Binomial Theorem And Its Simple Applications JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Binomial Theorem And Its Simple Applications

1. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is_____ **(+4, -1)**

---

2. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n, x \neq 0 n \in N$, be 376. Then the coefficient of x^4 is ______ **(+4, -1)**

---

3. If $\left({}^{30}C_1\right)^2 + 2\left({}^{30}C_2\right)^2 + 3\left({}^{30}C_3\right)^2 + \ldots + 30\left({}^{30}C_{30}\right)^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to : **(+4, -1)**

[24-Jan-2023 Shift 2]

a. 60

b. 10

c. 15

d. 30

---

4. Coefficient of x^{18} in $(x^4 - \frac{1}{x^3})^{15}$ **(+4, -1)** **[27-Jan-2024 Shift2]**

---

5. In the expansion of $(2^{\frac{1}{4}} + 3^{-\frac{1}{4}})^n$, the ratio of 5^{th} term from start and 5^{th} term form end is $\sqrt{6} : 1$, then find 3^{rd} term **(+4, -1)**

[31-Jan-2023 Shift1]

a. $30\sqrt{3}$

b. $60\sqrt{3}$

c. 30

d. $50\sqrt{3}$

---

6. $(2x^3 - \frac{1}{3}x^8)^5 \rightarrow$ coefficient of x^4 **(+4, -1)**

[27-Jan-2024 Shift2]

a. $\frac{-80}{3}$

b. $\frac{80}{3}$

c. $\frac{40}{3}$

d. $\frac{-40}{3}$

7. If the coefficients of three consecutive terms in the expansion of (1+x)n are in the ratio 1:5:20, then the coefficient of the fourth term of the expansion is? **(+4, -1)**

[24-Jan-2023 Shift2]

a. 3654

b. 3658

c. 3600

d. 1000

8. The coefficient of x and x^2 in $(1 + x)^p$ $(1 - x)^q$ are 4 and − 5, then 2p + 3q is **(+4, -1)**

9. In a binomial distribution $B(n, p)$, the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n + p - q)$ is equal to **(+4, -1)**

[26-Jun-2022-Shift-1]

a. 50

b. 53

c. 52

d. 51

10. $A(2, 6, 2), B(-4, 0, \lambda), C(2, 3, -1)$ and $D(4, 5, 0), |\lambda| \leq 5$ are the vertices of a quadrilateral $ABCD$ If its area is 18 square units, then $5 - 6\lambda$ is equal to________ **(+4, -1)**

[26-Jul-2022-Shift-2]

Answers

1. **Answer: 1080 - 1080**

Explanation:

The correct answer is 1080.

General term is $\sum \frac{5!(2x)^{n_1}\left(x^{-7}\right)^{n_2}\left(3x^2\right)^{n_3}}{n_1!n_2!n_3!}$

For constant term,

$n_1 + 2n_3 = 7n_2$

$\& n_1 + n_2 + n_3 = 5$

Only possibility $n_1 = 1, n_2 = 1, n_3 = 3$

$\Rightarrow$ constant term $= 1080$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

2. Answer: 405 - 405

Explanation:

The correct answer is 405

Given Binomial $\left(x - \frac{3}{x^2}\right)^n, x \neq 0, n \in N,$

Sum of coefficients of first three terms

$^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n - 10)(3n + 25) = 0$

$\Rightarrow n = 10$

Now general term $^{10}C_r x^{10-r} \left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r x^{10-r} (-3)^r \cdot x^{-2r}$

$= {}^{10}C_r (-3)^r \cdot x^{10-3r}$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$\Rightarrow r = 2$

$^{10}C_2 (-3)^2 = 405$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.

- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

3. **Answer: c**

Explanation:

The correct answer is (C) : 15

$S = 0 \cdot \left({}^{30}C_0\right)^2 + 1 \cdot \left({}^{30}C_1\right)^2 + 2 \cdot \cdot \left({}^{30}C_2\right)^2 + \ldots\ldots + 30 \cdot \left({}^{30}C_{30}\right)^2$

$S = 30. \left({}^{30}C_0\right)^2 + 29. \left({}^{30}C_1\right)^2 + 28. \left({}^{30}C_2\right)^2 + \ldots\ldots + 0. \left({}^{30}C_0\right)^2$

$2S = 30 \cdot \left({}^{30}C_0{}^2 + + {}^{30}C_1^2 + \ldots\ldots\ldots \cdot {}^{30}C_{30}{}^2\right)$

$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$

$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

Concepts:

1. **Oxidation Number:**

Oxidation number, also called **oxidation state**, the total number of electrons that an a tom either gains or loses in order to form a chemical bond with another atom.

