CH_2 \rightarrow$ Non polar

$\rightarrow$ Both have spherical symmetry

Polar

7. In which of the following compound central atom has +4 oxidation state?

   (1) SO_3
   (2) H_2SO_3
   (3) $H_2S_2O_7$
   (4) $BaSO_4$

Answer (2)

Sol. H_2SO_3

   $+1 \times 2 + x + (-2) \times 3 = 0$

   $2 + x - 6 = 0$

   $x = +4$

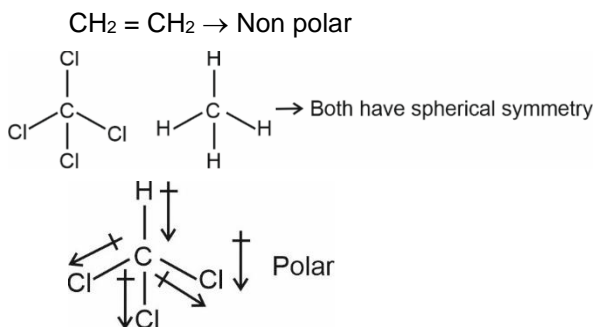   In H_2SO_3, sulphur present in +4 oxidation state.

8. It is given that radius of 3rd stationary orbit is r, find out radius of 4th stationary orbit.

   (1) $\dfrac{16r}{9}$
   (2) $\dfrac{6r}{16}$
   (3) $\dfrac{4r}{3}$
   (4) $\dfrac{3r}{4}$

Answer (1)

Sol. $r \propto \left(\dfrac{n^2}{z}\right)$

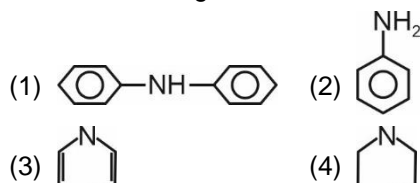 $r = \dfrac{(K)(3)^2}{1}$

   $K = \dfrac{r}{9}$

   $r_4 = \left(\dfrac{r}{9}\right)\left(\dfrac{16}{1}\right)$

   $= \dfrac{16r}{9}$

   Correct answer is option (1)

9. Select the strongest Bronsted base.

   (1) ⬡—NH—⬡
   (2) (aniline with NH_2)
   (3) (pyrrole)
   (4) (pyrrolidine)

Answer (4)

Sol. In case of 1, 2 and 3 the lone pair is delocalised due to resonance so the 4 has highest availability of lone pair and it is best proton acceptor.

10. The electronic configuration of Neodymium (60) (Nd) is

    (1) $[Xe]4f^46s^2$
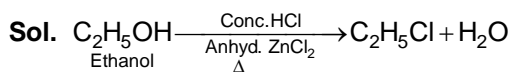   (2) $[Xe]5f^1$
    (3) $[Xe]4f^26s^2$
    (4) $[Xe]5f^44d^1$

Answer (1)

Sol. The electronic configuration of Neodymium is $4f^46s^2$

11. Ethanol shows turbidity with Lucas reagent (Conc. HCl + Anhyd. $ZnCl_2$).

    (1) Immediately
    (2) After 5 to 7 minutes
    (3) Upon heating
    (4) After 10-12 minutes

Answer (3)

Sol. $\underset{\text{Ethanol}}{C_2H_5OH} \xrightarrow[\underset{\Delta}{\text{Anhyd. ZnCl}_2}]{\text{Conc.HCl}} C_2H_5Cl + H_2O$

12. Which type of linkage is present in Nucleotide between base and sugar?

    (1) Peptide linkage
    (2) Glycosidic linkage
    (3) N-glycosidic linkage
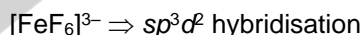   (4) Amide linkage

Answer (3)

Sol. The linkage between nitrogenous base and pentose sugar in nucleotide is N-glycosidic linkage.

13. A complex with maximum spin angular momentum

    (1) $[FeF_6]^{3-}$
    (2) $[Fe(CN)_6]^{3-}$
    (3) $[Fe(H_2O)_6]^{2+}$
    (4) $[V(H_2O)_6]^{2+}$

Answer (1)

Sol. $F^{\ominus}$ with Fe^{+3} behaves as WFL, Hence pairing does not take place, so it forms high spin complex.
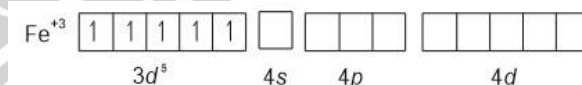
$[FeF_6]^{3-} \Rightarrow sp^3d^2$ hybridisation

