Memory Based

Answers & Solutions

for

JEE (Main)-2024 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs.

M.M. : 300

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt any 05 questions out of 10 Questions.

(6) Section - A (01 – 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

(7) Section - B (21 – 30) contains 10 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.


### Section - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Dimensional formula of Planck’s constant is
   - (1) \([M^2L^2T^{-1}]\)
   - (2) \([M^1L^2T^{-1}]\)
   - (3) \([M^2L^2T^{-2}]\)
   - (4) \([ML^2T^{-3}]\)

   Answer (2)

   Sol. \(E = h\nu\)

   \([h] = \frac{ML^2T^{-2}}{T^{-1}}\)

2. Find the magnitude of force \(F\), if the given system is in equilibrium.

   \(\begin{align*}
   F & \rightarrow 1 \text{ kg} \\
   \theta & = 45^\circ
   \end{align*}\)

   - (1) 10 N
   - (2) \(\frac{10}{\sqrt{2}}\) N
   - (3) 0 N
   - (4) \(\frac{1}{10\sqrt{2}}\) N

   Answer (2)

   Sol. \(T = 10 \text{ N}; T\sin45 = F\)

   \(F = \frac{10}{\sqrt{2}}\) N

3. The equivalent resistance between terminal \(A\) and \(B\) in the network shown.

   \(\begin{align*}
   \left(1\right) \frac{4R}{3} \\
   \left(2\right) \frac{8R}{3} \\
   \left(3\right) 3R \\
   \left(4\right) \frac{5R}{2}
   \end{align*}\)

   Answer (2)

   Sol. \(R_{AB} = \frac{8R}{3}\)

4. A nucleus at rest breaks into two parts with mass ratio 1 : 2. The ratio of their velocities and direction is
   - (1) Opposite direction 2 : 1
   - (2) Same direction 1 : 2
   - (3) Opposite direction 1 : 1
   - (4) Same direction 1 : 1

   Answer (1)

   Sol. By conservation of momentum

   \(m_1v_1 = m_2v_2\)

   \(\frac{v_1}{m_1} = \frac{m_2}{m_1}\)

   \(\frac{v_1}{v_2} = \frac{2}{1}\)
5. Two cars A and B are moving towards each other with speed 20 m/s each. When 300 m apart, they both apply breaks which causes deceleration of 2 m/s². The distance between them when they stop will be:

(1) 100 m
(2) 50 m
(3) 150 m
(4) 200 m

Answer (1)

Sol. \[ v_{AB} = 40 \hat{i} \text{ m/s} \]

\[ a_{AB} = -4 \hat{i} \text{ m/s} \]

\[ v^{2} = u^{2} + 2as \]

\[ 0 = 1600 - 8s \]

\[ s = 200 \text{ m} \]

Distance between them = 300 – 200 = 100 m

6. For a wire, original resistance was 50 Ω at initial temperature was 27°C. When temperature is increased, its resistance becomes 62 Ω. If the thermal coefficient of resistivity of wire is 2.4 \times 10^{-2} \text{ K}^{-1}, find final temperature.

(1) 45°C
(2) 32°C
(3) 37°C
(4) 48°C

Answer (3)

Sol. \[ R = R_{0}(1 + \alpha \Delta T) \]

\[ 62 = 50(1 + 2.4 \times 10^{-2} \Delta T) \]

\[ 1.24 = 1 + 2.4 \times 10^{-2} \Delta T \]

\[ \Delta T = 10 \]

\[ T = 37°C \]

7. Find work done by monatomic gas from A to B. Here temperature of gas (1 mole) changes from 330 K to 300 K.

\[ PV^{\gamma} = \text{constant} \]

(1) 125 J
(2) 250 J
(3) 500 J
(4) 625 J

Answer (1)

Sol. \[ w = \frac{\mu R \Delta T}{1 - \alpha} = \frac{25 \times 30}{3} = 125 \text{ J} \]

8. Two bubbles having radii \( r_A \) and \( r_B \) are having excess pressure \( P_A \) and \( P_B \) in them. If \( P_A = 3P_B \), find \( \frac{r_A}{r_B} \)

(1) 9 : 1
(2) 1 : 9
(3) 1 : 3
(4) 3 : 1

Answer (3)

Sol. \[ \Delta P = \frac{4T}{r} \]

\[ \frac{P_A}{P_B} = \frac{r_B}{r_A} \]

\[ \frac{r_A}{r_B} = \frac{1}{3} \]
9. Find the induced emf in the square loop of side 15 cm moving with 2 cm/s after 10 seconds.

\[ \text{At } t = 0 \]

\[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array} \]

\[ 50 \text{ cm} \]

(1) 0  
(2) 0.3 mV  
(3) 3 V  
(4) 9 V

**Answer (1)**

**Sol.** At \( t = 10 \) seconds, \( \phi = \text{Constant} \)

\[ \Rightarrow \frac{d\phi}{dt} = 0 \]

10. A spring exerts force on block \( F = -50x^2 \) where \( x \) is change in length of spring. Find time period of oscillations. \((m = 0.5 \text{ kg})\)

\[ r_p = \sqrt{\frac{m_p}{m}} \]

\[ r_d = \sqrt{\frac{m_d}{m}} \]

\[ \frac{q_d}{q_p} = \frac{1}{\sqrt{2}}. \]

(1) 0.63 sec.  
(2) 3.14 sec.  
(3) 1.57 sec.  
(4) 0.31 sec.

