IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt any 05 questions out of 10 Questions.

(6) Section - A (01 – 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

(7) Section - B (21 – 30) contains 10 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. For a given single electron atom, ratio of shortest wavelengths in Balmer and Lyman series is
   (1) 4 : 1  (2) 1 : 4  (3) 1 : 2  (4) 2 : 1

Answer (1)

Sol. \[ \frac{1}{\lambda_L} = RZ^2 \left(1 - \frac{1}{\infty}\right) \]
\[ \frac{1}{\lambda_B} = RZ^2 \left(1 - \frac{1}{4} - \frac{1}{\infty}\right) \]
\[ \frac{\lambda_B}{\lambda_L} = 4 \]

2. The value of unknown resistance \( x \) for which potential difference between point \( B \) and \( D \) is zero is

(1) 12 \( \Omega \)  (2) 6 \( \Omega \)  (3) 3 \( \Omega \)  (4) 2 \( \Omega \)

Answer (2)

Sol. \[ V_D - V_B = 0, \text{ i.e., it is condition of Wheatstone bridge.} \]
\[ \frac{12}{6 + x} = 0.5 \]
\[ x = 6 \Omega \]

3. Which of the following does not depend on the wave nature of light?
   A. Reflection  
   B. Diffraction  
   C. Photoelectric effect  
   D. Polarization  
   E. Interference

(1) C only  (2) A, B  (3) A, B, C  (4) D, E

Answer (1)

Sol. Theoretical

4. Four particles \( A, B, C, D \) have masses \( \frac{m}{2}, m, 2m \text{ and } 4m \). They have equal momentum.

The particle that has highest kinetic energy is

(1) \( A \)  
(2) \( B \)  
(3) \( C \)  
(4) \( D \)

Answer (1)

Sol. \[ KE = \frac{p^2}{2m} \]
\[ \Rightarrow KE \propto \frac{1}{m} \]
5. Which of the following is not a semiconductor?
   (1) Silicon    (2) Germanium
   (3) Copper oxide (4) Graphite

   **Answer (4)**
   **Sol.** Theoretical.

6. Find the truth table for the following circuit.

   ![Circuit Diagram]

   **Answer (2)**
   **Sol.** \( Y = AB + A' \)

7. A bullet of mass 50 gm enters a metal sheet with speed of 100 m/s and emerges with speed of 40 m/s. The loss in kinetic energy of bullet is
   (1) 105 J
   (2) 42 J
   (3) 210 J
   (4) 140 J

   **Answer (3)**
   **Sol.** \[ |\Delta K| = \frac{1}{2} \times \frac{50}{100} \times (100^2 - 40^2) = \frac{50}{2000} \times 140 \times 60 \]
   \[ = 210 \text{ J} \]

8. A ball of mass \( m \) and density \( \rho \) made to free fall into viscous liquid of density \( \rho_0 \). The viscous force on ball is
   (1) \( mg \left( 1 - \frac{\rho}{\rho_0} \right) \)
   (2) \( mg \left( 1 - \frac{\rho_0}{\rho} \right) \)
   (3) \( \frac{mg}{1 - \frac{\rho}{\rho_0}} \)
   (4) \( \frac{mg}{1 - \frac{\rho_0}{\rho}} \)

   **Answer (2)**
   **Sol.** \[ \vec{F}_v = W + B \]
   \[ = mg - \rho_0 v g \]
   \[ = mg - \rho_0 \frac{m}{\rho} g \]
   \[ = mg \left( 1 - \frac{\rho_0}{\rho} \right) \]

9. For a spring block system, the error in time period calculation is 2% and the error in mass calculation is 1%. Find the percentage error in spring constant \( k \).
   (1) 2%
   (2) 4%
   (3) 5%
   (4) 10%

   **Answer (3)**
   **Sol.** \[ k = 4\pi^2 \cdot \frac{m}{T^2} \]

   \[ \frac{dk}{k} \times 100 = \pm \left( \frac{dm}{m} \times 100 + 2 \cdot \frac{dT}{T} \times 100 \right) \]
   \[ \frac{dk}{k} \times 100 = 1 + 2 \times 2 = 5 \]
10. Match the dimensions:

<table>
<thead>
<tr>
<th></th>
<th>Torque</th>
<th>Magnetic moment</th>
<th>Magnetic field</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>i. $M^0L^2AT$</td>
<td>ii. $ML^2T^{-2}A^0$</td>
<td>iii. $MLT^{-3}A^{-2}$</td>
<td>iv. $ML^0T^{-2}A^{-1}$</td>
</tr>
</tbody>
</table>

(1) a-i, b-ii, c-iii, d-iv  
(2) a-ii, b-iv, c-i, d-iii  
(3) a-ii, b-i, c-iv, d-iii  
(4) a-ii, b-iii, c-i, d-iv

Answer (3)

Sol. \[ \tau = F' = ML^2T^{-2} \]
\[ \mu = iA = M^0L^2AT \]
\[ B = \frac{F}{qV} = \frac{MLT^{-2}}{ATLT^{-1}} = MT^{-1}A^{-1} \]
\[ \mu_0 = \frac{Br}{i} = \frac{MT^{-2}A^{-1}L}{AT} = MLT^{-3}A^{-2} \]

11. Kinetic energy to move a body of mass \( m \) from surface of earth to infinite distance form the earth is (\( g \) is acceleration due to gravity on surface of earth & \( R \) is radius of earth)

(1) \( 2mgR \)  
(2) \( \frac{1}{2}mgR \)  
(3) \( mgR \)  
(4) \( \frac{1}{4}mgR \)

