CAT 2021 QA Solution Slot 3

Q.1
Bank A offers 6% interest raw per annum compounded half yearly. Bank B and Bank C offer simple interest but the annual interest rate offered by Bank C Is W. that of Bank 8. Raju Invests a certain amount in Bank B for a certain period and Rupa Invests 10.000 in Bank C for twice that period. The interest that would accrue to RaJu during that period Is equal to the interest that would have accrued had he invested the same amount In Bank A for one year. The interest accrued, in INR. to Rupa is
1. 1436
2. 2436
3. 3436
4. 2346

Solution.
Let's solve the problem step by step:

Given:

For Bank A: 6% interest p.a compounded half-yearly, which means interest is 3% for every half year.

For Bank C: The annual interest rate is w times that of Bank B.

Raju invests P in Bank B for t time period and Rupa invests ₹10,000 in Bank C for 2t time period.

The interest that Raju earns from Bank B in t time = Interest from Bank A in 1 year.

1) Calculating interest for Bank A:

For compounded half-yearly, the amount after 1 year = P(1 + r/2)^2, where r is the rate of interest in decimal form.

Amount after 1 year = P(1 + 0.03)^2
= P(1.03)^2
= 1.0609P

Interest from Bank A for 1 year = 1.0609P - P = 0.0609P

2) Given that the interest Raju earns from Bank B for t time = 0.0609P (from the above calculation).

Let the interest rate of Bank B be R.
Interest = P * R * t = 0.0609P
=> R * t = 0.0609

3) Bank C interest rate = wR
Interest Rupa earns from Bank C = 10000 * wR * 2t
= 20000 * wR * t

Given, wR * t = 0.0609 (from the 2nd step)
=> wR * t = 0.0609 * P/P (where P cancels out)
=> wR = 0.0609/t

The interest Rupa earns = 20000 * 0.0609
= ₹1218

But this is only for t time period. Rupa invests for 2t, so the total interest is 2 * ₹1218 = ₹2436.

So, the correct answer is:
2. ₹2436.

Q.2

If $f(x) = x^2 - 7x$ and $g(x) = x + 3$, then the minimum value of f (g(x))-3x is

1. **-15**
2. **-20**
3. **-16**
4. **-12**

Solution.

To find the minimum value of $f(g(x)) - 3x$, we first need to find the expression for $f(g(x))$.

Given:

$$f(x) = x^2 - 7x$$

$$g(x) = x + 3$$

Now, $f(g(x))$ is:

$$f(g(x)) = (x+3)^2 - 7(x+3)$$

$$f(g(x)) = x^2 + 6x + 9 - 7x - 21$$

$$f(g(x)) = x^2 - x - 12$$

Now, we are interested in:

$$f(g(x)) - 3x = x^2 - x - 12 - 3x$$

$$f(g(x)) - 3x = x^2 - 4x - 12$$

To find the minimum value of the above expression, we can find the derivative and set it to zero.

$$\frac{d}{dx} (x^2 - 4x - 12) = 2x - 4$$

Setting it to zero, $2x - 4 = 0$, gives x = 2.

Plugging x = 2 into $f(g(x)) - 3x$:

$$f(2+3) - 3(2) = 2^2 - 4(2) - 12 = 4 - 8 - 12 = -16$$

So, the minimum value of $f(g(x)) - 3x$ is **-16**.

Q.3
In a tournament, a team has played 40 matches so far and won 30% of them. If they win SOS of the remaining matches. Pair overall win percentage will be 60%. Suppose they win 90% of the remaining Matches. then the total number of matches won by the warn In the tournament will be
1. 86
2. 78
3. 80
4. 84

Solution.
Given:

The team has played 40 matches and won 30% of them.

Number of matches won = $0.30 \times 40 = 12$.

Let the remaining number of matches be x.

Given that if they win 50% of the remaining matches, their overall win percentage will be 60%.

Thus, total matches = $40 + x$.

Now, they win 0.50x of the remaining matches.

So, total matches won = 12 + 0.50x.

Given that, the win percentage:

$$\frac{(12 + 0.50x)}{40 + x} = \frac{60}{100}$$
$$\Rightarrow 12 + 0.50x = 0.60(40 + x)$$
$$\Rightarrow 12 + 0.50x = 24 + 0.60x$$
$$\Rightarrow 0.10x = 12$$
$$\Rightarrow x = 120$$

Thus, they play a total of $40 + 120 = 160$ matches in the tournament.

Now, if they win 90% of the remaining 120 matches, they win 0.90 × 120 = 108 matches.

Thus, total matches won = 12 (from the first 40) + 108 = 120.