Number of unpaired electron = 5

$\left[Fe(CN)_6\right]^{3-} \Rightarrow d^2sp^3$ hybridisation

$Fe^{+3} = 3d^5$

Number of unpaired electron = 1

$[Fe(H_2O)_6]^{2+} \Rightarrow sp^3d^2$

$Fe^{+2} = 3d^6$

Number of unpaired electron = 4

$[V(H_2O)_6]^{+2} \Rightarrow d^2sp^3$ hybridisation

$V^{+2} = 3d^3$
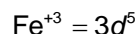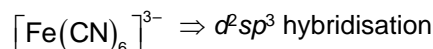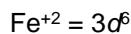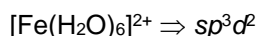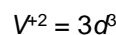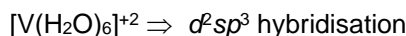
Number of unpaired electron = 3

Spin angular momentum = $\sqrt{S(S+1)}\, \dfrac{h}{2\pi}$

S = total spin quantum no.

More the number of unpaired electron more will be spin angular momentum. $[FeF_6]^{3-}$ has 5 unpaired electron hence maximum spin angular momentum value.

14. Calculate the temperature (in K) at which kinetic energy of mono-atomic gaseous molecule is equal to 0.414 eV

(1) 3199 K
(2) 319.8 K
(3) 2500 K
(4) **2900** K

Answer (1)

Sol. $(KE)_{atom} = \dfrac{3}{2}\left(\dfrac{R}{N_A}\right)(T)$

$0.414 \times 1.6 \times 10^{-19} = \dfrac{3}{2} \times \left(\dfrac{8.314}{6.022 \times 10^{23}}\right) \times T$

$T = \dfrac{(0.414) \times 1.6 \times 2 \times 6.022 \times 10^4}{3 \times 8.314}$

$\approx 3198.59\,K$

15. A solution of two volatile components showing negative deviation from Raoult's law shows:-

(1) Decrease in vapour pressure, boiling point increases

(2) Increase in vapour pressure boiling point decreases

(3) Decrease in vapour pressure, boiling point decreases

(4) Increase in vapour pressure boiling point increases

Answer (1)

Sol. In case of negative deviation from Raoult's law the vapour pressure is less than expected from Raoult's law and B.P. is more.

V.P

$x_A = 1$
$x_B = 0$

$x_A = 0$
$x_B = 1$

16. During S_N1 reaction which of the following statement is correct

(1) Inversion occurs
(2) Retention occurs
(3) Almost racemization
(4) 100% racemization

Answer (3)

Sol. During S_N1 reaction, attack of nucleophile on carbocation is slightly favoured from opposite side of leaving group due to intimate ion pair.

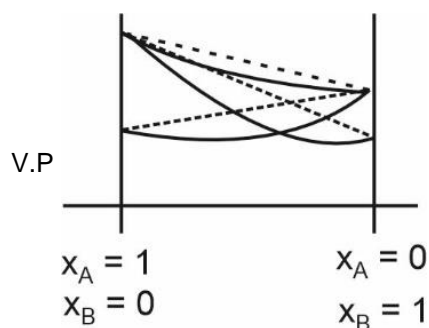17. **Assertion :** Boron is hard element.

Reason : Boron has unusually high melting point.

(1) Assertion is correct. Reason is correct and reason explains assertion.

(2) A is correct. R is correct R does not explains A.

(3) A is correct but R is incorrect.

(4) A is incorrect but R is correct.

Answer (2)

Sol. Both assertion and reason are true but reason is not the correct explanation of assertion.

18. $PbCrO_4 \xrightarrow{NaOH} Complex$
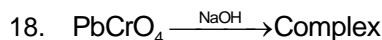
Complex is

(1) Dianionic with CN = 6
(2) Dianionic with CN = 4
(3) Neutral with CN = 4
(4) Trianionic with CN = 6

Answer (2)

Sol. $PbCrO_4 + 4NaOH \longrightarrow Na_2CrO_4 + Na_2[Pb(OH)_4]$
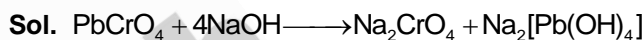
Complex is $Na_2[Pb(OH)_4]$ i.e. $[Pb(OH)_4]^{2-}$

Dianionic with CN = 4

19. Which of the following configuration has strongest metallic bonding?

(1) $[Ar]3d^74s^2$
(2) $[Ar]3d^54s^1$
(3) $[Ar]3d^64s^2$
(4) $[Ar]3d^84s^2$

Answer (2)

Sol. More the number of unpaired electrons, more strong the metallic bonding.