Answer (3)

Sol. \( K + U = 0 \)
\[ \Rightarrow K = \frac{GMm}{R} = mgR \]

12. Find the ratio of root mean square speed of oxygen and helium molecules at same temperature.

(1) \( \frac{2\sqrt{2}}{1} \)  
(2) \( \frac{1}{2\sqrt{2}} \)  
(3) \( \frac{1}{4} \)  
(4) \( \frac{1}{32} \)

Answer (2)

Sol. \[ v_{rms} = \sqrt{\frac{3RT}{M}} \]
\[ \frac{v_{rms}}{O_2}{v_{rms}}_{He} = \frac{M_{O_2}}{M_{He}} = \sqrt{\frac{4}{32}} \]
\[ = \frac{1}{2\sqrt{2}} \]

13. The specific heat capacity for a gas following the relation \( PV^2 = RT \) is (\( C_v \) is heat capacity at constant volume and \( R \) is gas constant)

(1) \( C_v \)  
(2) \( C_v + R \)  
(3) \( \frac{R}{3} + C_v \)  
(4) \( R \)

Answer (1)

Sol. \[ \frac{PV^2}{PV} = C \Rightarrow V = constant \]
\[ \Rightarrow C_v \]

14. A screw gauge has circular scale 100 divisions with pitch 1 mm. Upon keeping a wire between studs, main scale reading is 1 mm and circular scale divisions 42th coincide with reference line. Find the diameter of circular cross-section wire in mm.

(1) 1.42  
(2) 1.40  
(3) 1.38  
(4) 0.39

Answer (1)

Sol. Diameter = Main scale reading + circular scale reading \times least count
\[ d = 1 \text{ mm} + (42 \times 0.01) \text{ mm} \]
\[ d = 1.42 \text{ mm} \]

15.  
16.  
17.  
18.  
19.  
20.
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Ratio of angle of prism and minimum deviation is one for a prism whose refractive index is $\sqrt{3}$ then angle of prism (in degrees) is _______.

Answer (60)

Sol. $A = \delta$

$$\frac{\sin \left( \frac{A + \delta}{2} \right)}{\sin \frac{A}{2}} = \frac{\sin A}{\sin \frac{A}{2}} = \sqrt{3}$$

$$2\cos \frac{A}{2} = \sqrt{3}$$

$$\Rightarrow A = 60^\circ$$

22. Time period of a simple harmonic motion is 3.14 seconds, with amplitude 0.06 m. The maximum velocity of particle is $k \times 10^{-2}$ m/s. Find the value of $k$.

Answer (12)

Sol. $V_{\text{max}} = A\omega = A \frac{2\pi}{T}$

$$V_{\text{max}} = 0.06 \times \frac{2\pi}{3.14}$$

$$V_{\text{max}} = 0.12 \text{ m/s}$$

23. A body uniformly accelerates [starting from rest] to speed of 80 km/hr in time $t$ and then maintains this speed for time interval of 3$t$. Average speed for whole motion is _______ km/hr.

Answer (70)

Sol. $\langle v \rangle = \frac{40 \times t + 80 \times 3t}{4t} = \frac{40 + 240}{4} = 70$

24. Radiation of energy 3.5 eV is incident on a metal. The stopping potential required is 0.5 V. The work function of the metal is _______ eV.

Answer (3)

Sol. $hf - \phi = eV$

$$\Rightarrow \phi = 3.5 - 0.5 = 3 \text{ eV}$$

25. If the radius of earth is reduced to $\frac{3}{4}$ of its original radius, then the time period of earth’s rotation becomes $K$ hours 30 minutes. Find the value of $K$.

Answer (13)

Sol. $\tau_{\text{ext}} = 0 \Rightarrow$ Angular momentum is conserved

$$\frac{2}{5} mR^2 \cdot \omega = \frac{2}{5} m \left( \frac{3R}{4} \right)^2 \cdot \omega_1$$

$$\omega_1 = \frac{16\omega}{9}$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega} \cdot \frac{9}{16} = 24 \times \frac{9}{16} \text{ hours}$$

$$T_1 = 13 \text{ hours 30 minutes}$$

26. Two masses $m_1$ and $m_2$ are attached through a thin string passing over frictionless and massless pulley. The acceleration of masses is as shown.

Then $\frac{m_1}{m_2}$ is _______.

Answer (2)

Sol. $a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{g}{3}$

$$\Rightarrow \frac{m_1}{m_2} = 2$$

27.

28.

29.

30.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Among the given molecules, identify the one which undergoes nucleophilic addition reaction at fastest rate?
   (1) HCHO
   (2) CH₃CHO
   (3) CH₃CH₂CHO
   (4) CH₃CH₂CH₂CHO

   Answer (1)
   Sol. Rate of N.A.R. α electrophilicity of C-atom

   \[ \frac{1}{\text{Steric Hinderance}} \]

2. Which compound will absorb light of highest frequency?
   (1) [Cr(H₂O)₆]³⁺
   (2) [CrCl₆]³⁻
   (3) [Cr(CN)₆]³⁻
   (4) [CrCl₃(H₂O)₃]

   Answer (3)
   Sol. More the crystal field splitting energy more will be the frequency of absorbed light.