Q.4
If 3x + 2|y|+ y = 7 and x + lx| + 3y = 1, then x + 2y is

Solution.
Let's solve for x and y using the given equations:

Given:
1) $3x + 2|y| + y = 7$
2) $x + |x| + 3y = 1$

From the second equation:
$x + |x| = 1 - 3y$

Case 1: $x \geq 0$
In this case, $|x| = x$, so:
$x + x = 1 - 3y$
$2x = 1 - 3y$
$x = 0.5 - 1.5y$ ... (i)

Case 2: $x < 0$
In this case, $|x| = -x$, so:
$x - x = 1 - 3y$
This gives us 0 = 1 - 3y, which is not possible. Hence, the first case is our valid scenario.

Substitute the value of x from equation (i) into the first equation:
$3(0.5 - 1.5y) + 2|y| + y = 7$

Expanding:
$1.5 - 4.5y + 2|y| + y = 7$

$1.5 - 3.5y + 2|y| = 7$
$-3.5y + 2|y| = 5.5$

Now, for y:

Case 1: $y \geq 0$
In this case, $|y| = y$:
$-3.5y + 2y = 5.5$
$-1.5y = 5.5$
This gives a negative value for y, which is not possible in this case.

Case 2: $y < 0$
In this case, $|y| = -y$:
$-3.5y - 2y = 5.5$
$-5.5y = 5.5$
$y = -1$

Substituting this value of y in equation (i):
$x = 0.5 - 1.5(-1)$
$x = 0.5 + 1.5 = 2$

So, $x = 2$ and $y = -1$.

Finally,
$x + 2y = 2 + 2(-1) = 2 - 2 = 0$

Q.5
A shop owner bought a total of 64 shirts from a wholesale market that came in two sizes. small and large. The price of a small shirt was INR 50 less than that of 3 large shirts She paid a total of INR 5000 for the large shirts, and a total of INR 1800 for the small shirts. Then, the price of a large shirt and a small shirt together. in INR.

Solution.
Let's denote the number of small shirts as s and the number of large shirts as l.

From the problem, we know:

1) $s + l = 64$ ... (i)

Now, let's denote the cost of a large shirt as L and the cost of a small shirt as S.

From the problem, we're given:

$S = 3L - 50$ ... (ii)

She paid a total of ₹5000 for the large shirts:

$l \times L = 5000$ ... (iii)

She paid a total of ₹1800 for the small shirts:

$s \times S = 1800$ ... (iv)

From (iii), $L = \frac{5000}{l}$ ... (v)

Plug (v) into (ii):

$S = 3(\frac{5000}{l}) - 50$ ... (vi)

Using (iv) and (vi):

$s \times (3(\frac{5000}{l}) - 50) = 1800$

Expanding, we get:

$s = \frac{1800l}{(3 \times 5000) - 50l}$ ... (vii)

From (i) and (vii):

$64 = l + \frac{1800l}{(3 \times 5000) - 50l}$

Solving the above equation will give the value of \(l \). Once you have \(l \), you can find \(s = 64 - l \).

With \(l \) and \(s \) known, using equations (iii) and (iv) will give \(L \) and \(S \), respectively.

Finally, the price of a large shirt and a small shirt together is \(L + S \)
Given:
Price of one large shirt = \(L \) INR
Price of one small shirt = \(S \) INR

From equation (i): \(S \) = \(3L \) - 50

Given the total amount spent on large shirts = 5000 INR:
\(L \) * \(x \) = 5000 ... (ii)

Given the total amount spent on small shirts = 1800 INR:
\(S \) * (64 - \(x \)) = 1800 ... (iii)

From equation (ii), \(x \) = \(\frac{5000}{L} \)

Using equation (i) in equation (iii), we get:
(3L - 50) * (64 - \(\frac{5000}{L} \)) = 1800

Expanding:
192L - 50(64) - 3L^2 + 15000 = 1800L

Which simplifies to:
3L^2 - 1708L + 3200 = 0

Now, let's break this quadratic equation into its factors:
3L^2 - 1500L - 208L + 3200 = 0

Grouping terms:
3L(L - 500) - 208(L - 500) = 0
(3L - 208)(L - 500) = 0

From which, L = 500 or L = 208/3.

However, given the constraints of the problem, L cannot be a fraction. Therefore, L = 500.

Using the value of L in equation (i), we get:
S = 3(500) - 50 = 1450 - 50 = 1400.

Therefore, the price of a large shirt is 500 INR and the price of a small shirt is 1400 INR.

Sum of the prices of a large and a small shirt = 500 + 1400 = 1900 INR.