Maximum unpaired e^- in $[Ar]3d^54s^1$

$\Rightarrow$ 6 unpaired e^-

20. **Assertion :** All *s*-block elements are found in nature.

Reason : 4*f* and 5*f* series are kept below periodic table.

(1) Assertion and reason, both are true and reason is correct explanation of assertion.

(2) Assertion and reason, both are true and reason is not correct explanation of assertion.

(3) Assertion is true, but reason is false.

(4) Assertion is false but reason is true.

Answer (2)

Sol. All *s*-block elements have some abundance in nature Lanthanides and Actinoids are kept below periodic table.

SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find out sum of bond order of CO & NO⁺.

Answer (6)

Sol. CO and NO⁺ both are isoelectronic and each of them is having bond order 3 that can be explained by molecular orbital theory. The sum of bond order of CO & NO⁺ will be 6.

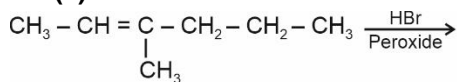22. Calculate mass of CH_4 consumed for the formation of 22 g CO_2.

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$
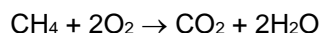
Answer (8)

Sol. Mass of CO_2 produced = 22 g

Moles of CO_2 produced $= \dfrac{22}{44}$ mol
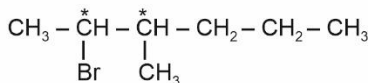
$= 0.5$ mol

1 mol of CO_2 produced by 1 mol of CH_4

0.5 mol of CO_2 can be produced by 0.5 mol of CH_4

Mole of CH_4 consumed = 0.5 mol

Mass of CH_4 consumed = 0.5 × 16 g

$= 8$ g

23. Find out number of stereoisomers obtained when 3-methylhex-2-ene reacts with HBr in presence of peroxide.

Answer (4)

$$CH_3 - CH = C - CH_2 - CH_2 - CH_3 \xrightarrow[\text{Peroxide}]{\text{HBr}}$$
$$\underset{CH_3}{|}$$

Sol.

$$CH_3 - \overset{*}{C}H - \overset{*}{C}H - CH_2 - CH_2 - CH_3$$
$$\underset{Br}{|} \quad \underset{CH_3}{|}$$

4-Isomers are possible.

24. Among the following number of meta directing groups are:

$-CN, -NO_2, -COOH, -CH_3, -SO_3H, -\overset{+}{N}H_3, -F$

Answer (5)

Sol.

$\left.\begin{array}{l} -CN \\ -NO_2 \\ -COOH \\ -SO_3H \\ -NH_3^{\oplus} \end{array}\right\}$ Deactivating and meta directing

$\left.\begin{array}{l} -CH_3 \\ -F \end{array}\right\}$ o-p, directing
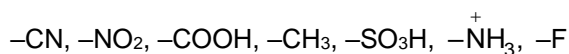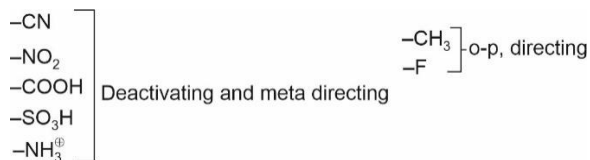
25. We are given with following information about concentration of reactant with initial rate of reaction.

Initial concentration	Initial rate
0.005 M	7.5×10^{-4}
0.02 M	3×10^{-3}

Find out order of reaction with respect to that reactant.

Answer (01.00)

Sol. Rate becomes 4 times on increasing concentration of reactant 4 times.

$$\left(\dfrac{3 \times 10^{-3}}{7.5 \times 10^{-4}}\right) = \left(\dfrac{0.02}{0.005}\right)^n$$

$n = 1$

∴ Correct answer is 1.

26. How many of the following are aromatic compounds?
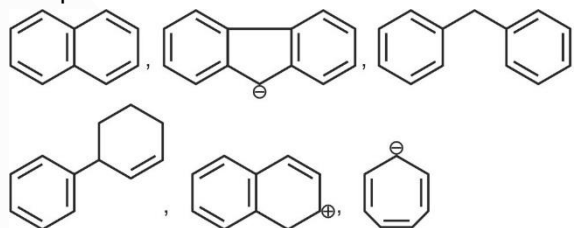
Answer (05.00)

Sol. is antiaromatic as it has 8π electrons. The remaining 5 compounds are aromatic as they have $4n + 2$ delocalising π-electrons associated to that ring.