Oxidation number of an atom is defined as the charge that an atom appears to have on forming ionic bonds with other heteroatoms. An atom having higher electronegativity (even if it forms a covalent bond) is given a negative oxidation state.

The definition, assigns oxidation state to an atom on conditions, that the atom –

1. Bonds with heteroatoms.
2. Always form ionic bonding by either gaining or losing electrons, irrespective of the actual nature of bonding.

Oxidation number is a formalized way of keeping track of oxidation state.

Read More: Oxidation and Reduction

Way To Find Oxidation Number Of An Atom?

Oxidation number or state of an atom/ion is the number of electrons an atom/ion that the molecule has either gained or lost compared to the neutral atom. Electropositive metal atoms, of group I, 2 and 3 lose a specific number of electrons and have always constant positive oxidation numbers.

In molecules, more electronegative atom gain electrons from a less electronegative atom and have negative oxidation states. The numerical value of the oxidation state is equal to the number of electrons lost or gained.

Oxidation number or oxidation state of an atom or ion in a molecule/ion is assigned by:

1. Summing up the constant oxidation state of other atoms/molecules/ions that are bonded to it and
2. Equating, the total oxidation state of a molecule or ion to the total charge of the molecule or ion.

---

4. Answer: 6 - 6

Explanation:

The answer is 6

$T_{r+1} = ^{15}C_r(x^4)^{15-r}(-\frac{1}{x^3})^r$

$T_{r+1} = ^{15}C_r(-1)^r x^{60-7r}$

$60 - 7r = 18 \Rightarrow r = 6$

$T_7 = ^{15}C_6(-1)^6 x^{18}$

$T_7 = ^{15}C_6 x^{18}$

So, the Coefficient of x^{18} is $^{15}C_6$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

5. **Answer: b**

Explanation:

$$\frac{^nC_4(2^{\frac{1}{4}})^{n-4}(3^{\frac{-1}{4}})^4}{^nC_4(2^{\frac{-1}{4}})^{n-4}(3^{\frac{1}{4}})^4} = \sqrt{6}$$

$$(\frac{2^{\frac{1}{4}}}{3^{\frac{-1}{4}}})^{(n-8)} = \sqrt{6}$$

$$(6)^{\frac{n-8}{4}} = \sqrt{6}$$

$$n - 8 = 2$$

$$n = 10$$

$$T_3 = {}^{10}C_2(2^{\frac{1}{4}})^8(3^{\frac{-1}{4}})^2$$

$$= {}^{10}C_2 \times (\sqrt{2})^4 \times \frac{1}{\sqrt{3}} = 60\sqrt{3}$$

So, the correct answer is (B): $60\sqrt{3}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

6. **Answer: a**

Explanation:

The correct option is (A): $\frac{-80}{3}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.

- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

7. **Answer: a**

Explanation:

The correct option is (A): 3654

Concepts:

1. Binomial Expansion Formula:

The binomial expansion formula involves binomial coefficients which are of the form

(n/k)(or) ^nC_k and it is calculated using the formula, ^nC_k =n! / [(n - k)! k!]. The binomial expansion formula is also known as the binomial theorem. Here are the binomial expansion formulas.

- When powers are natural numbers :

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_nx^0y^n$$

(OR)

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \ldots + y^n$$

- When powers are rational numbers :

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \infty$$

Here, $|x| < 1$.

This binomial expansion formula gives the expansion of $(x + y)^n$ where 'n' is a natural number. The expansion of $(x + y)^n$ has (n + 1) terms. This formula says:

We have $(x + y)^n = nC_0\, x_n + nC1\, x_{n-1} \cdot y + nC_2\, x_{n-2} \cdot y_2 + \ldots + nC_n\, y_n$

General Term = $T_{r+1} = nCr\, x_{n-r} \cdot y_r$

- General Term in $(1 + x)^n$ is $nC_r\, x_r$
- In the binomial expansion of $(x + y)^n$, the rth term from end is $(n - r + 2)^{th}$.