Q.6
Mira and Amal walk along a circular track. starting from the same point at the same time. If they walk in the same direction, then in 46 minutes. Amal completes exactly 3 more rounds than Mira. If they walk in opposite directions. then they mat for the first time exactly after 3 minutes. The number of rounds Mira walks in one hour

Solution.
Let's break the problem down step by step.

Let r be the rate at which Mira walks and a be the rate at which Amal walks. Let the circumference of the circular track be C.

1) Walking in the same direction:

In 46 minutes, the relative distance covered by Amal with respect to Mira (since they're moving in the same direction) is equivalent to 3 rounds.

So, $46(a - r) = 3C$

From this, $a - r = \frac{3C}{46}$ ... (i)

2) Walking in opposite directions:

When moving in opposite directions, their relative speed gets added.

So, in 3 minutes, they've covered a distance equivalent to the circumference of the track (because they meet after Amal has walked a full circle more than Mira).

This means $3(a + r) = C$

From this, $a + r = \frac{C}{3}$ ... (ii)

Now, summing equations (i) and (ii):

$2a = \frac{3C}{46} + \frac{C}{3}$

To get Mira's speed, subtract (i) from (ii):

$2r = \frac{C}{3} - \frac{3C}{46}$
$r = \frac{C}{6} - \frac{3C}{92}$
$r = \frac{11C}{46}$

This means Mira covers a distance equivalent to $\frac{11}{46}$ of the track in one minute.

In 60 minutes (1 hour), she covers $\frac{11 \times 60}{46} = 14.35$ times the circumference of the track.

So, **Mira walks 14 rounds in one hour** (because we'll only consider the complete rounds).

Q.7
If a certain weight of an alloy of silver and copper is mixed with 3 kg of pure silver. the resulting alloy will have 90% silver by weight. If the same weight of the Initial alloy is mixed with 2 kg of another

alloy which has 90% silver by weight, the resulting alloy will have 84% silver by weight. Then the weight of the initial alloy, in kg. is

1. **3.5**
2. **4**
3. **3**
4. **2.5**

Solution.
Let the initial weight of the alloy be x kg.
Let the percentage of silver in the initial alloy be y (in decimal form).

Case 1: Mixing with pure silver
When the alloy is mixed with 3 kg of pure silver, the resulting alloy will have 90% silver by weight.

Silver from the initial alloy = xy kg.
Silver from 3 kg pure silver = 3 kg.

Total silver in the mixture = $xy + 3$ kg.
Total weight of the mixture = $x + 3$ kg.
Percentage of silver = $\frac{xy + 3}{x + 3} = 0.9$

From this, $xy + 3 = 0.9(x + 3)$

Equation (1): $xy = 0.9x + 0.3$

Case 2: Mixing with 90% silver alloy
When the same weight of the initial alloy is mixed with 2 kg of another alloy which has 90% silver by weight, the resulting alloy will have 84% silver by weight.

Silver from the initial alloy = xy kg.
Silver from 2 kg of 90% silver alloy = 1.8 kg.

Total silver in the mixture = $xy + 1.8$ kg.
Total weight of the mixture = $x + 2$ kg.

Percentage of silver = $\frac{xy + 1.8}{x + 2} = 0.84$

From this, $xy + 1.8 = 0.84(x + 2)$

Equation (2): $xy = 0.84x + 0.32$

Subtracting (2) from (1):

$0.06x = -0.02$
or
$x = -\frac{0.02}{0.06}$
or
x = -1/3

Q.8
If n is a positive integer such that $(\sqrt[7]{10})(\sqrt[7]{10})^2 \ldots(\sqrt[7]{10})^n > 999$, then the smallest value of n is

Solution.
Given that:

$$(7\sqrt{10})(7\sqrt{10^2}).....(7\sqrt{10^n}) > 999$$

This implies:

$$10^{1/7} \times 10^{2/7} \times .... \times 10^{n/7} > 999$$

By multiplying powers with the same base, you add the exponents:

$$10^{(1/7 + 2/7 + ... + n/7)} > 999$$
$$10^{(n(n+1)/14)} > 999$$

Now, we know $10^3 = 1000$ and that's the closest power of 10 to 999.

So,
$$n(n+1)/14 > 3$$

$$n(n+1) > 42$$

To find the smallest value of n for which this is true:

For $n = 6$, $6*7 = 42$

For $n = 7$, $7*8 = 56$ which is greater than 42.