**Answer (1)**

**Sol.** \( T = 2\pi \sqrt{\frac{0.5}{50}} = \frac{2\pi}{10} = \frac{\pi}{5} \) sec.

11. A proton and deuteron, having same kinetic energy, enters a transverse uniform magnetic field. Radius of circular paths for proton and deuteron are in ratio of

(1) \( \sqrt{2} \)  
(2) \( \frac{1}{2\sqrt{2}} \)  
(3) \( \frac{1}{\sqrt{2}} \)  
(4) \( 2\sqrt{2} \)

**Answer (3)**

**Sol.** \( r = \frac{\sqrt{2mk}}{qB} \)

\[ \frac{r_p}{r_d} = \sqrt{\frac{m_p}{m_d}} \]

\[ \frac{q_d}{q_p} = \frac{1}{\sqrt{2}}. \]

12. A satellite of mass \( 10^3 \text{ kg} \) is orbiting in an orbit of radius \( 2r \) from centre of the planet of radius \( r \). If satellite is given energy \( E = \frac{GM}{6r} \), then find new radius of orbit in which satellite will revolve.

\((M = \text{mass of planet})\)

(1) \( 14r \)  
(2) \( 6r \)  
(3) \( 8r \)  
(4) \( 12r \)
Answer (2)

Sol. \( E_1 = -\frac{GMm}{4r} \)

\[ E_f = -\frac{GMm}{4r} + \frac{GMm}{6r} = -\frac{GMm}{2x} \]

\[
-\frac{1}{12r} = -\frac{1}{2x}
\]

\[ x = 6r \]

13. For which of the following is the \( I-V \) characteristics shown below is possible?

![\( I \) vs. \( V \) graph]

(1) Transistor
(2) Zener diode
(3) Solar cell
(4) Diode used as rectifier

Answer (2)

Sol. As Zener diode operates in reverse bias, it is for Zener diode.

14. For the circuit shown, the truth table

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( x )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( y )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

![Logic circuit diagram]

Answer (4)

Sol. \( Y = \overline{AB} + \overline{A}B \)

15. A ball of radius \( 10^{-4} \) m and density \( 10^5 \) kg/m\(^3\) is dropped from a height \( h \) into water (viscosity = \( 9.8 \times 10^{-6} \text{ Pa}\cdot\text{s} \)) such that after falling into liquid, its speed does not change. Find the approximate value of \( h \).

(1) \( 2200 \text{ m} \)
(2) \( 2350 \text{ m} \)
(3) \( 2470 \text{ m} \)
(4) \( 2520 \text{ m} \)

Answer (3)

Sol. Velocity just before entering water = Terminal velocity

\[
\sqrt{2gh} = \frac{2}{9}r^2g\frac{(\rho - \sigma)}{\eta}
\]

\[
\sqrt{2gh} = \frac{2}{9} \times 10^{-8} \times g \times \frac{(10^5 - 10^3)}{9.8 \times 10^{-6}}
\]

\[ h \approx 2470 \text{ m} \]

16. 
17. 
18. 
19. 
20.
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In given ray diagram, find distance \( u \) (in cm) between two convex lenses.

Answer (25)

Sol. \( f_1 + f_2 = L = 25 \text{ cm} \)

22. Find the work done (in J) by force \( F = 3x^2 + 2x - 5 \) in moving a particle from \( x = 2 \) to \( x = 4 \).

Answer (58)

Sol. \( W = \frac{1}{2} \int F \cdot dx \)

\[ = [x^3 + x^2 - 5x]_2^4 \]

\[ = 58 \text{ J} \]

23. There is an imaginary cube of side 2 m where edges are along axes. The electrostatic field varies as \( E(x) = 2x \hat{i} \), then flux through cube in Nm\(^2\)/C is

Answer (16)

Sol. \( E_1 = 4 \)

\[ E_2 = 8 \]

\[ \Rightarrow \Delta\phi = (8 - 4) \cdot 2^2 = 16 \]

24. If work function of a metal is 2.13 eV and energy per photon of incident light is 3.13 eV, then maximum kinetic energy of photoelectrons (in eV) will be______.

Answer (1)

Sol. \( KE_{\text{max}} = h\nu - \phi_0 \)

\[ = (3.13 - 2.13) \text{ eV} \]

\[ = 1 \text{ eV} \]

25. A photon of energy of 10.2 eV is incident on hydrogen atom in ground state. Thereafter number of emitted lines will be______.

