

Binomial Theorem And Its Simple Applications JEE Main PYQ - 2

Total Time: 25 Minute

Total Marks: 40

Instructions

Instructions

- 1. Test will auto submit when the Time is up.
- 2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
- 3. The clock in the top right corner will display the remaining time available for you to complete the examination.

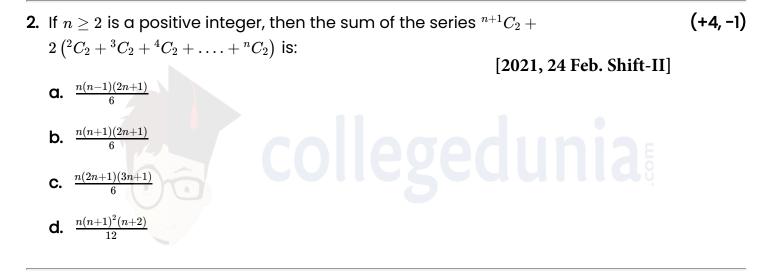
Navigating & Answering a Question

- 1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
- 2. To deselect your chosen answer, click on the clear response button.
- 3. The marking scheme will be displayed for each question on the top right corner of the test window.



Binomial Theorem And Its Simple Applications

- 1. Let $(x + 10)^{50} + (x 10)^{50} = a_0 + a_1 x + a_2 x^2 + \dots + a_{50} x^{50}$, for all $x \in R$, the $\frac{a_2}{a_0}$ is (+4, -1) equal to :- [1-Feb-2023 Shift 1]
 - **a.** 12.5
 - **b.** 12
 - **c.** 12.75
 - **d.** 12.25



- **3.** The sum of coefficients of integral powers of x in the binomial expansion (1 (+4, -1)) $2\sqrt{x})^{50}$ is [2015]
 - **a.** $\frac{1}{2}(3^{50})$
 - **b.** $\frac{1}{2}(3^{50}+1)$
 - **C.** $\frac{1}{2}(3^{50}-1)$
 - **d.** $\frac{1}{2}(2^{50}+1)$
- 4. The term independent of x in the expansion of $\left(\frac{1}{60} \frac{x^8}{81}\right) \cdot \left(2x^2 \frac{3}{x^2}\right)^6$ is equal (+4, -1) to: [NA April 12, 2019 (II)]



- **b.** -108
- **c.** -72
- **d.** -36

5. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + (+4, -1)$ $({}^{21}C_{10} - {}^{10}C_{10})$ is : [April 9, 2019 (I)] a. $2^{21} - 2^{10}$ b. $2^{20} - 2^{9}$ c. $2^{20} - 2^{9}$ d. $2^{21} - 2^{11}$ 6. If 6^{th} term in the expansion of $[-\frac{1}{8/3} + {}^{2}\log_{10}]^{8}$ is 5600, then is equal to (+4, -1) a. (A) 5 [2021, 24 Feb Shift-II] b. (B) 4 c. (C) 8

d. (D) 10

7. If the coefficient of x^{15} in the expansion of $(ax^3 + \frac{1}{bx^{1/3}})^{15}$ is equal to the (+4, -1) coefficient of x^{-15} in the expansion of $(ax^{1/3} - \frac{1}{bx^3})^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b): [30-Jan-2023 Shift1]

- **a.** a = 3b
- **b.** a = b
- **C.** ab = 1
- **d.** ab = 3



- 8. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is_____ (+4, -1)
- 9. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^{3}}\right)^{22}$ is 7315, then $|\alpha|$ is equal to (+4, -1)
- **10.** Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a (+4, term $\beta x^{-a}, \beta \in N$.Then α is equal to _____.[31-Jan-2023 Shift1]





Answers

1. Answer: d

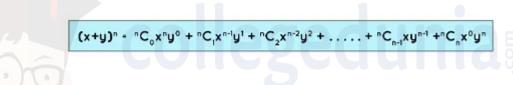
Explanation:

 $egin{aligned} &(10+x)^{50}+(10-x)^{50}\ \Rightarrow a_2 = 2.^{50}C_2 10^{48}, a_0 = 2.10^{50}\ rac{a_2}{a_0} = rac{5^0C_2}{10^2} = 12.25 \end{aligned}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is



Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

2. Answer: b

Explanation:



$${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2\right) \, {}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2\right) \, \left\{ \begin{array}{l} {\rm use} \, {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r \right\} = {}^{n+1}C_2 + 2\left({}^4C_3 + {}^4C_2 + {}^5C_3 + \dots + {}^nC_2\right) \, {}^{n+1}C_2 + 2\left({}^5C_3 + {}^5C_2 + \dots + {}^nC_2\right) \, \left[\begin{array}{l} {\rm e} \, {}^{n+1}C_2 + 2\left({}^nC_3 + {}^nC_2\right) = {}^{n+1}C_2 + 2 \cdot {}^{n+1}C_3 = \frac{(n+1)n}{2} + 2 \cdot {}^{n+1}C_3 + {}^{n+1}C_3$$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

 $(x+y)^{n} = {}^{n}C_{0}x^{n}y^{0} + {}^{n}C_{1}x^{n-1}y^{1} + {}^{n}C_{2}x^{n-2}y^{2} + \ldots + {}^{n}C_{n-1}xy^{n-1} + {}^{n}C_{n}x^{0}y^{n}$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of (x+y)ⁿ.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

3. Answer: b

Explanation:

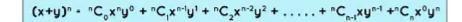
for sum of integral power of x put x = 1 in $\frac{(1-2\sqrt{x})^{50}+(1+2\sqrt{x})^{50}}{2} \Rightarrow \frac{3^{50}+1}{2}$.

Concepts:

1. Binomial Theorem:



The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is



Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
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- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

4. Answer: d

Explanation:

 $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{1}{81}x^8\left(2x^2-\frac{3}{x^2}\right)^6 \text{ its general term } \frac{1}{60}^6C_r2^{6-r}(-3)^rx^{12-r} - \frac{1}{81}^6C_r2^{6-r}(-3)^r12^{20-4r} \text{ for term independent of } x,r \text{ for } I^{st} \text{ expression is } 3 \text{ and } r \text{ for second expression is } 5 \therefore \text{ term independent of } x = -36$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

 $(x+y)^{n} - {}^{n}C_{0}x^{n}y^{0} + {}^{n}C_{1}x^{n-1}y^{1} + {}^{n}C_{2}x^{n-2}y^{2} + \ldots + {}^{n}C_{n-1}xy^{n-1} + {}^{n}C_{n}x^{0}y^{n}$



Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
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5. Answer: c

Explanation:

$$=^{21} C_1 + ^{21} C_2 + ... + ^{21} C_{10} \frac{1}{2} \left\{ {}^{21} C_0 + {}^{21} C_1 + ... + {}^{21} C_{21} \right\} - 1 = 2^{20} - 1$$

$$\left({}^{10} C_1 + {}^{10} C_2 + ... + {}^{10} C_{10} \right) = 2^{10} - 1 \therefore \text{ Required sum} = \left(2^{20} - 1 \right) - \left(2^{10} - 1 \right) = 2^{20} - 2^{10} - 1$$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are xⁿ and yⁿ respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.

• The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

6. Answer: d

Explanation:

Explanation:

Using binomial theorem, the 6 term of $(+)^8$ is 56 ^{3 5} Now substituting the given terms, we get, $[\frac{1}{\frac{8}{3}} + {}^2 \log_{10}]$ General term, ${}_{+1} = {}^8 [\frac{1}{\frac{8}{3}}]^- ({}^2 \log_{10})$ For 6 term, take $= 5_6 = {}^8 [\frac{1}{\frac{8}{3}}]^3 [{}^2 \log_{10}]^5 5600 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{1}{8} \times {}^{10} (\log_{10})^5 [{}^8]_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}]$ $10^2 = {}^2 (\log_{10})^5 [(\log_{10} = 1)]^2 = 10^2 = 10$ Hence, the correct option is (D).