So, the smallest value of n for which the inequality holds is n = 7

Q.9
The number of distinct pairs of integers (m, n) satisfying |1+mn| < |m+n | < 5 is

Solution.
Given:
$$|1+mn| < |m+n| < 5$$

To find the distinct pairs (m, n) that satisfy the above conditions, we'll tackle each inequality separately.

1) For $|1+mn| < |m+n|$:
This inequality is satisfied if either of the following conditions hold:
a) $1+mn > 0$ and $1+mn < m+n$
b) $1+mn < 0$ and $1+mn > -(m+n)$

2) For $|m+n| < 5$:
This inequality gives us four possible conditions:
a) $m+n < 5$
b) $m+n > -5$
c) $-(m+n) < 5$ or $m+n > -5$ (which is the same as the above condition)
d) $-(m+n) > -5$ or $m+n < 5$ (which is also the same as the first condition)

Considering the range for |m+n| which is (-5, 5), we can make a rough estimate:

For m = 0, n can range from -5 to 4.
For m = 1, n can range from -6 to 3.
Similarly, for m = 2, n can range from -7 to 2.
This pattern continues until the value of m+n reaches 5 or m+n reaches -5.

Now, considering the first inequality \(|1+mn| < |m+n| \), we'll check for values within our defined range.

By testing pairs, we can derive the following pairs that satisfy both inequalities:

(0,1), (0,2), (0,3), (0,4), (1,0), (1,-1), (1,-2), (1,-3), (1,2), (1,3), (-1,0), (-1,1), (-1,2), (-1,-3), (2,1), (2,-1), (2,-2), (2,-4), (-2,1), (-2,-1), (-2,2), (-2,-4), (3,0), (3,-1), (3,-2), (3,-5), (-3,0), (-3,-1), (-3,2), (-3,-5), (4,-1), (4,-2), (4,-3), (-4,-1), (-4,2), (-4,-3)

There are 36 pairs in total that satisfy both inequalities. So, the number of distinct pairs (m, n) is 36.

Q.10
One day. Rahul started a work at 9 AM and Gautam joined him two hours later. They then worked together and completed the work at 5 PM the same day. If both had started at 9 AM and worked together. the work would have been completed 30 minutes earlier. Working alone, the time Rahul would have taken, in hours, to complete the work is

Solution.
Let's denote the work rate of Rahul as \(R \) work/hour and the work rate of Gautam as \(G \) work/hour.

1) When both started together at 9 AM:

The total time they worked together was 8 hours (from 9 AM to 5 PM). They would have finished the work in 7.5 hours (30 minutes earlier). So, their combined work rate when they started together would be:
$$Total \ Work = (R + G) \times 7.5$$

2) On the day Rahul started at 9 AM and Gautam joined him 2 hours later:
Rahul worked for 2 hours alone, and then they worked together for the next 6 hours (from 11 AM to 5 PM). This gives:
$$Total \ Work = 2R + 6(R + G)$$

Since the total work done in both scenarios is the same, we can equate the two expressions:
$$2R + 6(R + G) = 7.5(R + G)$$

Expanding and simplifying:
$$2R + 6R + 6G = 7.5R + 7.5G$$
$$8R + 6G = 7.5R + 7.5G$$
$$0.5R = 1.5G$$
$$R = 3G$$

Now, let's use the combined work rate from the first scenario:
$$Total \ Work = (R + G) \times 7.5$$
Using the relationship $R = 3G$, we get:
$$Total \ Work = (3G + G) \times 7.5$$
$$Total \ Work = 4G \times 7.5$$
$$Total \ Work = 30G$$

Now, using this total work with Rahul's individual work rate for the time he worked alone:
$$2R = 2(3G) = 6G$$

Subtracting this from the total work to get the work done by both together:
$$30G - 6G = 24G$$

This means that both of them, working together for 6 hours, did $24G$ of the work:

$$6(R + G) = 24G$$

Using $R = 3G$:

$$6(4G) = 24G$$

The relationship holds true.

Now, to find the time Rahul would take to complete the entire work by himself:

Using $R = 3G$:

$$Total \ Work = 30G$$

If Rahul does the entire work:

$$Time \ for \ Rahul = \frac{Total \ Work}{R} = \frac{30G}{3G} = 10 \ hours$$

Rahul would take 10 hours to complete the work by himself.

Q.11

Anil can paint a house in 12 days while Rerun can paint it in 16 days. Anil,Barun and Chandu undertake to paint the house for ₹ 24000 and the three of them together complete the painting in 6 days. If Chandu is paid in proportion to the work done by him. then the amount in INR received by him is

Solution.

Let's first determine the work rates for Anil and Barun.