   Crystal field splitting energy depends on ligand strength here

   Order of ligand strength
   Cl⁻ < H₂O < CN⁻; So splitting energy will also follow same order

3. Find the ratio of shortest wavelengths in Lyman and Balmer series of H-atom.
   (1) \( \frac{1}{4} \)
   (2) \( \frac{4}{1} \)
   (3) \( \frac{1}{2} \)
   (4) \( \frac{2}{1} \)

   Answer (1)
   Sol. \( \lambda_\text{shortest} \) Lyman = \( \frac{1}{R} \); \( \lambda_\text{shortest} \) Balmer = \( \frac{4}{R} \)

   \[ \frac{\lambda_\text{Lyman}}{\lambda_\text{Balmer}} = \frac{1}{4} \]

4. Which of the following is not the intermediate observed in Reimer Tiemann Reaction?
   (1)
   (2) :CCl₂
   (3)
   (4) CHCl₃

   Answer (4)
   Sol. CHCl₃ + NaOH \( \rightarrow \) :CCl₂

   CHCl₃ is the reagent.
5. Correct metamer of the following compound is

(1) 
(2) 
(3) 
(4) 

Answer (3)

Sol. Metamers have the same functional group but differ in the alkyl/aryl groups attached to it keeping the molecular formula unchanged. Therefore, metamer of the given compound is

(3) 

6. How many of the following do not belong to Lanthanoids?

Eu, Er, Lu, Cm, Yb, Tb

(1) 5
(2) 4
(3) 3
(4) 1

Answer (4)

Sol. The correct answer is given as (4).

63Eu, 65Tb, 68Er, 70Yb and 71Lu belong to Lanthanoids.

96Cm belongs to actinoids.

7. Density of x M solution of NaOH is 1.12 g/mL and molality is 3 m, then the value of x is

(1) 3
(2) 2.8
(3) 3.8
(4) 3.5

Answer (1)

Sol. Given molality of NaOH = 3 m

It means 3 moles of NaOH present in 1000 g of solvent.

Mass of solute (NaOH) = 3 × 40 = 120 g

Mass of solution = 1000 + 120 = 1120 g

Density of solution = 1.12 = \frac{1120}{\text{volume}}

Volume of solution = \frac{1120}{1.12} = 1000 mL

Molarity of solution = \frac{3}{1000} \times 1000

= 3 M

8. Which of the following is not a semiconductor?

(1) Si
(2) Graphite
(3) CuO
(4) Ge

Answer (2)

Sol. Graphite is not a semiconductor, it is an allotrope of carbon and a good conductor of electricity.

CuO is a p-type semiconductor.

Si and Ge are also semiconductors.
9. Match List-I with List-II and choose the correct option.

<table>
<thead>
<tr>
<th>List-I (Reagent)</th>
<th>List-II (Radical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) dil. HCl</td>
<td>(A) Pb(^{2+})</td>
</tr>
<tr>
<td>(ii) NH(_4)Cl + NH(_4)OH + (NH(_4))(_2)CO(_3)</td>
<td>(B) Al(^{3+})</td>
</tr>
<tr>
<td>(iii) NH(_4)Cl + NH(_4)OH + H(_2)S</td>
<td>(C) Mn(^{2+})</td>
</tr>
<tr>
<td>(iv) NH(_4)Cl + NH(_4)OH</td>
<td>(D) Sr(^{2+})</td>
</tr>
</tbody>
</table>

(1) (i)-(A), (ii)-(D), (iii)-(C), (iv)-(B)
(2) (i)-(D), (ii)-(A), (iii)-(C), (iv)-(B)
(3) (i)-(A), (ii)-(D), (iii)-(B), (iv)-(C)
(4) (i)-(B), (ii)-(C), (iii)-(D), (iv)-(A)

Answer (1)

Sol. (i) dil. HCl
   Group-I: Pb\(^{2+}\)
   (ii) NH\(_4\)Cl + NH\(_4\)OH + (NH\(_4\))\(_2\)CO\(_3\)
   Group-V: Ba\(^{2+}\), Sr\(^{2+}\), Ca\(^{2+}\)
   (iii) NH\(_4\)Cl + NH\(_4\)OH + H\(_2\)S
   Group-IV: Co\(^{2+}\), Ni\(^{2+}\), Mn\(^{2+}\), Zn\(^{2+}\)
   (iv) NH\(_4\)Cl + NH\(_4\)OH
   Group-III: Al\(^{3+}\), Fe\(^{3+}\)

10. Choose the correct option based on matching.

<table>
<thead>
<tr>
<th>Hybridization</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>sp(^3)</td>
</tr>
<tr>
<td>B</td>
<td>sp(^3)d</td>
</tr>
<tr>
<td>C</td>
<td>sp(^2)</td>
</tr>
<tr>
<td>D</td>
<td>sp(^3)d(^2)</td>
</tr>
</tbody>
</table>

(1) A(I); B(II); C(III); D(IV)
(2) A(II); B(III); C(IV); D(I)
(3) A(II); B(III); C(I); D(IV)
(4) A(III); B(II); C(IV); D(I)

Answer (2)

Sol.  

