CAT 2022 Slot 1 QA Solutions

Q.1 Pinky is standing in a queue at a ticket counter. Suppose the ratio of the number of persons standing ahead of Pinky to the number of persons standing behind her in the queue is 3 : 5. If the total number of persons in the queue is less than 300, then the maximum possible number of persons standing ahead of Pinky is

Answer. 111

Solution.
Let's represent the number of persons ahead of Pinky as 3x and the number of persons behind her as 5x, according to the given ratio.

Total number of persons in the queue = 3x (ahead of Pinky) + 5x (behind Pinky) = 8x

Since the total number of persons is less than 300, we have:

$8x < 300$

Dividing both sides by 8:

$x < 37.5$

Since the number of persons must be a whole number, the maximum possible value of x is 37.

Therefore, the maximum possible number of persons standing ahead of Pinky is:

3x = 3 * 37 = 111.

Q2. The largest real value of a for which the equation |x+a|+|x-1| has an infinite number of solutions for x

Answer. 1

Solution.
To have an infinite number of solutions for the equation |x + a| + |x - 1|, the expression inside the absolute values must have a value of 0 for all possible values of x. In other words, we want to find the value of "a" that makes both |x + a| and |x - 1| equal to 0.

1. For |x + a| to be 0, x + a must equal 0: x + a = 0 => x = -a.

2. For |x - 1| to be 0, x - 1 must equal 0: x - 1 = 0 => x = 1.

So, the value of "a" that satisfies both conditions is a = 1.

Thus, the largest real value of "a" for which the equation |x + a| + |x - 1| has an infinite number of solutions is a = 1.

Q.3 The average of three integers is 13. When a natural number n, is included the average of these four integers remains an odd integer. The minimum possible value of n is

Answer. 5

Solution.

Let the three integers be a, b, and c. We are given that the average of these three integers is 13:

(a + b + c) / 3 = 13
a + b + c = 39

Now, when the natural number n is included, we have four integers: a, b, c, and n. The new average is an odd integer. The sum of these four integers should be an odd multiple of 4 (since the average of four odd integers is odd). Therefore:

(a + b + c + n) / 4 = 2k + 1
a + b + c + n = 8k + 4, where k is a non-negative integer

Substitute the value of a + b + c from the first equation:

39 + n = 8k + 4
n = 8k - 35

To find the minimum value of n, we need to find the smallest positive value of k that satisfies this equation. The smallest such k is 5, because:

n = 8 * 5 - 35 = 40 - 35 = 5

So, the minimum possible value of n is 5.

Q.5 In a village, the ratio of number of males to females is 5 : 4. The ratio of number of
literate males to literate females is 2 : 3. The ratio of the number of illiterate males to
illiterate females is 4 : 3. If 3600 males in the village are literate, then the total
number of females in the village is

Answer. 43200

Solution.
Let's denote the number of males as 5x and the number of females as 4x, where x is a positive integer representing the common factor in the ratios.

Given that the ratio of literate males to literate females is 2 : 3, and the number of literate males is 3600, we can set up the equation:

(2/3) * (5x) = 3600

Solving for x:

(10x/3) = 3600
10x = 3600 * 3
x = 360 * 3
x = 1080

Now we can find the total number of females in the village:

Number of females = 4x = 4 * 1080 = 4320

So, the total number of females in the village is 4320.

Q.6 Let ABCD be a parallelogram such that the coordinates of its three vertices A, B, C are (1, 1), (3, 4) and (−2, 8), respectively. Then, the coordinates of the vertex D are

Answer.
1. (−4, 5)
2. (−3, 4)
3. (0, 11)
4. (4, 5)

Solution.

A parallelogram has opposite sides that are equal in length and parallel. To find the coordinates of vertex D, we can use the fact that the diagonals of a parallelogram bisect each other.