27. Calculate number of electron for which $n = 4$ and $S = +\dfrac{1}{2}$

Answer (16)

Sol. For $n = 4$

$l = 0, 1, 2, 3$

Possible subshells are

$4s$, $4p$, $4d$ and $4f$

Number of electron have $S = +\dfrac{1}{2}$

$4s = 1$

$4p = 3$

$4d = 5$

$4f = 7$

Total number of electron with $S = +\dfrac{1}{2}$ for $n = 4$

$= 16$

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

   $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$, $\vec{a} \cdot \vec{c} = 3$

   $\vec{a} \times \vec{c} = \vec{b}$ then $\vec{a} \cdot \left[(\vec{c} \times \vec{b}) - \vec{b} - \vec{c} \right] =$

   (1) 24   (2) 38

   (3) 10   (4) None of these

Answer (1)

Sol. $\vec{a} \cdot \left[(\vec{c} \times \vec{b}) - \vec{b} - \vec{c} \right]$

$\Rightarrow \vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$ ...(i)

Now $\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} = \vec{b} \cdot \vec{b} = |\vec{b}|^2 = 27$

$\Rightarrow \left[\vec{a} \, \vec{c} \, \vec{b} \right] = 27$

From (i)

$27 - 0 - 3 = 24$

∴ Option (1) is correct

2. The vertices of a triangle ABC are $A(1, 2)$, $B(-3, 4)$, $C(5, 8)$, then orthocentre of $\triangle ABC$ is

   (1) $\left(\dfrac{2}{3}, 1 \right)$ (2) $\left(-\dfrac{7}{3}, 2 \right)$

   (3) $(2, 3)$ (4) $\left(\dfrac{3}{2}, 1 \right)$

Answer (4)

Sol.
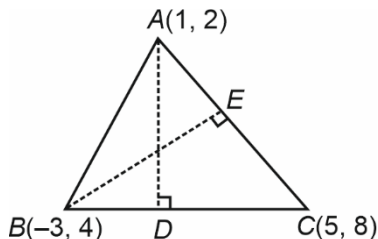
$AD : (y - 2) = -2(x - 1)$

$BE : (y - 4) = -\dfrac{2}{3}(x + 3)$

Intersection of AD and BE : $H\left(\dfrac{3}{2}, 1 \right)$

3. $S_1 = 3, 9, 15, \ldots 25$ terms

   $S_2 = 3, 8, 13, \ldots 37$ terms

   Number of common terms in S_1, S_2 is equal to

   (1) 3   (2) 4

   (3) 5   (4) 6

Answer (3)

Sol. $S_1 = 3, 9, 15 \ldots 147$

$d_1 = 6$, $a_1 = 3$

$S_2 = 3, 8, 13, \ldots 183$

$d_2 = 5$, $a_2 = 3$

LCM $(d_1, d_2) = 30$

∴ $3, 33, \ldots 123$

∴ $123 = 3 + (n - 1)30$

$\Rightarrow n = 5$

∴   Number of common terms = 5

4. $\displaystyle\int_0^1 \dfrac{1}{\sqrt{3 + x} + \sqrt{1 + x}} dx = a + b\sqrt{2} + c\sqrt{3}$, then

   $2a - 3b - 4c$ is equal to

   (1) 10   (2) 0

   (3) 12   (4) 20

Answer (3)

Sol. $I = \displaystyle\int_0^1 \dfrac{1}{\sqrt{x + 3} + \sqrt{x + 1}} dx$

On rationalization

$I = \displaystyle\int_0^1 \dfrac{\sqrt{x + 3} - \sqrt{x + 1}}{2} dx$

$I = \displaystyle\int_0^1 \dfrac{(x + 3)^{1/2}}{2} dx - \int_0^1 \dfrac{(x + 1)^{1/2}}{2} dx$

$I = \dfrac{(x + 3)^{3/2}}{3} \Big|_0^1 - \dfrac{(x + 1)^{3/2}}{3} \Big|_0^1$

$= \dfrac{1}{3}\left[4^{3/2} - 3^{3/2} \right] - \dfrac{1}{3}\left[2^{3/2} - 1^{3/2} \right]$

$= \dfrac{8 - 3\sqrt{3} - 2\sqrt{2} + 1}{3} = 3 - \sqrt{3} - \dfrac{2}{3}\sqrt{2}$