Answer (1)

Sol. \( \Delta E = 10.2 \text{ eV} \)

\( e^- \) will be excited to \( n = 2 \)

26.

27.

28.

29.

30.
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Correct order of bond angle of following compounds is BF$_3$, PF$_3$, ClF$_3$
   (1) BF$_3$ > PF$_3$ > ClF$_3$
   (2) PF$_3$ > ClF$_3$ > BF$_3$
   (3) ClF$_3$ > PF$_3$ > BF$_3$
   (4) BF$_3$ > ClF$_3$ > PF$_3$

   Answer (1)

   Sol. BF$_3$ ⇒ $sp^2$ ⇒ Bond angle = 120°
   PF$_3$ ⇒ $sp^3$ ⇒ Bond angle ≈ 109°28’
   ClF$_3$ ⇒ $sp^3d$ ⇒ Bond angle ≈ 90°

2. Identify the correct electronic configuration of Einstenium is
   (1) [Rn] 5$f^{14}6d^67s^2$
   (2) [Rn] 5$f^{17}7s^2$
   (3) [Rn] 5$f^{10}6d^67s^2$
   (4) [Rn] 5$f^{11}6d^67s^2$

   Answer (2)

   Sol. Es (Z = 99) ⇒ [Rn] 5$f^{11}7s^2$

3. The product obtained in the following reaction is:

   Answer (3)

   Sol.

4. Ca$^{2+}$ makes which type of complex with EDTA?
   (1) Trigonal bipyramidal
   (2) Square planar
   (3) Tetrahedral
   (4) Octahedral

   Answer (4)

   Sol. Co-ordination number of Ca$^{2+}$ with EDTA is 6
   Hybridisation = $sp^3d^2$
   Shape = Octahedral

5. Consider the following reaction and identify the major product P.

   Answer (1)

   Sol. The reaction is benzylic oxidation reaction
6. Match the complexes given in List-I with the hybridisation of central metal atom/ion given in List-II and choose the correct option.

<table>
<thead>
<tr>
<th>List-I (Complexes)</th>
<th>List-II (Hybridisation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $K_2[\text{Ni(CN)}_4]$</td>
<td>(I) $sp^3$</td>
</tr>
<tr>
<td>(B) $[\text{Ni(CO)}_4]$</td>
<td>(II) $sp^3d^2$</td>
</tr>
<tr>
<td>(C) $[\text{Co(NH}_3)_6]\text{Cl}_3$</td>
<td>(III) $dsp^2$</td>
</tr>
<tr>
<td>(D) $\text{Na}_3[\text{CoF}_6]$</td>
<td>(IV) $d^2sp^3$</td>
</tr>
</tbody>
</table>

(1) (A)-(I); (B)-(II) (C)-(III); (D)-(IV)
(2) (A)-(III); (B)-(I) (C)-(IV); (D)-(II)
(3) (A)-(IV); (B)-(III) (C)-(II); (D)-(I)
(4) (A)-(I); (B)-(II) (C)-(IV); (D)-(III)

Answer (2)

Sol.

(A) $K_2[\text{Ni(CN)}_4]$
Ni$^{2+}$ : $3d^8$

$dsp^2$ hybridisation as CN$^-$ is strong field ligand

(B) $[\text{Ni(CO)}_4]$
Ni$^0$ : $3d^44s^2$

$sp^3$ hybridisation as CO is strong field ligand

(C) $[\text{Co(NH}_3)_6]\text{Cl}_3$
Co$^{3+}$ : $3d^6$

$d^2sp^3$ hybridisation as NH$_3$ is strong field ligand

(D) $\text{Na}_3[\text{CoF}_6]$
$[\text{CoF}_6]^{3-}$; Co$^{3+}$ : $3d^6$

$sp^3d^2$ hybridisation as F$^-$ ion is a weak field ligand

7. Major product is

8. Which of the following is correct for strong electrolyte ($B > 0$)

(1) $\lambda_m - \lambda_m^0 - B\sqrt{C} = 0$
(2) $\lambda_m + \lambda_m^0 - B\sqrt{C} = 0$
(3) $\lambda_m - \lambda_m^0 + B\sqrt{C} = 0$
(4) $\lambda_m + \lambda_m^0 + B\sqrt{C} = 0$

Answer (3)

Sol. $\lambda_m = \lambda_m^0 - B\sqrt{C}$

$\lambda_m - \lambda_m^0 + B\sqrt{C} = 0$
9. Which one of the following statements regarding glucose is incorrect?
   (1) Glucose is one of the monosaccharides of sucrose
   (2) Glucose dissolves in water because it has aldehyde group.
   (3) Glucose has six carbon atoms in its structure
   (4) Glucose is an aldose

Answer (2)

Sol. Glucose is an aldohexose having molecular formula C₆H₁₂O₆. It is soluble in water due to number of hydroxyl groups which can form H-bonds with water. α(D) Glucose condenses with β(D) fructose to form sucrose.