<table>
<thead>
<tr>
<th>Hybridization</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>sp(^3)</td>
<td>Tetrahedral</td>
</tr>
<tr>
<td>sp(^3)d</td>
<td>Trigonal bipyramidal</td>
</tr>
<tr>
<td>sp(^2)</td>
<td>Trigonal planar</td>
</tr>
<tr>
<td>sp(^3)d(^2)</td>
<td>Octahedral</td>
</tr>
</tbody>
</table>

11. Which of the following will have positive electron gain enthalpy?
(A) Na + e\(^{-}\) \(\rightarrow\) Na\(^{-}\)
(B) O + 2e\(^{-}\) \(\rightarrow\) O\(^{2-}\)
(C) Be + e\(^{-}\) \(\rightarrow\) Be\(^{-}\)
(D) F + e\(^{-}\) \(\rightarrow\) F\(^{-}\)
(E) N + e\(^{-}\) \(\rightarrow\) N\(^{-}\)

(1) (B, C, E) (2) (A, B, E) (3) (A, C, D) (4) (A, B, C)

Answer (1)

Sol. Be & N have stable fully filled & half-filled electronic configuration respectively

\[ O + e^{-} \rightarrow O^{-} \quad \Delta H_{g1} = -141 \text{ kJ/mol} \]
\[ O^{-} + e^{-} \rightarrow O^{2-} \quad \Delta H_{g2} = +780 \text{ kJ/mol} \]
\[ O + 2e^{-} \rightarrow O^{2-} \quad \Delta H = +639 \text{ kJ/mol} \]

12. Consider the given reaction:

\[ H_2 + I_2 \rightleftharpoons 2HI \]

If equal number of molecules of H\(_2\), I\(_2\) and HI are present at equilibrium. Then \( K_p = t \times 10^{-1} \)

Find out t

(1) 10 (2) 0.01 (3) 0.1 (4) 1

Answer (1)
**JEE (Main)-2024 : Phase-2 (06-04-2024)-Morning**

**Sol.** $H_2 + I_2 \rightleftharpoons 2HI$

$$K_p = \frac{(P_{HI})^2}{(P_{H_2})(P_{I_2})} = 1$$

$$K_p = t \times 10^{-1}$$

$t = 10$

13. **Statement-I:** Gallium has low melting point, so it is used in thermometers.

**Statement-II:** A substance having 253 K can be measured by Ga thermometer.

(1) Statement-I and Statement-II both correct

(2) Statement-I and Statement-II both incorrect

(3) Statement-I correct and Statement-II incorrect

(4) Statement-II correct and Statement-I incorrect

**Answer (3)**

**Sol.** Melting point of Gallium is nearly 302 K so, it can’t measure temperature of 253 K.

14. Choose the correct option regarding the following statements:

**Statement-I:** 2, 4, 6-Trinitrotoluene is picric acid.

**Statement-II:** Reaction of 4-hydroxybenzene-1,3-disulphonic acid with conc. HNO$_3$ gives picric acid.

(1) Both statement-I and statement-II are true

(2) Both statement-I and statement-II are false

(3) Statement-I is true but statement-II is false

(4) Statement-I is false but statement-II is true

**Answer (4)**

**Sol.** 2, 4, 6-Trinitrotoluene is an explosive and not picric acid. Therefore statement-I is false.

2, 4, 6-Trinitrophenol is called picric acid. It is synthesised by the reaction of 4-hydroxybenzene-1,3-disulphonic acid with conc. HNO$_3$.

**Answer (2)**

**Sol.** Uracil is present in RNA. Instead of uracil, thymine is present in DNA.

15. Among the following which is not a base of DNA?

(1) Adenine

(2) Uracil

(3) Guanine

(4) Cytosine

**Answer (2)**

**Sol.** Uracil is present in RNA. Instead of uracil, thymine is present in DNA.

16. Identify the correct match among the given species and respective shape of molecule

<table>
<thead>
<tr>
<th>Species</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH$_4^+$</td>
<td>Tetrahedral</td>
</tr>
<tr>
<td>SF$_4$</td>
<td>T-shaped</td>
</tr>
<tr>
<td>ClF$_3$</td>
<td>Square planar</td>
</tr>
<tr>
<td>XeF$_6$</td>
<td>Distorted Octahedral</td>
</tr>
</tbody>
</table>

**Answer (3)**

**Sol.** NH$_4^+$ : Tetrahedral

ClF$_3$ : T-shaped

SF$_4$ : See-Saw

XeF$_6$ : Distorted Octahedral

---

**Aakashians Conquer JEE (Main) 2024**

**SESSION-1**

**Our Stars**

*As per student response sheet and NTA answer key.*
17. Which of the following statement is incorrect?
   (1) Glycerol is purified by vacuum distillation
   (2) Aniline is purified by steam distillation
   (3) Chloroform and aniline can be separated by distillation
   (4) Ethanol and water are azeotropic mixture can be separated by distillation

Answer (4)
Sol. Ethanol and water are azeotropic mixture and can’t be separated by distillation

18. ...

19. ...

20. ...

SECTION - B
Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find out ratio of \( t_{99} \) and \( t_{90} \) for first order

Answer (2)
Sol. \( t_{99} = \frac{2.303 \log\frac{100}{k}}{1} \)
\( t_{90} = \frac{2.303 \log\frac{100}{10}}{1} \)

22. How many of the following show(s) H-bonding.
   (i) \( \text{O} \)  (ii) \( \text{H}_2\text{O} \)  (iii) \( \text{CH}_3\text{OH} \)  
   (iv) \( \text{HF} \)  (v) \( \text{NH}_3 \)  (vi) \( \text{C}_2\text{H}_6 \)

Answer (6)
Sol. The system having suitable electropositive pole H and suitable electronegative pole can show H-bonding. Suitable positive pole H: F—H, O—H, N—H, suitable negative pole: Electronegative element F, O, N with sufficient negative charge.

Hence \( \text{H}_2\text{O}, \text{CH}_3\text{OH}, \text{HF} \) and \( \text{NH}_3 \) can show intermolecular H bonding and Can show intermolecular H bonding.

