Binomial Theorem And Its Simple Applications JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Binomial Theorem And Its Simple Applications

1. Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+.....+a_{50}\ x^{50}$, for all $x \in R$, the $\frac{a_2}{a_0}$ is equal to :- **(+4, -1)**

[1-Feb-2023 Shift 1]

a. 12.5

b. 12

c. 12.75

d. 12.25

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2. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2\left({}^2C_2+{}^3C_2+{}^4C_2+....+{}^nC_2\right)$ is: **(+4, -1)**

[2021, 24 Feb. Shift-II]

a. $\frac{n(n-1)(2n+1)}{6}$

b. $\frac{n(n+1)(2n+1)}{6}$

c. $\frac{n(2n+1)(3n+1)}{6}$

d. $\frac{n(n+1)^2(n+2)}{12}$

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3. The sum of coefficients of integral powers of x in the binomial expansion $(1-2\sqrt{x})^{50}$ is **(+4, -1)**

[2015]

a. $\frac{1}{2}(3^{50})$

b. $\frac{1}{2}(3^{50}+1)$

c. $\frac{1}{2}(3^{50}-1)$

d. $\frac{1}{2}(2^{50}+1)$

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4. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right).\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to: **(+4, -1)**

[NA April 12, 2019 (II)]

a. 36

b. -108

c. -72

d. -36

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5. The value of $({}^{21}C_1 -{}^{10}C_1) + ({}^{21}C_2 -{}^{10}C_2) + ({}^{21}C_3 -{}^{10}C_3) + ({}^{21}C_4 -{}^{10}C_4) + ... + ({}^{21}C_{10} -{}^{10}C_{10})$ is : **(+4, -1)**

[April 9, 2019 (I)]

a. $2^{21} - 2^{10}$

b. $2^{20} - 2^{9}$

c. $2^{20} - 2^{10}$

d. $2^{21} - 2^{11}$

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6. If 6th term in the expansion of $[\frac{1}{8/3} + {}^{2}\log_{10}]^{8}$ is 5600, then is equal to **(+4, -1)**

[2021, 24 Feb Shift-II]

a. (A) 5

b. (B) 4

c. (C) 8

d. (D) 10

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7. If the coefficient of x^{15} in the expansion of $(ax^3 + \frac{1}{bx^{1/3}})^{15}$ is equal to the coefficient of x^{-15} in the expansion of $(ax^{1/3} - \frac{1}{bx^3})^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) : **(+4, -1)**

[30-Jan-2023 Shift1]

a. $a = 3b$

b. $a = b$

c. $ab = 1$

d. $ab = 3$

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8. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is_______ **(+4, -1)**

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9. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315 , then $|\alpha|$ is equal to ___ **(+4, -1)**

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10. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-a}, \beta \in N$. **(+4, -1)**

Then α is equal to _________. **[31-Jan-2023 Shift1]**

Answers

1. **Answer: d**

Explanation:

$$(10+x)^{50}+(10-x)^{50}$$
$$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48}, a_0 = 2.10^{50}$$
$$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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2. **Answer: b**

Explanation:

${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2\right)$ ${}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2\right)$ $\{$ use ${}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r\}$ $= {}^{n+1}C_2 + 2\left({}^4C_3 + {}^4C_2 + {}^5C_3 + \ldots\ldots + {}^nC_2\right)$ ${}^{n+1}C_2 + 2\left({}^5C_3 + {}^5C_2 + \ldots\ldots + {}^nC_2\right) \vdots = {}^{n+1}C_2 + 2\left({}^nC_3 + {}^nC_2\right) = {}^{n+1}C_2 + 2 \cdot {}^{n+1}C_3 = \frac{(n+1)n}{2} + 2 \cdot \frac{(n+1)(n)(n-1)}{2.3} = \frac{n(n+1)(2n+1)}{6}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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3. **Answer: b**

Explanation:

for sum of integral power of x put $x = 1$ in $\frac{(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}}{2} \Rightarrow \frac{3^{50}+1}{2}$.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
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4. **Answer: d**

Explanation:

$\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}.x^8\left(2x^2 - \frac{3}{x^2}\right)^6$ its general term $\frac{1}{60}{}^6C_r2^{6-r}(-3)^r\,x^{12-r} - \frac{1}{81}{}^6C_r2^{6-r}(-3)^r12^{20-4r}$ for term independent of x, r for I^{st} expression is 3 and r for second expression is 5 $\therefore$ term independent of $x = -36$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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5. **Answer: c**