$\Rightarrow 2a = 6$

$3b = -2$

$4c = -4$

$2a - 3b - 4c = 6 + 2 + 4 = 12$

5. If $^{n-1}C_r = (k^2 - 8)\,^nC_{r+1}$ then

   (1) $k \in \left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

   (2) $k \in \left[-4, -2\sqrt{3}\right) \cup \left(2\sqrt{3}, 4\right]$

   (3) $k \in \left[-2\sqrt{3}, 4\right]$

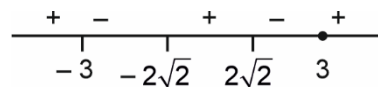   (4) $k \in \left[3, 2\sqrt{3}\right]$

Answer (1)

Sol. $^{n-1}C_r = (k^2 - 8) \times \dfrac{n}{r+1}\,^{n-1}C_r$

$\dfrac{1}{k^2 - 8} = \dfrac{n}{r+1}$ $\qquad (n \geq r+1)$

$\Rightarrow \dfrac{1}{k^2 - 8} \geq 1$

$\dfrac{1 - (k^2 - 8)}{k^2 - 8} \geq 0$

$\dfrac{k^2 - 9}{k^2 - 8} \leq 0$

$k \in \left[-3, 2 - \sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

6. The value of k for $(2k, 3k)$, $(0, 0)$, $(1, 0)$ and $(0, 1)$ to be on the circle is

   (1) $\dfrac{2}{13}$ $\qquad$ (2) $\dfrac{5}{13}$

   (3) $\dfrac{1}{13}$ $\qquad$ (4) $\dfrac{2}{13}$

Answer (2)

Sol. Circle passing through $(0, 0)$, $(1, 0)$ and $(0, 1)$ will be a circle having $(1, 0)$ and $(0, 1)$ as the end points of diameter.

$C: (x - 1)x + y(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

Now $(2k, 3k)$ lies on C.

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$k(13k - 5) = 0$

$\Rightarrow k = \dfrac{5}{13}$

7. Shortest distance between the parabola $y^2 = 4x$ and $x^2 + y^2 - 4x - 16y + 64 = 0$ is equal to

   (1) $2\sqrt{3} - 2$ $\qquad$ (2) $3\sqrt{2} - 3$

   (3) $4\sqrt{5} - 2$ $\qquad$ (4) $2\sqrt{5} - 2$

Answer (4)

Sol. $N : y = mx - 2am - am^3$

$N : y = mx - 2m - m^3$

It passes through $(2, 8)$

$8 = 2m - 2m - m^3$

$\Rightarrow m = -2$

$\therefore\ N : y + 2x = 12$

Point of intersection of normal with $y^2 = 4x$ is $(4, 4)$

$\therefore$ Shortest distance : $\sqrt{(4-2)^2 + (4-8)^2} - \sqrt{4 + 64 - 64}$

$= \sqrt{20} - 2$

$= 2\sqrt{5} - 2$

8. Let $f(x) = \begin{cases} 2^{\frac{\sin(x-3)}{x - [x]}} & x < 3 \\[2mm] \dfrac{a\left|-x^2 - 12 + 7\right|}{b\left(x^2 + 12x - 7\right)} & x > 3 \\[2mm] b & x = 3 \end{cases}$

   is continuous at $x = 3$, then (a, b) is

   (1) $(2, 3)$ $\qquad$ (2) $(1, 2)$

   (3) $(2^{\sin 2}, 3)$ $\qquad$ (4) None of these

Answer (4)

Sol. $\displaystyle\lim_{x \to 3^-} f(x)$

$\displaystyle\lim_{x \to 3^-} 2^{\frac{\sin(x-3)}{\{x\}}} = 2^0 = 1 = b = f(3)$

$\therefore\ b = 1$

$= \displaystyle\lim_{x \to 3^+} \dfrac{a}{b} \dfrac{\left|-x^2 - 12x + 7\right|}{\left(x^2 + 12x - 7\right)}$

$= \dfrac{a}{b}(1)$

$\displaystyle\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) = f(3)$

$\dfrac{a}{b} = 1 = b$

$\Rightarrow a = 1$

$\therefore\ (a, b) \equiv (1, 1)$

$\therefore\ $ 4th option is correct

9. If $f(x) - f(y) = \ln\left(\dfrac{x}{y}\right) + x - y$, then find $\displaystyle\sum_{k=1}^{20} f'\left(\dfrac{1}{k^2}\right)$

   (1) 2890
   (2) 2390
   (3) 1245
   (4) None of these

Answer (1)

Sol. Rearranging,

$f(x) - \ln(x) - x = f(y) - \ln y - y$

$\Rightarrow f(x) - \ln(x) - x = c$ (some constant)

$\Rightarrow f(x) = c + x + \ln x$

$f'(x) = 0 + 1 + \dfrac{1}{x}$

$f'\left(\dfrac{1}{k^2}\right) = 1 + \dfrac{1}{\frac{1}{k^2}} = (1 + k^2)$

$\displaystyle\sum_{k=1}^{20}(1 + k^2) = \sum_{k=1}^{20} 1 + \sum_{k=1}^{20} k^2$

$\qquad = 20 + \dfrac{20 \times 21 \times 41}{6}$

$\qquad = 20 + 2870 = 2890$

10. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

    Then

    S-I : $f(x).f(y) = f(x + y)$.

