# Binomial Theorem And Its Simple Applications JEE Main PYQ - 1

**Total Time:** 25 Minute **Total Marks:** 40

## Instructions

## Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

### Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

# Binomial Theorem And Its Simple Applications

1. The term independent of $x$ in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is **(+4, -1)**

**[28-Jun-2022-Shift-2]**

**a.** 4

**b.** 120

**c.** 210

**d.** 310

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2. If the number of terms in the expansion of $(1-\frac{2}{x}+\frac{4}{x^2})^n, x \neq 0$ , is $28$ , then the sum of the coefficients of all the terms in this expansion, is : **(+4, -1)**

**[2016]**

**a.** 64

**b.** 2187

**c.** 243

**d.** 729

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3. ______________If $\alpha$ and $\beta$ be the coefficients of $x^4$ and $x^2$ respectively in the expansion of $(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6$ , then : **(+4, -1)**

**[Jan. 8, 2020 (II)]**

**a.** $\alpha+\beta=-30$

**b.** $\alpha-\beta=-132$

**c.** $\alpha+\beta=60$

**d.** $\alpha-\beta=60$

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4. If $1+x^4+x^5=\sum_{i=0}^{5} a_i\ (1+x)^i$ , for all $x$ in $R$, then $a_2$ is: **(+4, -1)**

**[Online April 12, 2014]**

**a.** -4

b. 6

c. -8

d. 10

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5. If $(27)^{999}$ is divided by $7$, then the remainder is : **(+4, -1)**

**[Online April 8, 2017]**

a. 1

b. 2

c. 3

d. 6

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6. If some three consecutive in the binomial expansion of $(x+1)^n$ is powers of $x$ are in the ratio $2:15:70$, then the average of these three coefficient is :- **(+4, -1)**

**[April 09, 2019 (II)]**

a. 964

b. 625

c. 227

d. 232

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7. If $\sum^\limits{25}_{r=0} \left\{^{50}C_{r} . ^{50-r}C_{25-r}\right\}=K\left(^{50}C_{25}\right) $ , then $K$ is equal to : **(+4, -1)**

a. $2^{25}-1$

b. $(25)^2$

c. $2^{25}$

d. $2^{24}$

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8. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then $k$ is equal to : **(+4, -1)**

**[Jan. 9, 2019 (I)]**

**a.** 14

**b.** 6

**c.** 4

**d.** 8

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**9.** The coefficient of $x^{-6}$, in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$, is______ **(+4, -1)**

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**10.** If the constant term in the binomial expansion of $\left(\frac{x^{\frac{3}{2}}}{2}-\frac{4}{x^l}\right)^9$ is $-84$ and the coefficient of $x^{-3l}$ is $2^\alpha\beta$, where $\beta <$ is an odd number, then $|\alpha l-\beta|$ is equal to_____ **(+4, -1)**

**[31-Jan-2023 Shift2]**

# Answers

## 1. Answer: c

### Explanation:

$$\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$$

$$=\left[\frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1}-\frac{\{(\sqrt{x})^2\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$=\left[\frac{(x^{1/3}+1)(x^{2/3}+1-x^{1/3})}{x^{2/3}-x^{1/3}+1}-\frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$=\left[(x^{1/3}+1)-\frac{(\sqrt{x}+1)}{\sqrt{x}}\right]^{10} \quad =(x^{1/3}-x^{-1/2})^{10}$$

$\therefore$The general term is

$T_{r+1}= \, ^{10}C_r(x^{1/3})^{10-r} (-x^{-1/2})^r = \, {10}C_r(-1) ^r x^{\frac{10-r}{3}-\frac{r}{2}}$

For independent of x , put

$$\frac{10-r}{3}-\frac{r}{2}=0 \quad \Rightarrow \quad 20-2r-3r=0$$

$$\Rightarrow \qquad 20=5r \quad \Rightarrow \quad 20-2r-3r=0$$

$\therefore \, \, T_5= \, ^{10}C_4=\frac{10\times \, 9 \times \, 8 \times 7}{4 \times 3 \times 2 \times 1}=210$



### Concepts:

#### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are $x^n$ and $y^n$ respectively.

- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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## 2. Answer: d

### Explanation:

Number of terms $= \frac{(n+1)(n+2)}{2} = 28$

$\Rightarrow n = 6$

$\therefore\ a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + .... + \frac{a_2n}{x^{2n}} = \left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$

Put $x = 1, n = 6, a_0 + a_1 + a_2 + ... + a_{2n} = 3^6 = 729$

### Concepts:

#### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are $x^n$ and $y^n$ respectively.
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3. **Answer: b**

**Explanation:**

The correct answer is B:$\alpha - \beta = -132$
Given that;
$\alpha, \beta$ are coefficient of $x^4 and\ x^2$,and;
$x + \sqrt{(x^2-1)^6} + x - \sqrt{(x^2-1)^2}$
$2\left[{}^6C_0.x^6 +^6 C_2x^4\left(x^2-1\right) +^6 C_4x^2\left(x^2-1\right)^2 +^6 C_6\left(x^2-1\right)^3\right]$
$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^2 + x^6)]$
$= 64x^6 - 96x^4 + 36x^2 - 2$
as $\alpha, \beta$ are the coefficient of $x^4, x^2$
$\therefore \alpha = -96 \,\&\, \beta = 36$
$\therefore \alpha - \beta = -132$

6)

Sol$^n$: Given that:

$\alpha, \beta$ are coefficient of $x^4$ and $x^2$

and

$x+\sqrt{(x^2-1)}^6 + \left(x-\sqrt{(x^2-1)^2}\right)$

$$= 2\left[{}^6C_0\, x^6 + {}^6C_2\, x^4(x^2-1) + {}^6C_4\, x^2(x^2-1)^2 + {}^6C_6\,(x^2-1)^3\right]$$

$$= 2\left[x^6 + 15(x^6-x^4) + 15x^2(x^4-2x^2+1) + (-1+3x^2-3x^2+x^6)\right]$$

$$= 2(32x^6 - 48x^4 + 18x^2 - 1)$$

$$= 64x^6 - 96x^4 + 36x^2 - 2$$

$\therefore$ as $\alpha, \beta$ are coefficient of $x^4$ & $x^2$

$\therefore \alpha = -96, \ \beta = 36$

$\therefore \alpha - \beta = -96 - 36$

$\boxed{\alpha - \beta = -132}$ Ans

## Concepts:

1. **Binomial Theorem:**

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are $x^n$ and $y^n$ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
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### 4. Answer: a

### Explanation:

$1 + x^4 + x^5 = a_0 + a_1(1+x) + a_2(1+x)^2 +$
$a_3(1+x)^3 + a_4(1+x)^4 + a_5(1+x)^5$
$= a_0 + a_1(1+x) + a_2\left(1 + 2x + x^2\right) + a_3(1 + 3x$
$+3x^2 + x^3\right) + a_4\left(1 + 4x + 6x^2 + 4x^3 + x^4\right) + a_5(1$
$+5x + 10x^2 + 10x^3 + 5x^4 + x^5)$
So, Coeff. of $x^i$ in $LHS$ = Coeff. of $x^i$ on $RHS$
$i = 5 \Rightarrow 1 = a_5 \ldots$(i)
$i = 4 \Rightarrow 1 = a_4 + 5a_5 = a_4 + 5$
$\Rightarrow a_4 = -4 \ldots$(ii)
$i = 3 \Rightarrow 0 = a_3 + 4a_4 + 10a_5$
$\Rightarrow a_3 - 16 + 10 = 0$
$\Rightarrow a_3 = 6 \ldots$ (iii)
$i = 2 \Rightarrow 0 = a_2 + 3a_3 + 6a_4 + 10a_5$
$\Rightarrow a_2 + 18 - 24 + 10 = 0$
$\Rightarrow a_2 = -4$

Put $x=-1$

$1=a_0$

Now differentiate w.r.t. $x$.