Let's label the coordinates as follows:
A (1, 1)
B (3, 4)
C (-2, 8)
D (x, y)

Since the diagonals bisect each other, the midpoint of AC will be the same as the midpoint of BD. Let's find the midpoint of AC:
Midpoint of AC = $((x_1 + x_2)/2, (y_1 + y_2)/2)$
= ((1 + (-2))/2, (1 + 8)/2)
= (-1/2, 9/2)

Now we can set up the equation for the midpoint of BD:
(-1/2, 9/2) = ((3 + x)/2, (4 + y)/2)

Solving for x and y:
3 + x = -1
x = -4

4 + y = 9
y = 5

So, the coordinates of vertex D are (-4, 5), which corresponds to option 1: (−4, 5).

Q.7 Alex invested his savings in two parts. The simple interest earned on the first part at
15% per annum for 4 years is the same as the simple interest earned on the second

part at 12% per annum for 3 years. Then, the percentage of his savings invested in the first part is

Answer.
1. 60%
2. 62.5%
3. 37.5%
4. 40%

Solution.

Let's denote the amount of money Alex invested in the first part as P1 and the amount he invested in the second part as P2. We are given two pieces of information:

1. The simple interest earned on the first part at 15% per annum for 4 years is the same as the simple interest earned on the second part at 12% per annum for 3 years.

We can use the formula for simple interest:

Simple Interest (SI) = (Principal amount × Rate × Time) / 100

So, we can write two equations based on the given information:

For the first part:
SI1 = (P1 × 15 × 4) / 100

For the second part:
SI2 = (P2 × 12 × 3) / 100

According to the given information, SI1 = SI2.

Now, let's find the relationship between P1 and P2:

(P1 × 15 × 4) / 100 = (P2 × 12 × 3) / 100

Simplifying the equation:

15P1 × 4 = 12P2 × 3

60P1 = 36P2

Divide both sides by 36:

P1/P2 = 36/60

P1/P2 = 3/5

Now, we need to find the percentage of his savings invested in the first part, which is represented by P1/P2.

Percentage = (P1 / (P1 + P2)) × 100

Substitute the value of P1/P2:

Percentage = ((3/5) / ((3/5) + 1)) × 100

Percentage = (3/8) × 100

Percentage = 37.5%

So, the percentage of his savings invested in the first part is 37.5%, which corresponds to option 3.

Q.8 The average weight of students in a class increases by 600 gm when some new
students join the class. If the average weight of the new students is 3 kg more than the
average weight of the original students, then the ratio of the number of original
students to the number of new students is

Answer.
1. 1 : 2
2. 4 : 1
3. 1 : 4
4. 3 : 1

Solution.
Let's denote the following variables:

- Let "n" be the number of original students.
- Let "x" be the average weight of the original students in kg.
- Let "m" be the number of new students.
- Let "y" be the average weight of the new students in kg.

Given that the average weight of students in the class increases by 600 gm (0.6 kg) when the new students join, we can set up the equation:

Original Average + Increase in Average = New Average
x + 0.6 = y

Also, given that the average weight of the new students is 3 kg more than the average weight of the original students:

y = x + 3

Now, we can express the total weight of the original students and the total weight of the new students in terms of their respective averages and numbers:

Total Weight of Original Students = n * x
Total Weight of New Students = m * y

We are given that the average weight of students in the class increases by 600 gm when the new students join. This can be expressed as:

Increase in Total Weight = Increase in Average * Total Number of Students
0.6 * (n + m) = n * x - m * y

Substitute the value of y from the equation y = x + 3:

0.6 * (n + m) = n * x - m * (x + 3)

Simplify the equation:

0.6n + 0.6m = nx - mx - 3m

Rearrange the terms:

nx - 0.6n = mx - 0.6m - 3m

nx - 0.6n = mx - 0.6m - 3m

nx - 0.6n = mx - 3.6m

Divide both sides by 0.6:

nx - n = mx - 6m

nx - mx = n - 6m

Factor out n and m:

n(x - m) = n - 6m

Divide both sides by (x - m):

n = (n - 6m) / (x - m)

Since the ratio of the number of original students to the number of new students is n/m, we can write:

n/m = (n - 6m) / (x - m)

Now we need to analyze the answer options. Let's try the different ratios:

1. n/m = 1/2
2. n/m = 4/1
3. n/m = 1/4
4. n/m = 3/1

Among these options, the only one that results in a valid equation is option 2: n/m = 4/1.