7. Answer: c

Explanation:

Coefficient Of
$$x^{15}$$
 in $(ax^3 + \frac{1}{bx^{1/3}})^{15}$
 $T_{r+1} = {}^{15}C_r (ax^3)^{15-r} (\frac{1}{bx^{1/3}})^r$
 $45 - 3r - \frac{r}{3} = 15$
 $30 = \frac{10r}{3}$
 $r = 9$
Coefficient of $x^{15} = {}^{15}C_9a^6b^{-9}$
Coefficient of x^{-15} in $(ax^{1/3} - \frac{1}{bx^3})^{15}$
 $T_{r+1} = {}^{15}C_r (ax^{1/3})^{15-r} (-\frac{1}{bx^3})^r$
 $5 - \frac{r}{3} - 3r = -15$
 $\frac{10r}{3} = 20$
 $r = 6$
Coefficient $= {}^{15}C_6a^9 \times b^{-6}$
 $\Rightarrow \frac{a^9}{b^6} = \frac{a^6}{b^9}$
 $\Rightarrow a^3b^3 = 1 \Rightarrow ab = 1$

Concepts:

1. Binomial Theorem:



The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

 $(x+y)^n - {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
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8. Answer: 29 - 29

Explanation:

The correct answer is 29.

$$\begin{split} &(21+2)^{200} + (21-2)^{200} \\ &\Rightarrow 2 \left[{}^{100}C_0 21^{200} + 200 C_2 21^{198} \cdot 2^2 + \ldots + {}^{200}C_{198} 21^2 \ 2^{198} + 2^{200} \right] \\ &\Rightarrow 2 \left[49I_1 + 2^{200} \right] = 49I_1 + 2^{201} \\ &\text{Now} \ , 2^{201} = (8)^{67} = (1+7)^{67} = 49I_2 + {}^{67}C_0 {}^{67}C_1 \cdot 7 = \\ &49I_2 + 470 = 49I_2 + 49 \times 9 + 29 \\ &\therefore \text{ Remainder is } 29 \end{split}$$

Concepts:

1. Complex Number:

A Complex Number is written in the form



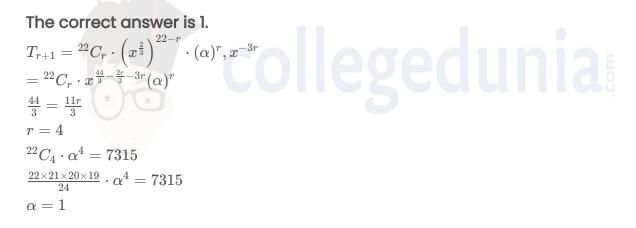
where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition $i^2 = -1$. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, –5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

9. Answer: 1 - 1

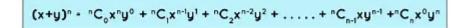
Explanation:



Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is



Properties of Binomial Theorem



- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
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- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

10. Answer: 2 - 2

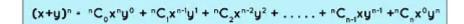
Explanation:

The correct answer is 2. $T_{r+1} = {}^{30}C_r (x^{2/3})^{30-r} (\frac{2}{x^3})^r$ $= {}^{30}C_r \cdot 2^r \cdot x^{\frac{60-11r}{3}}$ $= {}^{\frac{60-11r}{3}} < 0 \Rightarrow 11r > 60 \Rightarrow r > {}^{\frac{60}{11}} \Rightarrow r = 6$ $T_7 = {}^{30}C_6 \cdot 2^6 x^{-2}$ We have also observed $\beta = {}^{30}C_6(2)^6$ is a natural number. $\therefore \alpha = 2$

Concepts:

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