10. What is the work done on the gas in cyclic process ABCA
    (1) +773.7 J
    (2) −773.7 J
    (3) +4762.3 J
    (4) −4762.3 J

Answer (1)

Sol. \( W_{AB} = 0 \)

\[
W_{BC} = -10 \ (4 - 2) \\
= -20 \ \text{atm. Lit}
\]

\[
W_{CA} = 2.303(40) \ \log 2 \\
= 27.636 \ \text{atm. Lit}
\]

\[
W_{\text{total}} = 7.636 \ \text{atm. Lit} \\
= 773.7 \ \text{Joule}
\]

11. Which of the following compounds does not give Tollen’s test?
   (1) Formaldehyde
   (2) Formic acid
   (3) Benzaldehyde
   (4) Acetone

Answer (4)

Sol. Aldehyde and Formic acid can give Tollen’s test with ammoniacal silver nitrate solution.

12. Which of the following will give positive iodoform test?
   (1) \( CH_3 - CH_2 - CH_2 - CHO \)
   (2) \( CH_3 - CH - CH_3 \)
   (3) \( CH_3 - CH_2 - C - CH_2 - CH_3 \)
   (4) \( CH_3 - CH_2 - CH_2 - CH_2 - OH \)

Answer (2)

Sol. Molecules having groups as \( -C - CH_3 \) or \( CH - CH \) gives positive iodoform test.

13. Match the List and choose correct option.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Ni-Cd cell</td>
<td>(a) Rechargeable</td>
</tr>
<tr>
<td>(ii) Fuel cell</td>
<td>(b) Anode ( (Zn \rightarrow Zn^{2+} + 2e^-) )</td>
</tr>
<tr>
<td>(iii) Mercury cell</td>
<td>(c) Used in hearing aid</td>
</tr>
<tr>
<td>(iv) Leclanche cell</td>
<td>(d) Combustion energy in to electrical energy</td>
</tr>
</tbody>
</table>

(1) (i)-(a); (ii)-(d); (iii)-(c), (iv)-(b)
(2) (i)-(b); (ii)-(a); (iii)-(c), (iv)-(d)
(3) (i)-(d); (ii)-(a); (iii)-(c), (iv)-(b)
(4) (i)-(a); (ii)-(b); (iii)-(c), (iv)-(d)

Answer (1)

Sol. Ni-Cd cell is secondary cell and are rechargeable mercury cell is used in hearing aid.

14. What is the correct order of \( C - C \) bond length of ethane, ethene and ethyne?
   (1) Ethane > Ethene > Ethyne
   (2) Ethene > Ethane > Ethyne
   (3) Ethyne > Ethene > Ethane
   (4) Ethyne > Ethane > Ethene

Answer (1)

Sol. Correct order of \( C - C \) bond length is

\[
\text{Ethane} > \text{Ethene} > \text{Ethyne}
\]

\[
C - C > C = C > C \equiv C
\]

15. 

16. 
21. Fuming sulphuric acid has how many oxygen atoms?

**Answer (7)**

*Sol.* Fuming sulphuric acid is oleum (H$_2$SO$_7$), hence there are 7 O-atoms present in fuming sulphuric acid.

22. Total sum of number of electrons in $\pi^*$ orbitals of O$_2$, O$_2^+$, and O$_2^-$ is

**Answer (6)**

*Sol.* O$_2$ (16e$^-$): $\sigma_{1s}^2 \sigma_{2s}^2 \sigma_{2s}^2 \sigma_{2p}^2 \left( \pi_{2p}^2 \right) \left( \pi_{2p}^1 \right) \sigma_{2p}^2$

Total number of e$^-$ in $\pi^*$ orbitals of O$_2$, O$_2^+$, O$_2^-$

$= 2 + 1 + 3 = 6$

23. How many total number of stereoisomers are possible for the following structure?

**Answer (4)**

*Sol.* The structure has two stereogenic centres, one geometrical centre and one optical centre. Hence it has total 4 stereoisomers.

$2^2 = 4$

24. Among the elements – Sc, Ti, V, Cr, Mn find magnetic moment of element which have highest ionization enthalpy in +2 oxidation state. [Nearest integer]

**Answer (6)**

*Sol.* Sc$^{+2}$ Ti$^{+2}$ V$^{+2}$ Cr$^{+2}$ Mn$^{+2}$

Mn$^{+2}$ will have highest I.E. due to its stable half filled configuration.

$\text{Mn} \rightarrow \text{[Ar]} \, 4s^2 \, 3d^5 \rightarrow 5 \text{ unpaired e}^0$

$\mu_{\text{spin}} = \sqrt{5(5 + 2)} \text{ BM}$

$= \sqrt{35}$

$= 6$

25. How many of the following compounds will give Friedel Craft’s reaction?

**Answer (3)**

*Sol.* Friedel Craft’s reaction is not given by those aromatic compounds which have strong deactivating groups like –NO$_2$ group. Even aniline does not give Friedel Crafts reaction because the Lewis acid AlCl$_3$ will from co-coordinate bond with –NH$_2$ group thus converting it into strongly deactivating group, Friedel Crafts reaction is given by

by

by

26.