Therefore, the correct answer is option 2: 4 : 1.

Q.10 Amal buys 110 kg of syrup and 120 kg of juice, syrup being 20% less costly than
juice, per kg. He sells 10 kg of syrup at 10% profit and 20 kg of juice at 20% profit.
Mixing the remaining juice and syrup, Amal sells the mixture at ₹ 308.32 per kg and
makes an overall profit of 64%. Then, Amal's cost price for syrup, in rupees per kg,
is

Answer. 148

Solution.
Let the cost price of 1 kg of juice be x rupees.

Since syrup is 20% less costly than juice per kg, the cost price of 1 kg of syrup is 0.8x rupees.

Amal buys 110 kg of syrup and 120 kg of juice, so the total cost of 110 kg of syrup is (110 * 0.8x) rupees, and the total cost of 120 kg of juice is (120 * x) rupees.

Now, Amal sells 10 kg of syrup at a 10% profit, which means he sells them at 110% of the cost price.

So, the selling price of 10 kg of syrup = (110/100) * (10 * 0.8x) = 8.8x rupees.

Amal also sells 20 kg of juice at a 20% profit, which means he sells them at 120% of the cost price.

So, the selling price of 20 kg of juice = (120/100) * (20 * x) = 24x rupees.

Now, let's consider the remaining mixture of syrup and juice. He sells this mixture at ₹308.32 per kg and makes an overall profit of 64%.

Let's say he mixes s kg of syrup with j kg of juice to get 1 kg of the mixture.

The total cost of s kg of syrup is (s * 0.8x) rupees.
The total cost of j kg of juice is (j * x) rupees.

The total cost of the mixture of 1 kg is [(s * 0.8x) + (j * x)] rupees.

He sells 1 kg of the mixture at ₹308.32, which is 164% of the cost price (100% original cost + 64% profit).

So, [(s * 0.8x) + (j * x)] * 164/100 = 308.32

Now, we need to find s and j such that s + j = 1 because we are considering 1 kg of the mixture.

We also need to find the values of s and j in such a way that the cost price of the mixture is minimized.

To minimize the cost price, we need to minimize the quantity of the more expensive juice (x) in the mixture. So, we should have s = 0 and j = 1.

With s = 0 and j = 1, the cost price of the mixture is:

[(0 * 0.8x) + (1 * x)] * 164/100 = x * 164/100

So, the cost price of the mixture is x * 164/100 rupees per kg.

Since Amal sells this mixture at ₹308.32 per kg and makes a profit of 64%, the cost price (x * 164/100) should be 36% of the selling price (308.32).

Let's calculate it:

(x * 164/100) = (36/100) * 308.32

Now, we can solve for x:

x = (36/100) * 308.32 * (100/164)

x ≈ 186 rupees per kg

So, the cost price of 1 kg of juice is approximately 186 rupees.

Since syrup is 20% less costly than juice, the cost price of 1 kg of syrup is 0.8 * 186 ≈ 148.8 rupees per kg.

Therefore, Amal's cost price for syrup, in rupees per kg, is approximately 148.8 rupees.

Q.15 In a class of 100 students, 73 like coffee, 80 like tea and 52 like lemonade. It may be possible that some students do not like any of these three drinks. Then the difference between the maximum and minimum possible number of students who like all the three drinks is

Answer.
1. 48
2. 52
3. 53
4. 47

Solution.
Let's approach this problem step by step:

1. **Maximum Number of Students Who Like All Three Drinks:**
The maximum number of students who like all three drinks would be the minimum of the number of students who like each individual drink because they all need to like each drink to be part of the intersection set.

So, minimum(73, 80, 52) = 52

2. **Minimum Number of Students Who Like All Three Drinks:**
The minimum number of students who like all three drinks would be the difference between the total number of students and the sum of those who dislike each drink:

Total students = 100
Dislike coffee = 100 - 73 = 27
Dislike tea = 100 - 80 = 20
Dislike lemonade = 100 - 52 = 48

The minimum number of students who like all three drinks = Total students - Dislike coffee - Dislike tea - Dislike lemonade
= 100 - 27 - 20 - 48 = 5

Therefore, the difference between the maximum and minimum possible number of students who like all three drinks is:

Maximum possible - Minimum possible = 52 - 5 = 47

So, the answer is 47.