Anil's work rate is $\frac{1}{12}$ of the house per day since he can complete painting the house in 12 days. Similarly, Barun's work rate is $\frac{1}{16}$ of the house per day.

When Anil, Barun, and Chandu work together, they complete the house in 6 days. So, their combined work rate is $\frac{1}{6}$ of the house per day.

Given the work rates for Anil and Barun:

$Anil + Barun + Chandu = \frac{1}{6}$

Substituting in the known rates:
$$\frac{1}{12} + \frac{1}{16} + Chandu = \frac{1}{6}$$

Finding the least common denominator, which is 48:
$$4 + 3 + 48 \times Chandu = 8$$
$$48 \times Chandu = 1$$
$$Chandu = \frac{1}{48}$$

This means Chandu's work rate is $\frac{1}{48}$ of the house per day. Over 6 days, Chandu would have completed:
$$6 \times \frac{1}{48} = \frac{1}{8}$$
or 12.5% of the house.

So, Chandu's share of the payment is 12.5% of ₹24000:
$$0.125 \times 24000 = ₹3000$$

Chandu will receive ₹3000.

Q.12
The cost of fencing a rectangular plot is ₹200 per ft along one side. and ₹100 per ft along the three other sides. If the area of the rectangular plot M 60000 sq then the lowest possible cost of fencing all four sides,in INR, is
1. 120000
2. 100000
3. 160000
4. 90000

Solution.
Let's denote the length of the rectangular plot as l ft and the breadth as b ft.

Given: Area $A = l \times b = 60000$ sq ft ... (1)

Cost of fencing for the length = $200l$

Cost of fencing for the breadth = $100b$
Since there are 2 lengths and 2 breadths, the total cost is:
$$C = 2(200l) + 2(100b)$$
$$C = 400l + 200b$$

From equation (1), we have $b = \frac{60000}{l}$

Substituting this value in the equation for C, we get:
$$C(l) = 400l + 200 \times \frac{60000}{l}$$
$$C(l) = 400l + \frac{12000000}{l}$$

To minimize C, differentiate C with respect to l and equate to 0.
Taking the derivative and setting it to zero will give us the value of l for which C is minimum.

$$\frac{dC}{dl} = 400 - \frac{12000000}{l^2} = 0$$
$$400 = \frac{12000000}{l^2}$$
$$l^2 = \frac{12000000}{400}$$
$$l^2 = 30000$$
$$l = 173.2$$ (taking the positive square root because length cannot be negative)

Using equation (1), $b = \frac{60000}{173.2} = 346.4$

So, for the lowest possible cost of fencing:
$$C = 2(200 \times 173.2) + 2(100 \times 346.4)$$
$$C = 69440 + 69280 = ₹138720$$

The lowest possible cost of fencing all four sides is ₹138,720.

Q.13
AfoundIpltnumber Is formed by using only the digits 1. 2 and 3 such that both 2 and 3 appear at least once. The number of all such four-digit numbers is

Solution.
For the four-digit numbers using only the digits 1, 2, and 3 where both 2 and 3 appear at least once, we can break down the problem into cases:

Case 1: Two occurrences of 1, one of 2, and one of 3.
The number of ways to arrange two 1s, one 2, and one 3 = 4!/2! = 12 ways. (Divide by 2! because there are two 1s that are identical.)

Case 2: One occurrence of 1, two of 2, and one of 3.
The number of ways to arrange one 1, two 2s, and one 3 = 4!/2! = 12 ways.

Case 3: One occurrence of 1, one of 2, and two of 3.
The number of ways to arrange one 1, one 2, and two 3s = 4!/2! = 12 ways.

Summing up the number of ways from all the cases = 12 + 12 + 12 = 36.

So, there are 36 such four-digit numbers.

Q.14
In a triangle ABC, ∠BCA = 50°. D and E are points on AB and AC. respectively, such that AD = DE. If F is a point on BC such that BD = DF, then ∠FDE. in degrees, is equal to

Solution.
Given:

1. Triangle ABC with ∠BCA = 50°.
2. Points D on AB and E on AC with AD = DE.
3. Point F on BC with BD = DF.

To Find: ∠FDE.

Solution:

Since AD = DE and BD = DF, triangle BDF is isosceles with BD = DF.
So, $\angle BDF = \angle DBF$.

Also, since AD = DE, triangle ADE is isosceles.
Thus, $\angle ADE = \angle AED$.

Now, in triangle ABC, we have:
$\angle ABC + \angle BCA + \angle BAC = 180°$ (angles in a triangle sum to 180°).

So, $\angle ABC = 180° - 50° - \angle BAC$
$= 130° - \angle BAC$.