27.

28.

29.

30.
MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If \( \frac{z - 2i}{z + 2i} \) is purely imaginary, then maximum value of \(|z + 8 + 6i|\) is equal to

(1) 6  (2) 8  
(3) 10  (4) 12

Answer (4)

Sol. \( \frac{x + (y - 2)i}{x + (y + 2)i} \) = purely imaginary.
\( \Rightarrow x^2 + (y - 2)(y + 2) = 0 \)
\( \Rightarrow x^2 + y^2 = 4 \)
Maximum value = 10 + 2 = 12

2. \( \int_{\frac{1}{4}}^{\frac{3}{4}} \cos \left( 2\cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) \, dx = \)

(1) \( \frac{-1}{4} \)  (2) \( \frac{3}{2} \)
(3) \( \frac{1}{16} \)  (4) \( \frac{-4}{3} \)

Answer (1)

Sol. \( \int_{\frac{1}{4}}^{\frac{3}{4}} \cos \left( 2\cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) \, dx = \)
\( x = \cos 2\theta \)  \( \Rightarrow dx = (-\sin 2\theta) \, d\theta \)
\( = -2 \int_{\alpha}^{\beta} \cos \left( 2\cot^{-1} \tan \theta \right) \sin 2\theta \, d\theta \)
\( = -2 \left[ \int_{\alpha}^{\beta} \cos \theta \cdot \sin 2\theta \, d\theta \right] \)
\( = -2 \left[ \int_{\alpha}^{\beta} \cos \theta \cdot 2 \sin \theta \cos \theta \, d\theta \right] \)
\( = -2 \left[ \int_{\alpha}^{\beta} \cos^2 \theta \, d\theta \right] \)

3. \( \lim_{x \to 0} e^{-\left(1+2x\right)x} = \)

(1) \( e \)  (2) \( \frac{e}{4} \)
(3) \( \frac{e}{8} \)  (4) \( \frac{11}{24} e \)

Answer (1)

Sol. Using expansion
\( e - e \left[ 1 - \frac{2x}{2} + \frac{11x^2}{24} + \ldots \right] \)
\( \lim_{x \to 0} e - \frac{11x}{24} e \ldots \)
\( = e \)
4. In the given data

<table>
<thead>
<tr>
<th>x_i</th>
<th>f_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>2C</td>
<td>1</td>
</tr>
<tr>
<td>3C</td>
<td>1</td>
</tr>
<tr>
<td>4C</td>
<td>1</td>
</tr>
<tr>
<td>5C</td>
<td>1</td>
</tr>
<tr>
<td>6C</td>
<td>1</td>
</tr>
</tbody>
</table>

If \( \sigma^2 = 160 \). Find the value of \(|C|\).

(1) 7 (2) 5 (3) 6 (4) 4

Answer (1)

Sol.

\[
\begin{align*}
\sigma^2 &= E(x^2) - [E(x)]^2, \\
E(x) &= \Sigma x f(x) = 22C \\
E(x^2) &= \Sigma x^2 f(x) = 92C^2 \\
\end{align*}
\]

\[
\sigma^2 = 160 = 92C^2 - \left(\frac{22C}{7}\right)^2 \\
\Rightarrow C = \pm 7
\]

5. \[ \int_{-1}^{2} \log(x + \sqrt{x^2 + 1})dx \]

(1) \( \log[(2 + \sqrt{5})^2(\sqrt{2} - 1)] + \sqrt{5} + \sqrt{2} \)
(2) \( \log[(2 + \sqrt{5})(\sqrt{2} - 1)] + \sqrt{5} - \sqrt{2} \)
(3) \( \log[(2 + \sqrt{5})^2(\sqrt{2} - 1)] + \sqrt{5} + \sqrt{2} \)
(4) \( \log(2 + \sqrt{5})^2 + \sqrt{5} + \sqrt{2} \)

Answer (1)

Sol. \[ \int_{-1}^{2} \log(x + \sqrt{x^2 + 1})dx \]

\[ = x\log(x + \sqrt{x^2 + 1}) - \left[ x \int_{-1}^{2} \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} dx \right] \]

\[ = x\log(x + \sqrt{x^2 + 1}) - \int_{-1}^{2} \frac{x}{\sqrt{x^2 + 1}} dx \]

\[ = x\log(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1} \]

\[ = [2\log(2 + \sqrt{5}) - \sqrt{5}] - [-\log(\sqrt{2} - 1) - \sqrt{2}] \]

\[ = \log((2 + \sqrt{5})^2(\sqrt{2} - 1)) - \sqrt{5} + \sqrt{2} \]

6. The sum of coefficients of \( x^3 \) and \( x^5 \) in the binomial expansion of \( \left(\frac{x^3 + x^5}{2}\right)^6 \) is