23. \( \text{KMnO}_4 \rightarrow X \) (product having Mn)

What is the difference in spin only magnetic moment (in B.M.) between the given reactant and product. (Nearest integer)

Answer (6)
Sol. In acidic medium \( \text{KMnO}_4 \) goes to \( \text{Mn}^{2+} \) e.g. \( \text{MnSO}_4 \).

The \( \text{KMnO}_4 \) has zero unpaired electrons (hence it is diamagnetic) because it has Mn at +7 oxidation state (electronic configuration \( (n-1)d^0 ns^0 \)).

\( \text{Mn}^{2+} \) has 5 unpaired electrons as it has \( (n-1)d^5 ns^0 \) electronic configuration so it has 5.92 B.M. magnetic moment so difference will be 5.92 B.M. Nearest integer = 6.
24. 9.3 g of aniline was treated with NaNO₂ and HCl at 0°C to get product A which reacts with phenol to form a product B. Assuming 100% yield in each step, what is the weight of product B obtained?

Answer (20)

Sol. 

\[
\begin{align*}
\text{NH}_2 & \xrightarrow{\text{NaNO}_2, \text{HCl}, 0°C} \text{N}_2\text{Cl}^- \\
& \xrightarrow{\text{OH}} \text{A} \\
\text{A} & \xrightarrow{\text{Phenol}} \text{B}
\end{align*}
\]

\[
9.3 \text{ g} \quad \frac{9.3}{93} = 0.1 \text{ mole.}
\]

Final product formed = 0.1 mol.

\[
= 0.1 \times 198 \\
= 19.8 \text{ gm}
\]

= 20

25. Consider the following sequence of reactions

\[
\text{But} \xrightarrow{2-\text{yne}} \text{Na-Liq.NH}_3 \xrightarrow{\text{Cold, alkaline, dil. KMnO}_4} \text{A} \xrightarrow{\text{Cold, alkaline, dil. KmnO}_4} \text{B}
\]

Find the number of oxygen in B (1 molecule)

Answer (2)

Sol. 

\[
\begin{align*}
\text{Na-Liq.NH}_3 & \xrightarrow{\text{Bronz reduction}} \text{Trans-But-2-ene} \\
& \xrightarrow{\text{Cold, alkaline, dil. KmnO}_4} \text{OH} \quad \text{OH}
\end{align*}
\]

Hence the number of O atom in one molecule of B is 2.

26. According to the reaction,

\[
\text{HX}^{(aq)} \rightleftharpoons \text{H}^{+(aq)} + \text{X}^{-(aq)} \quad K_a = (1.2 \times 10^{-5})
\]

Find out osmotic pressure of 0.03 M HX solution at 300 K (in atm).

Answer (1)

Sol. 

\[
K_a = \frac{C\alpha^2}{1-\alpha}
\]

\[
1.2 \times 10^{-5} = (3 \times 10^{-2}) (\alpha^2)
\]

\[
\alpha^2 = 0.4 \times 10^{-3}
\]

\[
\alpha^2 = 4 \times 10^{-4}
\]

\[
\alpha = 0.02
\]

\[
\pi = i\text{CRT}
\]

\[
i = 1 + \alpha = 1.02
\]

\[
\pi = (1.02)(0.03)(0.0821)(300)
\]

\[
= 0.7536 \text{ atm}
\]

\[
\approx 1
\]
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If \( A_r = 2r \begin{vmatrix} 1 & \frac{n^2}{2} + \alpha \\ 2 & n^2 - \beta \\ 3r - 1 & 3 \frac{n^2}{2} (3n - 1) \end{vmatrix} \) then the value of \( 2A_{10} - A_8 \) is equal to

   (1) \( 4\alpha + 2\beta \)  
   (2) \( 2n \)  
   (3) \( 0 \)  
   (4) \( 2\alpha + 4\beta \)

Answer (1)

Sol. \( A_r = 2r \begin{vmatrix} 1 & \frac{n^2}{2} + \alpha \\ 2 & n^2 - \beta \\ 3r - 2 & 3 \frac{n^2}{2} - \frac{n}{2} \end{vmatrix} \)

\[ A_0 = \begin{vmatrix} 1 & \frac{n^2}{2} + \alpha \\ 2 & n^2 - \beta \\ 0 & 0 \end{vmatrix} = 0 \]

\[ A_1 = \begin{vmatrix} 1 & \frac{n^2}{2} + \alpha \\ 2 & n^2 - \beta \\ -2 & 3 \frac{n^2}{2} - \frac{n}{2} \end{vmatrix} = -2 \left[ (n^2 - \beta) - 2 \left( \frac{n^2}{2} + \alpha \right) \right] = -2 \left[ n^2 - \beta - \frac{n^2}{2} - 2\alpha \right] \]

\[ A_r = (\beta + 2\alpha) \cdot 2 \]

\[ 2A_{10} - A_8 = 4(\beta + 2\alpha) - 2(\beta + 2\alpha) \]

\[ = 2(\beta + 2\alpha) = 4\alpha + 2\beta \]

2. The value of \( \int_{\pi/4}^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} \, dx \) is equal to

   (1) \( \frac{1}{6} \)  
   (2) \( \frac{1}{3} \)  
   (3) \( \frac{1}{2} \)  
   (4) \( 1 \)

Answer (1)

Sol. \( \int_{\pi/4}^{\pi/4} \frac{\tan^2 x \sec^2 x}{(1 + \tan^3 x)^2} \, dx \)

Put \( 1 + \tan^3 x = t \)

\[ 3\tan^2 x \sec^2 x \, dx = dt \]

\[ \int \frac{1}{3} \frac{t^2}{1 - t} \, dt \]

\[ = \frac{1}{6} \]