Now, using the property of exterior angles:
$\angle ADE = \angle ABC + \angle BCA$
$= 130° - \angle BAC + 50°$
$= 180° - \angle BAC$.

But since triangle ADE is isosceles,
$\angle ADE = \angle AED$.

Thus, $\angle AED = 180° - \angle BAC$.

Now, moving to triangle BDF, we have:
$\angle DBF + \angle BFD + \angle FDB = 180°$.

Given that triangle BDF is isosceles, we have:
$\angle DBF = \angle BFD$.

Thus, $2\angle DBF + \angle FDB = 180°$.

But we know,
$\angle FDB = 180° - \angle BAC - \angle AED$
$= 180° - \angle BAC - (180° - \angle BAC)$
$= \angle BAC$.

Thus, $2\angle DBF + \angle BAC = 180°$

=> $2\angle DBF = 180° - \angle BAC$.

This implies,
$\angle DBF = 90° - (1/2)\angle BAC$.

We are to find $\angle FDE$.

Now, $\angle FDE = 180° - \angle ADE - \angle BDF$
$= 180° - (180° - \angle BAC) - (90° - (1/2)\angle BAC)$
$= 180° - 180° + \angle BAC - 90° + (1/2)\angle BAC$
$= \angle BAC - 90° + (1/2)\angle BAC$
$= (3/2)\angle BAC - 90°$.

$\angle FDE$ in terms of $\angle BAC$ is $(3/2)\angle BAC - 90°$.

Q.15 Consider a sequence of real numbers x_1, x_2, x_3.......such that $x_{n+1} = x_n + n-1$ for all $n \geq 1$. If $x_1=-1$ then x_{100} is equal to

Solution.
Given the recurrence relation:

\[x_{n+1} = x_n + n

Given:

\[x_{n+1} = x_n + n \]
\[x_1 = -1 \]

To find: \(x_{100} \)

Using the relation, we can determine:

\[x_2 = x_1 + 1 = -1 + 1 = 0 \]
\[x_3 = x_2 + 2 = 0 + 2 = 2 \]
\[x_4 = x_3 + 3 = 2 + 3 = 5 \]

And so on...

The pattern can be seen as:

$$ x_{n+1} = x_n + n $$

$$ x_n = x_{n-1} + (n-1) $$

By summing up the terms, you can deduce:

$$ x_n = x_1 + 1 + 2 + 3 + ... + (n-1) $$

Using the formula for the sum of the first n natural numbers:

$$ S_n = \frac{n(n-1)}{2} $$

Plugging this into our equation:

$$ x_n = x_1 + \frac{n(n-1)}{2} $$

$$ x_n = -1 + \frac{n(n-1)}{2} $$

Now, for x_{100}:

$$ x_{100} = -1 + \frac{100(99)}{2} $$

$$ x_{100} = -1 + 4950 = 4949 $$

So, $x_{100} = 4949$.

Q.16 A tea shop offers tea in cups of three different sizes. The product of the prices, in INR. of three different sixes is equal to 800. The prices of the smallest size and the medium size are in the ratio 2: 5. If the shop owner decides to increase the prices of the smallest and the medium ones by INR 6 keeping the price of the largest size unchange,. the product then changes to 3200. The sum of the original prices of three different sixes, in INR, is

Solution.

Let's denote the prices of the cups as:

- Small size = s
- Medium size = m
- Large size = l

Given:
1) $s \times m \times l = 800$
2) $s : m = 2 : 5$ or $m = \frac{5}{2} s$

Substituting the value of m in the first equation:
$s \times \frac{5}{2} s \times l = 800$
=> $5s^2l = 1600$
=> $s^2l = 320$ -----(i)

Now, when the prices of the smallest and the medium ones are increased by 6:
New price of small size = $s + 6$
New price of medium size = $m + 6$

Given:
$(s + 6) \times (m + 6) \times l = 3200$
Substituting the value of m:
$(s + 6) \times (\frac{5}{2} s + 6) \times l = 3200$

Expanding and rearranging:
$5s^2l + 15s^2 + 5s^2 + 15sl + 12l = 3200$

From equation (i):
$s^2l = 320$
=> $5s^2l = 1600$

Thus:
$30sl + 12l = 1600$
=> $l(30s + 12) = 1600$
=> $l(30s + 12) = 2 \times 800$

From this, since $s^2l = 320$ is constant from the first scenario, the only way $l(30s + 12)$ could become twice of 800 is if l is halved.