    S-II : $f(-x) = 0$ is invertible.

    (1) S-I True, S-II False
    (2) S-I True, S-II True
    (3) S-I False, S-II True
    (4) S-I False, S-II False

Answer (2)

Sol. $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

S-I:

$f(x).f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y)$

$\therefore$ S-I is true

Now,

$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\det(f(-x)) = \cos^2 x + \sin^2 x = 1$

$\Rightarrow |f(-x)| \neq 0$

$\therefore$ Non-singular

$\therefore$ S-II is true

11. If $\displaystyle\lim_{x \to 0} \dfrac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4} = A$ and

    $\displaystyle\lim_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}} = B$ then $AB^3 =$

    (1) 8
    (2) 32
    (3) 6
    (4) None of these

Answer (2)

Sol. $A = \displaystyle\lim_{x \to 0} \dfrac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$

$= \displaystyle\lim_{x \to 0} \dfrac{1+\sqrt{1+x^4} - 2}{x^4 \times \left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$

$= \displaystyle\lim_{x \to 0} \dfrac{1+x^4 - 1}{x^4 \times \left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4} + 1\right)}$

$A = \dfrac{1}{2\sqrt{2} \times 2} = \dfrac{1}{4\sqrt{2}}$

$B = \displaystyle\lim_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$

$= \displaystyle\lim_{x \to 0} \dfrac{\sin^2 x(\sqrt{2} + \sqrt{1+\cos x})}{2 - (1+\cos x)}$

$= \displaystyle\lim_{x \to 0} \dfrac{\sin^2 x}{1 - \cos x}(\sqrt{2} + \sqrt{1+\cos x})$

$= \displaystyle\lim_{x \to 0} \dfrac{\sin^2 x}{2\sin^2 \frac{x}{2}}(\sqrt{2} + \sqrt{1+\cos x})$

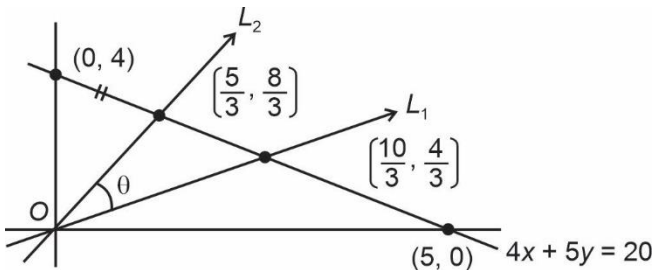$= \dfrac{1}{2 \times \frac{1}{4}} \times 2\sqrt{2}$

$B = 4\sqrt{2}$

$AB^3 = \dfrac{1}{4\sqrt{2}} \times \left(4\sqrt{2}\right)^3 = \left(4\sqrt{2}\right)^2 = 32$

12. Two lines L_1 and L_2 passing through origin trisecting the line segment intercepted by the line $4x + 5y = 20$ between the coordinate axes. Then the tangent of angle between the lines L_1 and L_2 is

(1) $\sqrt{3}$ 　　　　(2) $\dfrac{1}{\sqrt{3}}$

(3) 1 　　　　(4) $\dfrac{30}{41}$

Answer (4)

Sol.

$\Rightarrow \; m_1 = \dfrac{4/3}{10/3} = \dfrac{2}{5}$ (Slope of line 1)

$m_2 = \dfrac{8/3}{5/3} = \dfrac{8}{5}$ (Slope of line 2)

Angle between L_1 and L_2 is

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\dfrac{\dfrac{8}{5} - \dfrac{2}{5}}{1 + \dfrac{8}{5} \times \dfrac{2}{5}}\right|$

$= \left|\dfrac{\dfrac{6}{5}}{\dfrac{25+16}{25}}\right| = \dfrac{30}{41}$

13. If $S = \{z : |z + i| = |z - i| = |z - 1|, \; z \in c\}$, then number of elements in set S is equal to

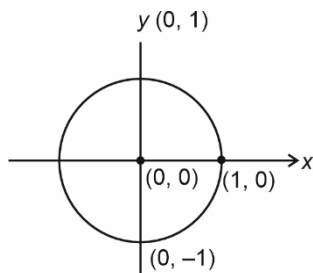(1) 01 　　　　(2) 02

(3) 03 　　　　(4) 04

Answer (01)