(1) \( \frac{9C_4}{2^5} \) (2) \( \frac{9C_6}{2^4} \) (3) 0 (4) \( \frac{63}{8} \)

Answer (1)

Sol. \[ T_{r+1} = \binom{6}{r} \left(\frac{x^3}{2}\right)^r \left(\frac{x^5}{2}\right)^{6-r} \]

\[ = \frac{9C_r}{2^r} \left(\frac{x^3}{2}\right)^r \left(\frac{x^5}{2}\right)^{6-r} \]

Coefficient of \( x^3 \)

\[ \Rightarrow 90 - 16r = 10 \]

\[ \Rightarrow r = 5 \]
Coefficient of $\frac{x^2}{3}$ $\Rightarrow$ $6 \cdot \frac{16r}{15} = \frac{2}{5}$
$\Rightarrow$ $90 - 16r = 6$
$\Rightarrow$ $r = \frac{84}{16} \neq I$
$\Rightarrow$ Sum = \binom{9}{5} \cdot \frac{1}{2^8} + 0$
$\Rightarrow$ $63 \cdot \frac{1}{16}$

7. Dice is thrown 3 times, then find the probability that $x_1 < x_2 < x_3$. (Here $x_1$, $x_2$, $x_3 \in [1, 6]$)
(Where $x_1$, $x_2$, $x_3$ are outcomes on dice)
(1) $\frac{7}{54}$
(2) $\frac{5}{54}$
(3) $\frac{11}{54}$
(4) $\frac{17}{54}$

Answer (2)
Sol. Given condition is $x_1 < x_2 < x_3$
So, $n(E) = \binom{6}{3}$
$n(s) = 6^3 = 216$
Then required probability $= \frac{\binom{6}{3}}{216}$
$= \frac{20}{216} = \frac{5}{54}$

8. If $f'(x) = 3f(x) + x$ and $f(0) = 1$, then $f(x)$ is
(1) $-\frac{x}{3} + \frac{10}{9}e^{-3x}$
(2) $-\frac{x}{3} + \frac{10}{9}e^{3x}$
(3) $-\frac{x}{3} - \frac{10}{9}e^{-3x}$
(4) $-\frac{x}{3} - \frac{10}{9}e^{3x}$

Answer (2)
Sol. $\frac{dy}{dx} = 3y + x$
$\Rightarrow \frac{dy}{dx} - 3y = x$
IF $= e^{\int -3dx} = e^{-3x}$
$y \cdot e^{-3x} = \int e^{-3x} \cdot x + c$
$\Rightarrow y \cdot e^{-3x} = -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + c$
$y = -\frac{1}{3}x - \frac{1}{9} + c \cdot e^{3x}$
$y(0) = 1$
$1 = -\frac{1}{9} + c \Rightarrow c = \frac{10}{9}$
$y = -\frac{x}{3} + \frac{10}{9}e^{3x}$

9. Find the area bounded by ellipse $x^2 + 3y^2 = 18$ below the line $y = x$ is (in first quadrant)
(1) $3\pi - \frac{1}{4}$
(2) $\sqrt{3}\pi$
(3) $3\pi - \frac{3}{4}$
(4) $3\pi + \frac{1}{4}$

Answer (2)
Sol. Area $= \int_0^{3\sqrt{2}} xdx + \int_0^{3\sqrt{2}} \sqrt{\frac{18 - x^2}{3}}dx$
$= \frac{1}{2}(x^2)_{x=0}^{x=3\sqrt{2}} + \frac{1}{\sqrt{3}} \int \frac{x}{\sqrt{18 - x^2}} + 9\sin^{-1}\left(\frac{x}{3\sqrt{2}}\right)_{x=0}^{x=3\sqrt{2}}$
\[
\frac{1}{2} \left( \frac{9}{2} \right) + \frac{1}{\sqrt{3}} \left[ 9 \sin^{-1}(1) - \frac{3 \sqrt{3}}{2 \sqrt{2}} - 9 \sin^{-1} \left( \frac{1}{2} \right) \right] \\
= \frac{9}{4} + \frac{1}{\sqrt{3}} \left( \frac{9 \pi}{2} - \frac{9 \sqrt{3}}{4} - \frac{9 \pi}{6} \right) \\
= \sqrt{3} \pi \\
\]

10. \( x^2 - \sqrt{2} x - \sqrt{3} = 0 \) and \( P_n = \alpha^n + \beta^n \). The value of \( P_{12} - \sqrt{2} P_{11} - \sqrt{3} P_{10} + P_{11} - \sqrt{2} P_{10} \) is

(1) \( \sqrt{3} P_9 \)  
(2) \( 2 + \sqrt{5} \) \( P_9 \)  
(3) \( \sqrt{5} P_9 \)  
(4) \( 3 + \sqrt{5} \) \( P_9 \)

Answer (1)

Sol. \( x^2 - \sqrt{2} x - \sqrt{3} = 0 \)