So, the new $l = \frac{l}{2}$

Old $l = 2 \times \frac{l}{2} = l$

Now, using $s \times m \times l = 800$:
$s \times \frac{5}{2} s \times l = 800$
=> $5s^2l = 1600$

Given $s^2l = 320$:
=> $s^2 = \frac{320}{l}$
=> $5s^2 = \frac{1600}{l}$

From which $l = 5$

Substituting in the ratio of $s : m$:
$s = 10$ and $m = 25$

Sum of the original prices = $s + m + l = 10 + 25 + 5 = 40$

So, the sum of the original prices of the three different sizes is INR 40.

Q.17
1 Fora real number a, if ($\log_{15} a + \log_{32} a$)/($\log_{15} a$)($\log_{32} a$)= 4 then a must lie in the range
1. 4 < a <5
2. 3 <a <4
3. 2 <a <3
4. a > 5

Solution.
Given the equation:
$\frac{\log_{15} a + \log_{32} a}{(\log_{15} a)(\log_{32} a)} = 4$

First, let's express the logarithm with base 32 in terms of base 15 using change of base formula:

$log_{32} a = \frac{log_{15} a}{log_{15} 32}$

Substitute this expression into the given equation:

$\frac{log_{15} a + \frac{log_{15} a}{log_{15} 32}}{log_{15} a \times \frac{log_{15} a}{log_{15} 32}} = 4$

Now, multiplying through by $log_{15} 32$:

$log_{15} 32 \times log_{15} a + log_{15} a = 4 \times log_{15} a \times log_{15} 32$

Expanding:

$log_{15} 32 \times log_{15} a + log_{15} a = 4 \times log_{15} a^2$

Now, in order to solve for 'a', we need to consider the properties of logarithm functions:

For $0 < a < 1$, $log_{15} a < 0$.
For $a = 1$, $log_{15} a = 0$.
For $a > 1$, $log_{15} a > 0$.

If $2 < a < 3$, then both $log_{15} 32$ and $log_{15} a$ are positive but less than 1. The product will thus be less than 1, which doesn't satisfy the equation.

If $a > 5$, then both $log_{15} 32$ and $log_{15} a$ are greater than 1. Their sum is thus greater than their product, which fits the given equation.

Thus, from the given options, the correct choice is:
a > 5

Q.18
A park Is shaped like a rhombus and has area 96 sq rn If 40 m of fencing is needed to enclose the park,the cost,in INR. of laying electric wires along its two diagonals, at the rate of ₹125 per m, is

Solution.

Q.19
One pad of a hostel's monthly expenses is fixed. and the other part is proportional to the number of its boarders. The hostel collects If ₹1600 per month from each boarder. When the number of boarders is 50. the profit of the hostel is ₹ 200 per boarder. and when the number of boarders Is 75, the profit of the hostel Is ₹250 per boarder. When the number of boarders is 60. the total profit of the hostel. In INR. will be Ans A
1. 20800
2. 20500
3. 20200
4. 70000

Solution.
Let's break down the problem.

Let the fixed cost of the hostel be F and the variable cost per boarder be V.

1) When there are 50 boarders:
Total collection = $50 \times 1600 = 80000$
Total expenses = $F + 50V$
Profit for 50 boarders = $80000 - (F + 50V) = 200 \times 50 = 10000$
=> $F + 50V = 70000$ ...(i)

2) When there are 75 boarders:
Total collection = $75 \times 1600 = 120000$
Total expenses = $F + 75V$
Profit for 75 boarders = $120000 - (F + 75V) = 250 \times 75 = 18750$

=> \(F + 75V = 101250 \) ...(ii)

From equation (i), we can express \(F \) in terms of \(V \):
\[F = 70000 - 50V \] ...(iii)

Substitute the value of \(F \) from (iii) into (ii):
\[70000 - 50V + 75V = 101250 \]
=> \(25V = 31250 \)
=> \(V = 1250 \)

Substitute \(V \) in (iii) to get:
\[F = 70000 - 50 \times 1250 \]
\[F = 70000 - 62500 = 7500 \]

Now, for 60 boarders:
Total collection = \(60 \times 1600 = 96000 \)
Total expenses = \(7500 + 60 \times 1250 = 7500 + 75000 = 82500 \)
Profit for 60 boarders = \(96000 - 82500 = 13500 \)

Therefore, when the number of boarders is 60, the total profit of the hostel is ₹13,500.

Q.20
Let ABCD be a parallelogram. The lengths of the side AD and the diagonal AC are 10 cm and 20 cm. respectively. If the angle ∠ADC is equal to 30° then the area of the parallelogram in sq. cm, is

Solution.
Given that ABCD is a parallelogram, and AD = 10 cm, AC = 20 cm, and ∠ADC = 30°.

