

Binomial Theorem And Its Simple Applications JEE Main PYQ - 1

Total Time: 25 Minute

Total Marks: 40

Instructions

Instructions

- 1. Test will auto submit when the Time is up.
- 2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
- 3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

- 1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
- 2. To deselect your chosen answer, click on the clear response button.
- 3. The marking scheme will be displayed for each question on the top right corner of the test window.



Binomial Theorem And Its Simple Applications

1. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is (+4, -1) [28-Jun-2022-Shift-2] **a**. 4 **b.** 120 **c.** 210 **d**. 310 2. If the number of terms in the expansion of $\left(1-rac{2}{x}+rac{4}{x^2}
ight)^n, x
eq 0$, is 28 , then the (+4, -1) sum of the coefficients of all the terms in this expansion, is : [2016] **a**. 64 **b.** 2187 **c.** 243 **d.** 729 **3.** ______If α and β be the coefficients of x^4 and x^2 respectively in (+4, -1) the expansion of $\left(x+\sqrt{x^2-1}
ight)^6+\left(x-\sqrt{x^2-1}
ight)^6$, then : [Jan. 8, 2020 (II)] **a.** $\alpha + \beta = -30$ **b.** $\alpha - \beta = -132$ C. $\alpha + \beta = 60$ **d.** $\alpha - \beta = 60$ **4.** If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i \ (1+x)^i$, for all x in R, then a_2 is: (+4, -1) [Online April 12, 2014] **a.** -4



	b. 6	
	c. -8	
	d. 10	
5.	If $(27)^{999}$ is divided by 7, then the remainder is : [Online April 8, 2017] a. 1	(+4, -1)
	b. 2	
	c. 3	
	d. 6	
6.	If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :-	(+4, -1)
	a. 964	
	b. 625	
	c. 227	
	d. 232	
7.	If \$\sum^\limits{25}_{r=0} \left\{^{50}C_{r} . ^{50-r}C_{25- r}\right\}=K\left(^{50}C_{25}\right) \$, then <i>K</i> is equal to :	(+4, -1)
	a. $2^{25} - 1$	
	b. $(25)^2$	
	C. 2^{25}	
	d. 2^{24}	

8. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(+4, -1)



- a. 14
 b. 6
 c. 4
- **d.** 8

9. The coefficient of x^{-6} , in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is ____ (+4, -1)

10. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{3}{2}}}{2} - \frac{4}{x^l}\right)^9$ is -84 and the **(+4,** coefficient of x^{-3l} is $2^{\alpha}\beta$, where $\beta <$ is an odd number, then $|\alpha l - \beta|$ is equal to_____

[31-Jan-2023 Shift2]





Answers

1. Answer: c

Explanation:

$$\begin{bmatrix} \frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{(x-1)}{x-x^{1/2}} \end{bmatrix}^{10}$$

$$= \begin{bmatrix} \frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1} - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)} \end{bmatrix}^{10}$$

$$= \begin{bmatrix} (x^{1/3}+1)(x^{2/3}+1-x^{1/3}) - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)} \end{bmatrix}^{10} = (x^{1/3}-x^{-1/2})^{10}$$

$$\therefore \text{The general term is}$$

$$T_{r+1} = \, \wedge\{10\}C_r(x\wedge\{1/3\})\wedge\{10-r\}(-x\wedge\{-1/2\})\wedge r = \, \{10\}C_r(-1)\wedge r \times^{\{\frac\{10-r\}\}} \{3\}-\sqrt{frac}\{r\}\{2\}\}$$
For independent of x, put

$$\frac{10-r}{2} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow 20 = 5r \Rightarrow 20 - 2r - 3r = 0$$

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Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is



- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are xⁿ and yⁿ respectively.



- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

2. Answer: d

Explanation:

Number of terms $= \frac{(n+1)(n+2)}{2} = 28$ $\Rightarrow n = 6$ $\therefore a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \dots + \frac{a_2n}{x^{2n}} = \left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$ Put $x = 1, n = 6, a_0 + a_1 + a_2 + \dots + a_{2n} = 3^6 = 729$

Concepts:

1. Binomial Theorem:

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(x+y)^{n} - {}^{n}C_{0}x^{n}y^{0} + {}^{n}C_{1}x^{n-1}y^{1} + {}^{n}C_{2}x^{n-2}y^{2} + \ldots + {}^{n}C_{n-1}xy^{n-1} + {}^{n}C_{n}x^{0}y^{n}
```

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- There are (n+1) terms in the expansion of $(x+y)^n$.
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- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.



3. Answer: b

Explanation:

The correct answer is B: $\alpha - \beta = -132$ Given that; α, β are coefficient of x^4 and x^2 , and; $x + \sqrt{(x^2 - 1)^6} + x - \sqrt{(x^2 - 1)^2}$ $2 \left[{}^6C_0 \cdot x^6 + {}^6C_2 x^4 (x^2 - 1) + {}^6C_4 x^2 (x^2 - 1)^2 + {}^6C_6 (x^2 - 1)^3 \right]$ $= 2 [x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^2 + x^6)]$ $= 64x^6 - 96x^4 + 36x^2 - 2$ as α, β are the coefficient of x^4, x^2 $\therefore \alpha = -96 \& \beta = 36$ $\therefore \alpha - \beta = -132$



Date Para solo. Given jest x, B are coefficient of x and x2 and $x + (x = 1)b + (x - \sqrt{(x^2 - 1)^2})$ $b_{c_0} x^6 + b_{c_2} x^4 (x^2 - 1) + b_{c_4} x^2 (x^2 - 1)^2$ $+ b_{C_c}(x^2 - 1)^3$ (x6-x4)+15x (x4 + [-1+32-32+ 2 3226--18x2-1] 6 4x - 96x + 36x - 2 are coefficient of 24 × 2 -Pf -9%, B= 36 $\alpha =$ x-B= -96-36 2.1 x-B = -132 me

1. Binomial Theorem:



The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

 $(x+y)^n - {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

4. Answer: a

```
1 + x^{4} + x^{5} = a_{0} + a_{1}(1 + x) + a_{2}(1 + x)^{2} + a_{3}(1 + x)^{3} + a_{4}(1 + x)^{4} + a_{5}(1 + x)^{5} = a_{0} + a_{1}(1 + x) + a_{2}(1 + 2x + x^{2}) + a_{3}(1 + 3x + 3x^{2} + x^{3}) + a_{4}(1 + 4x + 6x^{2} + 4x^{3} + x^{4}) + a_{5}(1 + 5x + 10x^{2} + 10x^{3} + 5x^{4} + x^{5})
So, Coeff. of x^{i} in LHS = Coeff. of x^{i} on RHS
i = 5 \Rightarrow 1 = a_{5} \dots (i)
i = 4 \Rightarrow 1 = a_{4} + 5a_{5} = a_{4} + 5
\Rightarrow a_{4} = -4 \dots (ii)
i = 3 \Rightarrow 0 = a_{3} + 4a_{4} + 10a_{5}
\Rightarrow a_{3} - 16 + 10 = 0
\Rightarrow a_{3} = 6 \dots (iii)
i = 2 \Rightarrow 0 = a_{2} + 3a_{3} + 6a_{4} + 10a_{5}
\Rightarrow a_{2} + 18 - 24 + 10 = 0
\Rightarrow a_{2} = -4
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```
Put x = -1

1 = a_0

Now differentiate w.r.t. x.

4x^3 + 5x^4 = a_1 + 2a_2(1 + x) + 3a_3(1 + x)^2 + \dots

Put x = -1

\Rightarrow 1 = a_1

Again differentiate w.r.t. x

12x^2 + 20x^3 = 2xa_2 + 6a_3(1 + x)