$4x^3+5x^4=a_1+2a_2(1+x)+3a_3(1+x)^2+\dots.$

Put $x=-1$

$\Rightarrow 1=a_1$

Again differentiate w.r.t. $x$

$12x^2+20x^3=2xa_2+6a_3(1+x)$

Put $x=-1$

$12-20=2a_2$

$\Rightarrow a_2=-4$

## Concepts:

### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \dots\dots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

# Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are $x^n$ and $y^n$ respectively.
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### 5. Answer: d

### Explanation:

$\frac{(28-1)^{999}}{7} = \frac{28\lambda-1}{7} \Rightarrow \frac{28\lambda-7+1-1}{7} = \frac{7(4\lambda-1)+6}{7}$

∴ Rem = 6

### Concepts:

#### 1. Binomial Theorem:

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## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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### 6. Answer: d

### Explanation:

$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$

$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$

$\frac{r}{n-r+1} = \frac{2}{15}$

$15r = 2n - 2r + 2$

$17r = 2n + 2$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$

$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$

$\frac{r+1}{n-r} = \frac{3}{14}$

$14r + 14 = 3n - 3r$

$3n - 17r = 14$

$\frac{2n-17r=-2}{n=16}$

$17r = 34, r = 2$

${}^{16}C_1, {}^{16}C_2, {}^{16}C_3$

$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$

$\frac{680+16}{3} = \frac{696}{3} = 232$

### Concepts:

#### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

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## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
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### 7. Answer: c

### Explanation:

$$\sum_{r=0}^{25} {}^{50}C_r . {}^{50-r}C_{25-r}$$
$$= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \times \frac{(50-r)!}{(25)!(25-r)!}$$
$$= \sum_{r=0}^{25} \frac{50!}{25!25!} \times \frac{25!}{(25-r)!(r!)}$$
$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = \left(2^{25}\right)^{50} C_{25}$$
$$\therefore K = 2^{25}$$

## Concepts:

### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

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# Properties of Binomial Theorem



- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are $x^n$ and $y^n$ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
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## 8. Answer: d

## Explanation:

$$\frac{2^{403}}{15} = \frac{2^3.\left(2^4\right)^{100}}{15} = \frac{8}{15}\left(15+1\right)^{100}$$
$$= \frac{8}{15}\left(15\lambda+1\right) = 8\lambda + \frac{8}{15}$$

$\because 8\lambda$ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15} \frac{8}{15} \Rightarrow k = 8$

### Concepts:

#### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
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9. **Answer: 5040 - 5040**

### Explanation:

$\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$

Now, $T_{r+1} = {}^9C_r \cdot \left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$

$= {}^9C_r \cdot \left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r \cdot x^{9-3r}$

Coefficient of $x^{-6}$ i.e. $9-3r=-6 \Rightarrow r=5$

So, Coefficient of $x^{-6} = {}^9C_5\left(\frac{4}{5}\right)^4 \cdot \left(\frac{5}{2}\right)^5 = 5040$

So, the correct answer is 5040.

### Concepts:

#### 1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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## 10. Answer: 98 - 98

### Explanation:

In, $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^{\ell}}\right)^9$

$T_{r+1} = {}^9C_r\frac{\left(x^{5/2}\right)^{9-r}}{2^{9-r}}\left(\frac{-4}{x^{\ell}}\right)^r$

$= (-1)^r\frac{{}^9C_r}{2^{9-r}}4^rx^{\frac{45}{2}-\frac{5r}{2}-r}$

$= 45 - 5r - 2lr = 0$

$r = \frac{45}{5+21}$ ....(1)

Now, according to the question, $(-1)^r\frac{{}^9C_r}{2^{9-r}}4^r = -84$

$= (-1)^{r\,9}C_r2^{3r-9} = 21 \times 4$

Only natural value of $r$ possible if $3r - 9 = 0$

$r = 3$ and ${}^9C_3 = 84$

$\therefore 1 = 5$ from equation (1)

Now, coefficient of $x^{-31} = x^{\frac{45}{2}-\frac{5r}{2}-\mathrm{lr}}$ at $1 = 5$, gives

$r = 5$

$\therefore {}^9c_5(-1)\frac{4^5}{2^4} = 2^{\alpha} \times \beta$

$= -63 \times 2^7$

$\Rightarrow \alpha = 7, \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

So, the correct answer is 98.

## Concepts:

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## Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
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