3. Let \( \alpha, \beta \) be the distinct roots of the quadratic equation \( x^2 - (t^2 - 5t + 6)x + 1 = 0 \), \( a_n = \alpha^n + \beta^n \), then the minimum value of \( a_{2023} + a_{2025} \) is \( a_{2024} \)

   (1) \( -\frac{1}{4} \)  
   (2) \( \frac{1}{4} \)  
   (3) \( -\frac{1}{2} \)  
   (4) \( \frac{1}{2} \)

Answer (1)

Sol. Given equation

\[ x^2 - (t^2 - 5t + 6)x + 1 = 0 \]

\[ \therefore a_{2025} - (t^2 - 5t + 6) a_{2024} + a_{2023} = 0 \]
5. \( R \) is defined on set \( X = \{1, 2, \ldots, 20\} \)
   \( R_1 = \{(x, y) : 2x - 3y = 2\} \)
   \( R_2 = \{(x, y) : 5x - 4y = 0\} \)
   If \( M, N \) represent the number of elements to be added to make \( R_1 \) and \( R_2 \) symmetric respectively. Then value of \( M + N \) equals to
   (1) 10
   (2) 8
   (3) 12
   (4) 11
   Answer (1)
   Sol. \( R \) is defined on \( X = \{1, 2, 3, \ldots, 20\} \)
   \( R_1 = \{(x, y) : 2x - 3y = 2\} \)
   \( R_2 = \{(x, y) : 5x + 4y = 0\} \)
   As \( 2x - 3y = 2 \)
   So \( 2x \) and \( 3y \) both has to be even or odd simultaneously and \( 2x \) can't be odd so \( 2x \) and \( 3y \) both will be even.
   So \( R_1 = \{(4, 3), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\} \)
   for symmetric we need to add 6 elements here as \((3, 4), (4, 7) \) and \((6, 10), (8, 13), (10, 16), (12, 19)\)
   So \( M = 6 \)
   For \( R_2 \) \( 5x - 4y = 0 \)
   So \( 5x \) and \( 4y \) has to be equal. \( 4y \) is always even so \( 5x \) will also be even
   \( R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\} \)
   So 4 elements \((5, 4), (10, 8), (15, 12), (20, 16)\) need to be added
   \( N = 4 \)
   \( M + N = 10 \)

6. If \( \frac{dy}{dx} = \frac{y}{x \ln x} \) and \( y(e^{-1}) = 0 \). Then \( y(e) \) equals to
   (1) \( \frac{e^2 + 1}{e} \)
   (2) \( \frac{e^2 - 1}{e} \)
   (3) \( \frac{e^2 + 2}{e} \)
   (4) \( \frac{e^2 - 2}{e} \)
   Answer (2)
Sol. \[ I.F = e^{\int \frac{1}{x/nx} \, dx} \]

Put \( nx = t \)
\[ \frac{1}{x} \, dx = dt. \]

\[ I.F = e^{\int \frac{1}{t} \, dt} = te^{nt} = t(nx). \]

\[ y \cdot nx = \int y \cdot nx \left( \frac{1}{x^2(nx)} \right) \, dx \]
\[ y \cdot nx = \int \frac{1}{x^2} \, dx \]
\[ y \cdot nx = \frac{1}{x} + c \] \( \therefore \)

Put \( x = e \)
\[ y = \frac{1}{e} + e \]
\[ y = \frac{e^2 - 1}{e} \]

Option (b) is correct

7. Interval in which \( x^4 \) is strictly increasing is

(1) \((0, \infty)\)
(2) \(\left[ 0, \frac{1}{e} \right] \)
(3) \(\left[ \frac{1}{e^2}, \infty \right)\)
(4) \(\left[ \frac{1}{e}, \infty \right)\)

Answer (4)

Sol. Let \( f(x) = e^x \)

\[ \Rightarrow f(x) = x^4(1 + \ln x) \]

For strictly increasing, \( f(x) > 0 \)

\[ \Rightarrow 1 + \ln x > 0 \]
\[ \Rightarrow x > \frac{1}{e} \]

8. If \[ \frac{dy}{dx} + \frac{y}{x^2} = e^{-\tan^{-1}x}, \] then which of the following is true

(1) \( ye^{-\tan^{-1}x} = \frac{x^2}{2} + c \)
(2) \( ye^{-\tan^{-1}x} = \frac{1}{x} + c \)
(3) \( ye^{-\tan^{-1}x} = x + c \)
(4) \( ye^{-\tan^{-1}x} = -x + c \)

Answer (3)

Sol. \[ \frac{dy}{dx} + \frac{y}{1 + x^2} = e^{-\tan^{-1}x} \]

I.F = \[ e^{\int \frac{1}{1+x^2} \, dx} = e^{\tan^{-1}x} \]

Now,
\[ y e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \cdot e^{-\tan^{-1}x} \, dx \]
\[ \Rightarrow ye^{\tan^{-1}x} = \int \, dx \]
\[ \Rightarrow ye^{\tan^{-1}x} = x + c \]

9. A company produces automobiles. It has two factories: factory 'A' produces 60% of the automobiles and rest is produced by factory B. 80% of the automobiles produced by 'A' is up to the standards and 90% of the automobiles by 'B' is up to the standards. If an automobile is selected and found to be standard, the probability it came from 'B' is \( P \). Then 126 \( P \) equals to

(1) 54
(2) 52
(3) 48
(4) 27

Answer (1)
Sol. \(P\) (standard automobile from A) \[\frac{6}{10} \times \frac{8}{10} = \frac{12}{25}\]