Explanation:

$$= {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} \frac{1}{2}\left\{{}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{21}\right\} - 1 = 2^{20} - 1$$

$\left({}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10}\right) = 2^{10} - 1$ $\therefore$ Required sum $= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

Concepts:

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The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + ... + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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6. Answer: d

Explanation:

Explanation:
Using binomial theorem, the 6 term of (+)^8 is 56 ^{3} ^{5}Now substituting the given terms, we get,[$\frac{1}{\frac{8}{3}}$ + ^{2}log$_{10}$]General term, $_{+1}$ = ^{8} [$\frac{1}{\frac{8}{3}}$] $^-$ (^{2}log$_{10}$) For 6 term, take = 5 $_6$ = ^{8} $_5$[$\frac{1}{\frac{8}{3}}$]^3[^{2}log$_{10}$]^5 5600 = $\frac{8\times7\times6}{3\times2\times1}$ × $\frac{1}{8}$ × ^{10}(log$_{10}$)^5 [^{8} $_3$ = $\frac{8\times7\times6}{3\times2\times1}$]
10^2 = ^{2}(log$_{10}$)^5 [(log$_{10}$ = 1)] ^{2} = 10^2 = 10Hence, the correct option is (D).

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7. Answer: c

Explanation:

Coefficient Ofx^{15} in $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$
$T_{r+1}={}^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$
$45-3r-\frac{r}{3}=15$
$30=\frac{10r}{3}$
$r=9$
Coefficient of $x^{15}={}^{15}C_9a^6b^{-9}$
Coefficient of x^{-15} in $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$
$T_{r+1}={}^{15}C_r\left(ax^{1/3}\right)^{15-r}\left(-\frac{1}{bx^3}\right)^r$
$5-\frac{r}{3}-3r=-15$
$\frac{10r}{3}=20$
$r=6$
Coefficient $={}^{15}C_6a^9\times b^{-6}$
$\Rightarrow\frac{a^9}{b^6}=\frac{a^6}{b^9}$
$\Rightarrow a^3b^3=1\Rightarrow ab=1$

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Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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8. **Answer: 29 - 29**

Explanation:

The correct answer is 29.

$(21+2)^{200}+(21-2)^{200}$

$\Rightarrow 2\left[{}^{100}C_0 21^{200}+200C_2 21^{198}\cdot 2^2+\ldots\ldots+{}^{200}C_{198}21^2\ 2^{198}+2^{200}\right]$

$\Rightarrow 2\left[49I_1+2^{200}\right]=49I_1+2^{201}$

$\text{Now },2^{201}=(8)^{67}=(1+7)^{67}=49I_2+{}^{67}C_0{}^{67}C_1\cdot 7=$

$49I_2+470=49I_2+49\times 9+29$

$\therefore\ \text{Remainder is } 29$

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- “a” is a real number
- “b” is an imaginary number

The Complex Number consists of a symbol “i” which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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9. Answer: 1 - 1

Explanation:

The correct answer is 1.

$$T_{r+1} = {}^{22}C_r \cdot \left(x^{\frac{2}{3}}\right)^{22-r} \cdot (\alpha)^r, x^{-3r}$$
$$= {}^{22}C_r \cdot x^{\frac{44}{3} - \frac{2r}{3} - 3r} (\alpha)^r$$
$$\frac{44}{3} = \frac{11r}{3}$$
$$r = 4$$
$${}^{22}C_4 \cdot \alpha^4 = 7315$$
$$\frac{22 \times 21 \times 20 \times 19}{24} \cdot \alpha^4 = 7315$$
$$\alpha = 1$$

Concepts:

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10. **Answer: 2 - 2**

Explanation:

The correct answer is 2.

$T_{r+1} = {}^{30}C_r \left(x^{2/3}\right)^{30-r} \left(\frac{2}{x^3}\right)^r$

$= {}^{30}C_r \cdot 2^r \cdot x^{\frac{60-11r}{3}}$

$\frac{60-11r}{3} < 0 \Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6$

$T_7 = {}^{30}C_6 \cdot 2^6 x^{-2}$

We have also observed $\beta = {}^{30}C_6(2)^6$ is a natural number.

$\therefore \alpha = 2$

Concepts:

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- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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