\( \alpha^2 - \sqrt{2} \alpha - \sqrt{3} = 0 \)

\( \alpha^{n+2} - \sqrt{2} \alpha^{n+1} - \sqrt{3} \alpha^n = 0 \) \( \ldots (i) \)

Similarly,

\( \beta^{n+2} - \sqrt{2} \beta^{n+1} - \sqrt{3} \beta^n = 0 \) \( \ldots (ii) \)

\( (\alpha^{n+2} + \beta^{n+2}) - \sqrt{2}(\alpha^{n+1} + \beta^{n+1}) - \sqrt{3}(\alpha^n + \beta^n) \)

\( P_{n+2} - \sqrt{2} P_{n+1} - \sqrt{3} P_n = 0 \) \( \ldots (iii) \)

Put \( n = 10 \)

\( P_{12} - \sqrt{2} P_{11} - \sqrt{3} P_{10} + P_{11} - \sqrt{2} P_{10} = 0 \)

Now in the expression

\( P_{12} - \sqrt{2} P_{11} - \sqrt{3} P_{10} + P_{11} - \sqrt{2} P_{10} = P_{11} - \sqrt{2} P_{10} \)

Put \( n = 9 \) in eq. (iii)

\( P_{11} - \sqrt{2} P_{10} - \sqrt{3} P_9 = 0 \)

\( P_{11} - \sqrt{2} P_{10} = \sqrt{3} P_9 \)

11. If range of function \( f(x) = \frac{1}{2 + \sin(3x) + \cos(3x)} \) is \([a, b]\). If \( \alpha \) and \( \beta \) be arithmetic and geometric mean of \( a, b \) then \( \left( \frac{\alpha}{\beta} \right) \) is equal to

(1) \( \frac{1}{\sqrt{2}} \)  
(2) \( \sqrt{2} \)  
(3) \( \frac{1}{2} \)  
(4) \( \sqrt{3} \)

Answer (2)

Sol. \( f(x) = \frac{1}{2 + \sin(3x) + \cos(3x)} \)

\( \sin(3x) + \cos(3x) \in [-\sqrt{2}, \sqrt{2}] \)

\( 2 + \sin(3x) + \cos(3x) \in [2 - \sqrt{2}, 2 + \sqrt{2}] \)

\( \Rightarrow \frac{1}{2 + \sin(3x) + \cos(3x)} \in \left[ \frac{1}{2 + \sqrt{2}}, \frac{1}{2 - \sqrt{2}} \right] \)

\( a = \frac{1}{2 + \sqrt{2}} = \frac{2 - \sqrt{2}}{2} \)

\( b = \frac{1}{2 - \sqrt{2}} = \frac{2 + \sqrt{2}}{2} \)

\( \alpha = \frac{a + b}{2} = \frac{1}{4}(2 - \sqrt{2})(2 + \sqrt{2}) = \frac{1}{2} = \frac{1}{2} \sqrt{2} \)

Then, \( \frac{\alpha}{\beta} = \sqrt{2} \)

12. If \( \int_0^x \left[ 1 - (y'(t))^2 \right] dt = \int_0^x y'(t) dt \) and \( 0 \leq x \leq 3, y \geq 0, y(0) = 0 \), then find \( y'' + 1 + y \).

(1) \( \frac{x}{\sqrt{2}} - 1 \)  
(2) \( \frac{x}{\sqrt{2}} + 1 \)  
(3) \( \frac{x}{2} + 1 \)  
(4) \( \frac{x}{2} - 1 \)

Answer (2)
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13. If \( y = e^{3\sin^{-1}x} \), then value of
\[
(1-x^2) \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} \text{ at } x = \frac{1}{2}
\]
equals to

(1) \( 9e^{\frac{\pi}{2}} \) (2) \( 3e^{\frac{\pi}{2}} \)
(3) \( 3e^{\frac{\pi}{2}} \) (4) \( e^{\pi} \)

Answer (3)

Sol. \( y = e^{3\sin^{-1}x} \)
\[
\frac{dy}{dx} = e^{3\sin^{-1}x} \cdot \frac{3}{\sqrt{1-x^2}}
\]
\[
\sqrt{1-x^2} \frac{dy}{dx} = 3y
\]

Differentiating
\[
\sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{2x}{2\sqrt{1-x^2}} \frac{dy}{dx} = 3y
\]
\[
(1-x^2) \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} = 3y
\]

\[
\therefore \text{At } x = \frac{1}{2}, \quad 3y = 3e^{\frac{\pi}{2}}
\]

Option (3) is correct

14. If \( A \) is a \( 2 \times 2 \) matrix such that \( AB^{-1} = A^{-1} \) where \( B = \begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix} \). If \( C = BAB^{-1} \) and \( C \) satisfy \( C^4 + \beta C^2 + \alpha I = 0 \) then \( 2(\beta - \alpha) \) is equal to

(1) 12
(2) 8
(3) 10
(4) 14

Answer (3)