Sol. z will be circumcentre of the triangle with vertices i, $-i$ and 1. (which is unique)

$\Rightarrow \; z = 0 + 0i$

Only one element exists in S

14. If $\cos 2x - a\sin x = 2a - 7$, then range of a is

(1) $-2 \le a \le 0$ 　　　(2) $2 \le a \le 6$

(3) $a \ge 6$ 　　　(4) $6 \le a \le 8$

Answer (2)

Sol. $1 - 2\sin^2 x - a\sin x = 2a - 7$

$2\sin^2 x + a\sin x + 2a - 8 = 0$

$\sin x = \dfrac{-a \pm \sqrt{a^2 - 8(2a-8)}}{4}$

$= \dfrac{-a \pm |a - 8|}{4}$

$= -2$ or $\dfrac{8 - 2a}{4}$

(-2) Not possible

$\therefore \; \sin x = \dfrac{8 - 2a}{4}$

$-1 \le \dfrac{8 - 2a}{4} \le 1$

$-4 \le 8 - 2a \le 4$

$2 \le a \le 6$

15. $A = \{1, 2, 3, \ldots 10\}$, S be the set of subset of A and $R = \{(a, b) : a, b \in S \text{ and } a \cap b \ne \phi\}$, then R is

(1) Reflexive only

(2) Symmetric only

(3) Symmetric and transitive

(4) Transitive only

Answer (2)

Sol. When $a = \phi$, then $a \cap a = \phi$,

hence $(a, a) \notin R$ $\therefore$ R is not reflexive

as $(a, a) \ne \phi$ $\forall$ $a \in S$ is not true

for $(a, b) \in R \Rightarrow (b, a) \in R$ $\forall$ $a, b \in S$

as $a \cap b \ne \phi \Rightarrow b \cap a \ne \phi$

$\therefore$ R is symmetric

For transitive,

Let $b = \{1, 2\}$, $c = \{1, 3, 4\}$, $d = \{4, 5, 6\}$

$b \cap c \ne \phi$, $c \cap d \ne \phi$, $b \cap d = \phi$

$\therefore$ $(b, c) \in R$, $(c, d) \in R \nRightarrow (b, d) \in R$ $\forall$ $b, c, d \in R$

$\therefore$ R is not transitive

Option (2) is correct.

16. The shortest distance between the line $\dfrac{x-1}{2} = \dfrac{y+1}{4} = \dfrac{z-1}{3}$ and $\dfrac{2x-1}{5} = \dfrac{y-2}{3} = \dfrac{z}{6}$ is equal to

(1) 10 unit

(2) $\dfrac{7}{10}$ unit

(3) 0 unit

(4) $\dfrac{34}{\sqrt{1045}}$ unit

Answer (4)

Sol. $\dfrac{x-1}{2} = \dfrac{y+1}{4} = \dfrac{z-1}{3}$ and

$\dfrac{x-\frac{1}{2}}{\left(\frac{5}{2}\right)} = \dfrac{y-2}{3} = \dfrac{z}{6}$,

$\vec{a_1} - \vec{a_2} = \dfrac{1}{2}\hat{i} - 3\hat{j} + \hat{k}$

$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 3 \\ \frac{5}{2} & 3 & 6 \end{vmatrix}$

$= 15\hat{i} - \dfrac{9}{2}\hat{j} + (-4k)$

$\vec{d} = \left| \dfrac{(\vec{a_1} - \vec{a_2}) \cdot (\vec{n_1} \times \vec{n_2})}{|\vec{n_1} \times \vec{n_2}|} \right|$

$= \left| \dfrac{\frac{15}{2} + \frac{27}{2} - 4}{\sqrt{225 + \frac{81}{4} + 16}} \right|$

$= \dfrac{\frac{34}{2}}{\sqrt{\frac{1045}{4}}} = \dfrac{34}{\sqrt{1045}}$ unit

17. If α is a root of $x^2 + x + 1 = 0$ satisfying $(1 + \alpha)^7 = a + b\alpha + c\alpha^2$, then the ordered triplet (a, b, c) is

(1) (2, 3, 4)

(2) (1, 3, 5)

(3) (3, 3, 2)

(4) (−1, 5, 4)

Answer (3)

Sol. $x^2 + x + 1 = 0$ ⟶ $\begin{matrix} \omega \\ \omega^2 \end{matrix}$ roots, $1 + \omega + \omega^2 = 0$, $\omega^3 = 1$