---

8. Answer: 63 - 63

Explanation:

The correct answer is 63

$(1+x)^p(1-x)^q$

$(1+px+\frac{p(p-1)}{2!}x^2+......)$

$(1-qx+\frac{q(q-1)}{2!}x^2-......)$

$p-q=4$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$p^2+q^2 - p - 2 - 2pq=-10$

$(q+4)^2 + q^2 - (q-4) - q - 2(4+q)q = -10$

$q^2 + 8q + 16 - q^2 - q - 4 - q - 8q - 2q^2 = -10$

-2q=-22

so , q=11 and p=15

∴ 2p + 3q = 2(15) + 3(11)

= 30 + 33

= 63

Concepts:

1. Binomial Expansion Formula:

The binomial expansion formula involves binomial coefficients which are of the form

(n/k)(or) ^nC_k and it is calculated using the formula, ^nC_k =n! / [(n - k)! k!]. The binomial expansion formula is also known as the binomial theorem. Here are the binomial expansion formulas.

- **When powers are natural numbers :**

$(x + y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_n x^0 y^n$

(OR)

$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!} x^{n-2} y^2 + \frac{n(n-1)(n-2)}{3!} x^{n-3} y^3 + \ldots + y^n$

- **When powers are rational numbers :**

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots \infty$

Here, $|x| < 1$.

This binomial expansion formula gives the expansion of $(x + y)^n$ where 'n' is a natural number. The expansion of $(x + y)^n$ has $(n + 1)$ terms. This formula says:

We have $(x + y)^n = nC_0\ x_n + nC1\ x_{n-1} \cdot y + nC_2\ x_{n-2} \cdot y_2 + \ldots + nC_n\ y_n$

General Term $= T_{r+1} = nCr\ x_{n-r} \cdot y_r$

- General Term in $(1 + x)^n$ is $nC_r\ x_r$
- In the binomial expansion of $(x + y)^n$, the rth term from end is $(n - r + 2)^{th}$.

---

9. **Answer: c**

Explanation:

$np + npq = 5, np \cdot npq = 6$
$np(1 + q) = 5, n^2p^2q = 6$
$n^2p^2(1 + q)^2 = 25, n^2p^2q = 6$
$\frac{6}{q}(1 + q)^2 = 25$
$6q^2 + 12q + 6 = 25q$
$6q^2 - 13q + 6 = 0$
$6q^2 - 9q - 4q + 6 = 0$
$(3q - 2)(2q - 3) = 0$
$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3}$ is accepted
$p = \frac{1}{3} \Rightarrow n \cdot \frac{1}{3} + n \cdot \frac{1}{3} \cdot \frac{2}{3} = 5$
$\frac{3n+2n}{9} = 5$

$n = 9$

So $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

Concepts:

1. Binomial Distribution:

A common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters is called the binomial distribution. It summarizes the number of trials when each trial has the same probability of attaining one specific outcome. The value of a binomial is acquired by multiplying the number of independent trials by the successes.

Criteria of Binomial Distribution:

Binomial distribution models the probability of happening an event when specific criteria are met. In order to use the binomial probability formula, the binomial distribution involves the following rules that must be present in the process:

1. Fixed trials
2. Independent trials
3. Fixed probability of success
4. Two mutually exclusive outcomes

---

10. Answer: 11 - 11

Explanation:

The correct answer is 11.

$A(2,6,2) \quad B(-4,0,\lambda), C(2,3,-1) D(4,5,0)$

$\text{Area } = \frac{1}{2}|\overrightarrow{BD} \times \overrightarrow{AC}| = 18$

$$\overrightarrow{AC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

$= (3\lambda + 15)\hat{i} - \hat{j}(-24) + \hat{k}(-24)$

$\overrightarrow{AC} \times \overrightarrow{BD} = (3\lambda + 15)\hat{i} + 24\hat{j} - 24\hat{k}$

$= \sqrt{(3\lambda + 15)^2 + (24)^2 + (24)^2} = 36$

$= \lambda^2 + 10\lambda + 9 = 0$

$= \lambda = -1, -9$
$|\lambda| \leq 5 \Rightarrow \lambda = -1$
$5 - 6\lambda = 5 - 6(-1) = 11$

Concepts:

1. Binomial Distribution:

A common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters is called the binomial distribution. It summarizes the number of trials when each trial has the same probability of attaining one specific outcome. The value of a binomial is acquired by multiplying the number of independent trials by the successes.

Criteria of Binomial Distribution:

Binomial distribution models the probability of happening an event when specific criteria are met. In order to use the binomial probability formula, the binomial distribution involves the following rules that must be present in the process:

1. Fixed trials
2. Independent trials
3. Fixed probability of success
4. Two mutually exclusive outcomes