\(P\) (standard automobile from B) \[\frac{4}{10} \times \frac{9}{10} = \frac{9}{25}\]

Required probability \[\frac{25}{12} + \frac{9}{25} = \frac{93}{217}\]

\[P = \frac{93}{217}\]

10. If \(\sigma = 4\) (standard deviation) and \(\bar{x} = 10\) (mean) of 20 observations. One term was taken wrong i.e., instead of 12 they have taken 8. Then the correct standard deviation

(1) \(1.8\)

(2) \(\sqrt{3.96}\)

(3) \(\sqrt{3.84}\)

(4) \(1.93\)

Answer (2)

Sol.

\[\text{Mean } = \bar{x} = 10\]

\[\sigma = 4, \quad n = 20\]

Take observations as \(x_1, x_2, \ldots, x_{20}\)

\[\frac{x_1 + x_2 + \ldots + x_{20}}{20} = 200\]

\[x_1 + x_2 + \ldots + x_{20} = 200\]

One term, say \(x_{20}\) is wrongly written as 8

So, \(x_1 + x_2 + \ldots + x_{19} = 200 - 8\)

\[x_1 + x_2 + \ldots + x_{19} = 192\]

Now \((x_{20})_{\text{new}} = 12\)

So, \(\bar{x}_{\text{new}} = \frac{192 + 12}{20} = \frac{204}{20} = \frac{102}{10} = 10.2\)

\[\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 = 4\]

\[\frac{x_1^2 + x_2^2 + \ldots + x_{19}^2}{20} = 4 + 100\]

\[\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_{19}^2 + 64}{20} = 104\]

\[\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_{19}^2 + 64}{20} = 2080\]

\[\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_{19}^2}{20} = 2016\]

Now \(\sigma_{\text{new}}^2 = \frac{2016 + 144 - (10,2)^2}{20}\)

\[\sigma_{\text{new}}^2 = 108 - 104.04\]

\[\sigma = \sqrt{3.96}\]

11. A point \(P(10, -2, -1)\) and \(R(1, 7, 6)\), if \(Q\) is a foot of perpendicular from \(R\) to the line joining points \((2, -5, 11)\) and \((6, -7, 5)\). Then \((PQ)^2\) is

(1) \(\frac{3509}{14}\)

(2) \(\frac{3600}{7}\)

(3) \(\frac{3509}{7}\)

(4) \(\frac{3409}{7}\)

Answer (1)

Sol.

\(R \perp L\)

\[\lambda = 1\]

\(PQ \perp L\)

\[RQ \perp L = 4\hat{i} - 2\hat{j} - 6\hat{k}\]

\[\therefore RQ \cdot L = 4(6\lambda + 2) - 2(-7\lambda - 5) - 6(\lambda + 1)\]

\[= 4(5\lambda + 1) - 2(-14\lambda - 12) - 6(-\lambda + 5)\]

\[= \frac{\lambda(20 + 28 + 6) + (4 + 24 - 30)}{\lambda + 1}\]

\[= 0\]
\[ Q = \left( \frac{6}{27}, \frac{-7}{27}, \frac{5}{27} + 11 \right) \]
\[ = \left( \frac{60}{28}, \frac{-142}{28}, \frac{302}{28} \right) \]
\[ = R = \left( \frac{15}{7}, \frac{71}{14}, \frac{151}{14} \right) \]
\[ \Rightarrow (QP) = \sqrt{\left(10 - \frac{15}{7}\right)^2 + \left(-2 - \frac{71}{14}\right)^2 + \left(-1 - \frac{151}{14}\right)^2} \]
\[ = \sqrt{3509} \]

12.
13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If \( \cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4} \), value of \( n \) is

Answer (47)

Sol. \( \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4} \)
\[ \Rightarrow \tan^{-1}\left(\frac{7}{12}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4} \]
\[ \Rightarrow \tan^{-1}\left(\frac{23}{24}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4} \]
\[ \Rightarrow \frac{23}{24} + \frac{1}{n} = 1 \]
\[ \Rightarrow \frac{23n + 24}{24n - 23} = 1 \]
\[ \Rightarrow n = 47 \]

22. If \( A = [100, 700] \), how many numbers are in \( A \) which are neither multiple of 3 nor 4?

Answer (300)

Sol. Let \( A_1 \) denotes multiple of 3,
\( B_1 \) denotes multiple of 4
\[ \Rightarrow \text{We need to find } A_1 \cap B_1 \]
\[ \Rightarrow |A_1 \cap B_1| = |A_1 \cup B_1| \]
\[ |A_1 \cup B_1| = n(A_1) + n(B_1) - n(A_1 \cap B_1) \]
\[ = 200 + 151 - 50 \]
\[ = 301 \]
\[ \Rightarrow A_1 \cup B_1 = 601 - 301 = 300 \]

23. If the second, third, fourth terms of the expression \( (x + y)^n \) is 135, 30, \( \frac{10}{3} \) respectively, then the value of \( 9(n^2 + x^2 + y) \) is

Answer (1153)

Sol. \( (x + y)^n = nC_0 x^0 y^n + nC_1 x^1 y^{n-1} + nC_2 x^2 y^{n-2} \)
First term \( \Rightarrow n C_1 x^1 y^{n-1} = 135 \ldots (1) \)

Second term \( \Rightarrow n C_2 x^2 y^{n-2} = 30 \ldots (2) \)

Third term \( \Rightarrow n C_3 x^3 y^{n-3} = \frac{10}{3} \ldots (3) \)