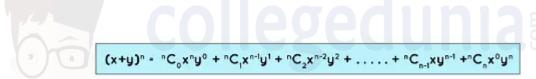
Put x = -1

12 - 20 = 2a_2

\Rightarrow a_2 = -4
```

1. Binomial Theorem:

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Explanation:

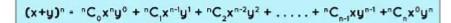
$$\frac{(28-1)^{999}}{7} = \frac{28\lambda - 1}{7} \Rightarrow \frac{28\lambda - 7 + 1 - 1}{7} = \frac{7(4\lambda - 1) + 6}{7}$$

:. Rem = 6

Concepts:

1. Binomial Theorem:

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Properties of Binomial Theorem

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6. Answer: d

$$\frac{\frac{n}{n}C_{r-1}}{\frac{n}{n}C_{r}} = \frac{2}{15}$$

$$\frac{\frac{(r-1)!(n-r+1)!}{r!(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$$15r = 2n - 2r + 2$$

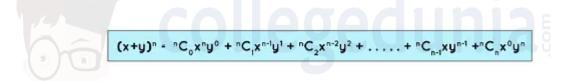
$$17r = 2n + 2$$



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\begin{split} & \frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{15}{70} \\ & \frac{r}{r!(n-r)!} \\ \hline \frac{r+1}{(r+1)!(n-r-1)!} = \frac{3}{14} \\ & 14r+14 = 3n-3r \\ & 3n-17r = 14 \\ & \frac{2n-17r=-2}{n=16} \\ & 17r = 34, r = 2 \\ & \frac{16}{C_{1}}, {}^{16}C_{2}, {}^{16}C_{3} \\ & \frac{16}{3}C_{1} + {}^{16}C_{2} + {}^{16}C_{3} \\ & \frac{680+16}{3} = \frac{696}{3} = 232 \end{split}
```

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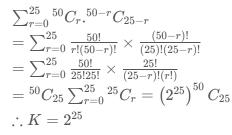


Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
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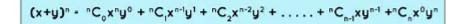
7. Answer: c





1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is



Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of (x+y)ⁿ.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

8. Answer: d

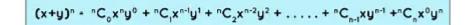
$$\frac{2^{403}}{15} = \frac{2^3 \cdot (2^4)^{100}}{15} = \frac{8}{15} (15+1)^{100}$$

= $\frac{8}{15} (15\lambda+1) = 8\lambda + \frac{8}{15}$
 $\therefore 8\lambda$ is integer
 \Rightarrow fractional part of $\frac{2^{403}}{15} \frac{8}{15} \Rightarrow k = 8$



1. Binomial Theorem:

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Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
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9. Answer: 5040 - 5040

Explanation:

 $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$ Now, $T_{r+1} = {}^9C_r \cdot \left(\frac{4x}{5}\right)^{9-r} \left(\frac{5}{2x^2}\right)^r$ $= {}^9C_r \cdot \left(\frac{4}{5}\right)^{9-r} \left(\frac{5}{2}\right)^r \cdot x^{9-3r}$ Coefficient of x^{-6} i.e. $9 - 3r = -6 \Rightarrow r = 5$ So, Coefficient of $x^{-6} = {}^9C_5 \left(\frac{4}{5}\right)^4 \cdot \left(\frac{5}{2}\right)^5 = 5040$ So, the correct answer is 5040.

Concepts:

1. Binomial Theorem:



The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

 $(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

10. Answer: 98 - 98

$$\begin{aligned} & \ln, \left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^{\ell}}\right)^{9} \\ & T_{r+1} = {}^{9}C_{r} \frac{\left(x^{5/2}\right)^{9-r}}{2^{9-r}} \left(\frac{-4}{x^{\ell}}\right)^{r} \\ &= (-1)^{r} \frac{{}^{9}C_{r}}{2^{9-r}} 4^{r} x^{\frac{45}{2} - \frac{5r}{2} - r} \\ &= 45 - 5r - 2lr = 0 \\ & r = \frac{45}{5+21} \dots (1) \\ & \text{Now, according to the question, } (-1)^{r} \frac{{}^{9}C_{r}}{2^{9-r}} 4^{r} = -84 \\ &= (-1)^{r9}C_{r} 2^{3r-9} = 21 \times 4 \\ & \text{Only natural value of } r \text{ possible if } 3r - 9 = 0 \\ & r = 3 \text{ and } {}^{9}C_{3} = 84 \\ & \therefore 1 = 5 \text{ from equation (1)} \\ & \text{Now, coefficient of } x^{-31} = x^{\frac{45}{2} - \frac{5r}{2} - \ln} \text{ at } 1 = 5, \text{ gives} \\ & r = 5 \\ & \therefore {}^{9}c_{5}(-1)\frac{4^{5}}{2^{4}} = 2^{\alpha} \times \beta \\ &= -63 \times 2^{7} \end{aligned}$$

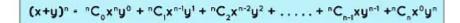


 $\Rightarrow \alpha = 7, \beta = -63$ $\therefore \text{ value of } |\alpha \ell - \beta| = 98$ So, the correct answer is 98.

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