Let's solve for the area of the parallelogram.

In triangle ADC, we can use the sine rule to find the altitude (height) from D to AC (let's call it h).

$$\sin(\angle ADC) = \frac{h}{AD}$$

Given that,
$$\sin(30°) = \frac{h}{10}$$
$$\frac{1}{2} = \frac{h}{10}$$
$$h = 5 \text{ cm}$$

Now, the area of a parallelogram is given by:
$$\text{Area} = \text{base} \times \text{height}$$
$$= AC \times h$$
$$= 20 \text{ cm} \times 5 \text{ cm}$$
$$= 100 \text{ cm}^2$$

So, the area of the parallelogram ABCD is 100 cm^2.

Q.21

The total of male and female populations in a city increased by 25% from 1970 to 1980. During the same period, the male population increased by 40% while the female population increased by 20%. From 1980 to 1990. the female population increased by 25%. In 1990, if the female population is twice the male population, then the percentage increase in the total of male and female populations in the city from 1970 to 1990 is

Solution.

Let the initial male population in 1970 be M and the initial female population in 1970 be F.

Given:
Total population in 1970 = $M + F$
From 1970 to 1980:
Male population increased by 40% -> Male population in 1980 = $1.4M$
Female population increased by 20% -> Female population in 1980 = $1.2F$

Total population in 1980 = $1.4M + 1.2F$
Given that the total population increased by 25% from 1970 to 1980:
$$1.4M + 1.2F = 1.25(M + F)$$
$$1.4M + 1.2F = 1.25M + 1.25F$$
$$0.15M = 0.05F$$
$$3M = F$$
This gives the relation between the male and female populations in 1970.

From 1980 to 1990:
Female population increased by 25% -> Female population in 1990 = $1.25 \times 1.2F = 1.5F$

Given that in 1990, the female population is twice the male population:
$$1.5F = 2 \times \text{Male population in 1990}$$
$$\text{Male population in 1990} = 0.75F$$

Since no information is provided about how the male population changed from 1980 to 1990, we'll use the relation from the previous decade:
Male population in 1980 = $1.4M$
Given that $M = F/3$
Male population in 1980 = $1.4 \times F/3$ = $0.4667F$

Now, comparing male populations from 1980 and 1990:
$$0.4667F \to 0.75F$$
The male population increased by $0.2833F$ or approximately 60.7% of the male population in 1980.

Total population in 1990 = $1.5F + 0.75F = 2.25F$

The total population in 1970 = $M + F$ = $F/3 + F = 4F/3$

Percentage increase from 1970 to 1990:
$$\frac{(2.25F - 4F/3)}{4F/3} \times 100$$
$$\frac{2.25F - 4F/3}{4F/3} \times 100$$
$$\frac{6.75F - 4F}{4F} \times 100$$

\[\frac{2.75F}{4F} \times 100 \]
\[= 68.75\% \]

Thus, the percentage increase in the total of male and female populations in the city from 1970 to 1990 is 68.75

Q.22
The arithmetic mean of scores of 25 students in an examination is 50. Five of these students top the examination with the same score. If the scores of the other students are distinct integers with the lowest being 30. then the maximum possible score of the toppers is Case

Solution.
Given:

1. The arithmetic mean of scores of 25 students is 50. So, the total score of all students combined is \(25 \times 50 = 1250\).
2. Five students have the same score (let's call this score \(T\)). So, the combined score of these five students is \(5T\).
3. The scores of the other 20 students are distinct integers, with the lowest being 30.

To maximize the score of the toppers, we need to minimize the scores of the other 20 students.

Let's calculate the total score of the smallest possible distinct integers for 20 students:

Starting with 30, the smallest 20 distinct integers are:
30, 31, 32, 33 ... up to 49.

The sum of these scores is:

\[30 + 31 + 32 + ... + 49\]

This is an arithmetic progression, where:
$a_1 = 30$, $d = 1$, and $n = 20$.

The sum of n terms of an arithmetic progression is given by:

$$S = \frac{n}{2} [2a_1 + (n-1) d]$$

Using the values provided:

$$S = \frac{20}{2} [2(30) + (20-1) 1]$$
$$S = 10 [60 + 19]$$
$$S = 10 \times 79 = 790$$

So, the combined score of the 20 students (with the smallest possible distinct integers) is 790.

Therefore, the combined score of the 5 toppers is:
$$1250 - 790 = 460$$

The score of one topper is:
$$T = \frac{460}{5} = 92$$

So, the maximum possible score of each topper is 92.