Sol. \( AB^{-1} = A^{-1} \) and \( C = BAB^{-1} = BA^{-1} \Rightarrow BA^{-1} = A \)
\[
C^4 + \beta C^2 + \alpha I = 0
\]
\[
C^2 = BA^{-1} \quad \Rightarrow \quad A^2 = B
\]
\[
A^2 = B \Rightarrow B \text{ satisfy characteristic eq.}
\]
\[
(1 - \lambda) (1 - \lambda) - 15 = 0 \Rightarrow \lambda^2 - 2\lambda - 14 = 0
\]
\[
B^2 - 2B - 14I = 0 \quad \Rightarrow \quad A^4 - 2A^2 - 14I = 0
\]
\[
\Rightarrow C^4 - 2C^2 - 14I = 0
\]
\[
\Rightarrow \beta = -2, \quad \alpha = -14
\]
\[
2\beta - \alpha = -4 + 14 = 10
\]
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Sum of infinite terms of \(a, ar, ar^2...\) and \(a^3r^3, a^3r^6, a^3r^9...\) is 57 and 9747 respectively, then \(a + 18r\) is

Answer (31)

Sol. \(a, ar, ar^2,...\)

\[
\frac{a}{1-r} = 57 \quad \text{...(1)}
\]

\(a^3r^3, a^3r^6, a^3r^9,...\)

\[
\frac{a^3}{1-r^3} = 9747 \quad \text{...(2)}
\]

Equation \((1)^3 \text{ given,}

\[
\frac{a^3}{1-r^3} \times (1-r) = 57^3 = 9747
\]

\[
\frac{a^3}{(1-r)^3} = 19
\]

\[
\Rightarrow \frac{1-r^3}{(1-r)^3} = 19
\]

\[
1 + r^2 + r = 19 + 19^2 - 38r
\]

\[
18r^2 - 39r + 18 = 0
\]

\[
\Rightarrow r = \frac{2}{3} \quad \text{and} \quad \frac{3}{2} \quad \text{rejected}
\]

\[
\therefore r = \frac{2}{3} \quad \text{and} \quad a = 19
\]

Now \(a + 18r = 19 + 12 = 31\)
22. The number of numbers between 100 to 1000 such that sum of their digits is 14, is

Answer (70.00)

Sol. Number in this range will be 3-digit number.

\[ N = abc \] such that \( a + b + c = 14 \)

Also, \( a \geq 1, \quad a, b, c \in \{0, 1, 2, ...9\} \)

Case I

All 3-digit same

\[ \Rightarrow 3a = 14 \text{ not possible} \]

Case II

Exactly 2 digits same:

\[ \Rightarrow 2a + c = 14 \]

\( (a, c) \in \{(3, 8), (4, 6), (5, 4), (6, 2), (7, 0)\} \)

\[ \Rightarrow \binom{3!}{2!} \text{ ways} \Rightarrow 5 \times 3 - 1 \]

\[ = 15 - 1 = 14 \]

Case III

All digits are distinct

\( a + b + c = 14 \)

without losing generality \( a > b > c \)

\( (a, b, c) \in \{(9, 5, 0), (9, 4, 1), (9, 3, 2), (8, 6, 0), (8, 5, 1), (8, 4, 2), (7, 6, 1), (7, 5, 2), (7, 4, 3), (6, 5, 3)\} \)

\[ \Rightarrow 8 \times 3! + 2(3! - 2!) = 48 + 8 = 56 \]

\[ = 0 + 14 + 56 = 70 \]

23. Find the number of solutions of \( 3\sin^{-1}x + 2\cos^{-1}x = \frac{2\pi}{5} \).

Answer (0)

Sol. \( \sin^{-1}x = \frac{2\pi}{5} - \pi = \frac{-3\pi}{5} \)

\[ \frac{-3\pi}{5} < \frac{-\pi}{2} \]

\[ \therefore \text{No real solution} \]

24. If \( f(x) = 2(2 - p)x - (p^2 - 6p + 8) \cos 4x + 7 \), then for what values of \( p \), does \( f(x) \) not have a vertical point?

Answer (4)

Sol. \( f(x) = 2(2 - p) + 4\sin 4x(p - 2)(p - 4) \)

\[ = (p - 2)(4\sin 4x(p - 4) - 2), \quad p \neq 2 \]

\[ 4\sin 4x(p - 4) - 2 \neq 0 \]

\[ \Rightarrow \sin 4x(p - 4) \neq \frac{1}{2} \]

\[ \sin 4x \neq \frac{1}{2} \]

\[ \frac{1}{2(p - 4)} > 1 \]

\[ \frac{1}{2(p - 4)} - 1 > 0 \quad \Rightarrow \quad y \in \left(4, \frac{9}{2}\right) \]

\[ \frac{1}{2(p - 4)} < -1 \quad \Rightarrow \quad p \in \left(7, \frac{9}{2}\right) \]

\[ \therefore \quad p = 4 \text{ is the only required value} \]