⟹ $(1 + \alpha)^7 = (1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2$

⟹ Since $1 + \omega + \omega^2 = 0$

⟹ $(1 + \alpha)^7 = \lambda(1 + \omega + \omega^2) - \omega^2 = a + b\omega + c\omega^2$

⟹ $\lambda + \lambda\omega + (\lambda - 1)\omega^2 = a + b\omega + c\omega^2$

⟹ $(\lambda, \lambda, \lambda - 1)$, will be ordered triplets where $\lambda \in R$.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, −00.33, −00.30, 30.27, −27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find P if

$$3 + \dfrac{1}{4}(3 + P) + \dfrac{1}{4^2}(3 + 2P) + ... \infty = 8$$

Answer (9)

Sol. $3 + \dfrac{1}{4}(3 + P) + \dfrac{1}{4^2}(3 + 2P) + ... \infty = 8$

$\left(3 + \dfrac{3}{4} + \dfrac{3}{4^2} + ...\right) + \underbrace{\left(\dfrac{P}{4} + \dfrac{2P}{4^2} + ...\right)}_{k} = 8$

$3\left[\dfrac{1}{1 - \frac{1}{4}}\right] + k = 8$

$4 + k = 8$

Now,

$k = \dfrac{P}{4} + \dfrac{2P}{4^2} + \dfrac{3P}{4^3} + ...$

$\dfrac{k}{4} = \qquad \dfrac{P}{4^2} + \dfrac{2P}{4^3} + ...$

$\overline{\dfrac{3k}{4} = \dfrac{P}{4} + \dfrac{P}{4^2} + \dfrac{P}{4^3} + ...}$

$\dfrac{3k}{4} = \dfrac{\frac{P}{4}}{1 - \frac{1}{4}}$

$k = \dfrac{4}{9}P$

$4 + \dfrac{4}{9}P = 8$

$P = 9$

22. If $f(x) = x^3 + 2x^2 f'(1) + x f''(2) + f'''(3)$. The value of $f'(10)$ is equal to _____.

Answer (218)

Sol. $f(x) = 3x^2 + 4x\,f'(1) + f''(2)$

$f'(1) = 3 + 4\,f'(1) + f''(2)$ ....(i)

$f'(x) = 6x + 4\,f'(1)$

$f''(2) = 12 + 4\,f'(1)$ ....(ii)

$f''(x) = 6 \Rightarrow f''(3) = 6$

using (i) and (ii)

$3f'(1) + f''(2) = -3$

$\Rightarrow 3f'(1) + 12 + 4\,f'(1) = -3$

$\Rightarrow 7f'(1) = -15$

$\Rightarrow \boxed{f'(1) = \dfrac{-15}{7}} \Rightarrow \boxed{f''(2) = 12 - \dfrac{60}{7} = \dfrac{24}{7}}$

$\Rightarrow f(x) = x^3 + 2x^2\left(-\dfrac{15}{7}\right) + x\cdot\dfrac{24}{7} + 6$

$f'(x) = 3x^2 - \dfrac{60}{7}x + \dfrac{24}{7}$

$f'(10) = 300 - \dfrac{600}{7} + \dfrac{24}{7}$

$f'(10) = 217.7142$

$f'(10) = 217.7$

23. Given $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $B = [B_1\ B_2\ B_3]$

Which satisfying the conditions

$A \cdot B_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

and $\alpha = |B|$, β = Diagonal sum of matrix B

Then the value of $\alpha^3 + \beta^3$ equals to.

Answer (117)

Sol. $B = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$, $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Rightarrow B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$A \cdot B_1 = \begin{bmatrix} 2x_1 + y_1 \\ z_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow 2x_1 + y_1 = 2$

$z_1 = 3$

$x_1 = 1$

$\Rightarrow x_1 = 1,\ y_1 = 0,\ z_1 = 3$

$A \cdot B_2 = \begin{bmatrix} 2x_2 + y_2 \\ z_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_2 + y_2 = 2,\ z_2 = 0,\ x_2 = 0$

$x_2 = 0$

$z_2 = 0$

$y_2 = 2$

$A \cdot B_3 = \begin{bmatrix} 2x_3 + y_3 \\ z_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$2x_3 + y_3 = 3,\ z_3 = 2,\ x_3 = 1$

$x_3 = 1$

$z_3 = 2$

$y_3 = 1$

$\Rightarrow B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 3 & 0 & 2 \end{bmatrix}$

$\alpha = |B| = 4 - 6 = -2$

$\beta = 5$

$\alpha^3 + \beta^3 = -8 + 125$

$= 117$

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