Dividing equation (1) by (2)

\[
\frac{2n}{n-1} \times \frac{1}{xy} = \frac{135}{30}
\]

\[
\frac{2}{(n-1)xy} = \frac{9}{2}
\]

\[
(n-1)xy = \frac{4}{9} \quad \ldots (4)
\]

Dividing equation (2) by (3)

\[
\frac{3}{(n-2)xy} = 9
\]

\[
(n-2)xy = \frac{1}{3} \quad \ldots (5)
\]

Dividing equation (4) by (5)

\[
\frac{n-1}{n-2} = \frac{4}{3}
\]

\[
\Rightarrow 3n - 3 = 4n - 8
\]

\[
\Rightarrow n = 5
\]

Now equation (1) becomes

\[
5 \times y^4 = 135 \Rightarrow x = \frac{27}{y^4} \quad \ldots (6)
\]

And equation (2) becomes

\[
10x^2 y^3 = 30
\]

\[
\Rightarrow x^2 y^3 = 3 \quad \ldots (7)
\]

From equation (6) and (7)

\[
\Rightarrow \frac{3^6}{y^3} xy^3 = 3
\]

\[
\Rightarrow y^6 = 3^5
\]

\[
\Rightarrow y = 3
\]

\[
\Rightarrow x = \frac{1}{3}
\]

\[
\Rightarrow 9(n^3 + x^2 + y)
\]

\[
= 9\left(\frac{5^3 + 1}{3^2} + 3\right)
\]

\[
= 9\left(\frac{1153}{9}\right)
\]

\[
= 1153
\]

24. Find the number of triangles formed whose vertices are also a vertices of regular octagon but the side of triangle is not common with sides of octagon is

Answer (56)

Sol.

\[
\Rightarrow \text{Total triangles} = 8 C_3
\]

Triangles with all 3 sides common with octagon = 0

Triangle with 2 sides common with octagon \( \Rightarrow \) Choose vertex

\[
\Rightarrow \binom{8}{1} = 8 \text{ triangles}
\]

Triangle with exactly 1 side common

\[
\Rightarrow \binom{8}{1}\text{ways to choose a side, remaining vertex can be selected in 4 vertices} = 4 C_1 \text{ways} = 8 C_1 \cdot 4 C_1
\]

\[
= 32
\]

\[
\Rightarrow 56 - (8 + 0 + 32) = 16 \text{ triangles}
\]

25. The number of real solutions of \( x |x + 5| + 2|x + 7| - 2 = 0 \) is equal to

Answer (3)

Sol. (I) \( x \geq -5 \)

\[
x^2 + 5x + 2x + 14 = 0
\]
\[ x^2 + 7x + 12 = 0 \]
\[ x = 3, 4 \]

(ii) \(-7 < x < -5\)
- \[ x^2 - 5x + 2x + 14 - 2 = 0 \]
- \[ x^2 - 3x + 12 = 0 \]
- \[ x^2 + 3x - 12 = 0 \]
\[ x = 2.275, -5.275, \text{ here } x \neq 2.275 \]

So, \( x = -5.275 \)

(iii) \( x \leq -7 \)
- \[ x^2 - 5x - 2x - 14 - 2 = 0 \]
- \[ x^2 - 7x - 16 = 0 \]
- \[ x^2 + 7x + 16 = 0 \]
\[ D < 0 \rightarrow \text{ no real roots} \]

Only 3 solutions possible

26. The number of points of discontinuities of \( f(x) = 2x^2 + [x^2] - [x] \) where \([x]\) is greatest integer function and \( x \in [-1, 2] \) is equals to

Answer (4)

Sol. \( f(x) = 2x^2 + [x^2] - [x], x \in [-1, 2] \)

This function may be discontinuous at \( x = -1, 0, 1, \sqrt{2}, \sqrt{3} \) and 2.

For continuity at \( x = -1 \)
\[ f(-1) = 4 \]
\[ \lim_{h \to 0} f(-1+h) = \lim_{h \to 0} 2(-1+h)^2 + [(-1+h)^2] - [-1+h] \]
\[ = 3 \]
\[ \therefore f(x) \text{ is discontinuous at } x = -1 \]

For continuity at \( x = 0 \), \( f(0) = 0 \)
\[ f(0^-) = 1 \]
\[ \therefore f(x) \text{ is discontinuous at } x = 0 \]

Continuity at \( x = 1 \)
\[ \text{L.H.L} = \lim_{h \to 0} f(1-h) = 2(1-h)^2 + [(1-h)^2] - [1-h] \]
\[ = 2 \]
\[ f(1) = 2(1)^2 + 1 - 1 = 2 \]
\[ \text{R.H.L} = \lim_{h \to 0} f(1+h) = 2(1+h)^2 + [(1+h)^2] - [1+h] \]
\[ = 2 \]
\[ \therefore f(x) \text{ is continuous at } x = 1 \]

For continuity, at \( x = \sqrt{2} \) and \( \sqrt{3} \) similarly it is discontinuous

For continuity at \( x = 2 \)
\[ f(2) = 2.2^2 + [2^2] - [2] = 10 \]
\[ \text{L.H.L} = \lim_{h \to 0} 2(2-h)^2 + [(2-h)^2] - [2-h] \]
\[ = 8 + 3 - 1 \]
\[ = 10 \]
\[ \therefore f(x) \text{ is discontinuous at } x = 2 \]
\[ f(x) \text{ is discontinuous at } x = -1, 0, \sqrt{2} \text{ and } \sqrt{3} . \]

No. of points of discontinuity = 4

27.
28.
29.
30.