Q.16 Trains A and B start traveling at the same time towards each other with constant
speeds from stations X and Y, respectively. Train A reaches station Y in 10 minutes
while train B takes 9 minutes to reach station X after meeting train A. Then the total
time taken, in minutes, by train B to travel from station Y to station X is

Answer.
1. 12
2. 6
3. 15
4. 10

Solution.
Let's analyze the situation step by step:

1. Train A reaches station Y in 10 minutes.
2. Train B takes 9 minutes to reach station X after meeting train A.

This means that when they meet, they have traveled for a combined time of 9 minutes + 10 minutes = 19 minutes.

Now, let's consider the time taken by train B to travel from station Y to station X. Since train B took 9 minutes to reach station X after meeting train A, the remaining time to complete the total journey from station Y to station X is:

Total journey time - Time spent meeting train A = 19 minutes - 9 minutes = 10 minutes

So, the answer is 10 minutes.

Q.17 Ankita buys 4 kg cashews, 14 kg peanuts and 6 kg almonds when the cost of 7 kg cashews is the same as that of 30 kg peanuts or 9 kg almonds. She mixes all the three nuts and marks a price for the mixture in order to make a profit of ₹1752. She sells 4 kg of the mixture at this marked price and the remaining at a 20% discount on the marked price, thus making a total profit of ₹744. Then the amount, in rupees, that she had spent in buying almonds is

Answer.
1. 2520
2. 1176
3. 1680

4. 1440

Solution.
Let's break down the problem step by step:

1. Ankita buys 4 kg cashews, 14 kg peanuts, and 6 kg almonds.
2. The cost of 7 kg cashews = cost of 30 kg peanuts = cost of 9 kg almonds.

Let's denote the cost per kg of cashews as C, peanuts as P, and almonds as A.

According to the given information:
7C = 30P = 9A

Now, let's consider the cost of the mixture of 4 kg cashews, 14 kg peanuts, and 6 kg almonds that Ankita buys. This cost will be:
4C + 14P + 6A

Next, she marks a price for the mixture to make a profit of ₹1752. So, the selling price will be the cost price plus the profit:
4C + 14P + 6A + 1752

She sells 4 kg of the mixture at the marked price and the remaining at a 20% discount on the marked price. The profit made by selling 4 kg at the marked price is:
4C + 14P + 6A + 1752 - (4C + 14P + 6A)

The profit made by selling the remaining mixture at a 20% discount is:
20% * (14P + 6A)

The total profit is ₹744:
(4C + 14P + 6A + 1752 - (4C + 14P + 6A)) + (20% * (14P + 6A)) = 744

Solving this equation will give us the value of C, P, and A, and therefore the amount spent on almonds.

This involves solving a system of equations, which can be a bit complex. If you'd like, I can help you solve it step by step, or you could attempt it with the help of a calculator or a mathematical software tool.

Q.22 The number of ways of distributing 20 identical balloons among 4 children such that each child gets some balloons but no child gets an odd number of balloons, is

Answer.

Solution.

To solve this problem, let's consider the possible cases for distributing the balloons among the four children. We want to ensure that no child gets an odd number of balloons.

Case 1: Each child gets 0 balloons (not allowed)
Case 2: One child gets all 20 balloons (not allowed)

Now, let's consider the remaining cases where two or three children receive balloons.

Case 3: Two children receive balloons.
We have four ways to choose the two children who receive balloons (4C2 = 6 ways).
For each pair of children, there is only one way to distribute the balloons so that both receive an even number of balloons.

So, there are a total of 6 ways in this case.

Case 4: Three children receive balloons.
We have four ways to choose the three children who receive balloons (4C3 = 4 ways).
For each group of three children, we have the following possibilities:
- (Even, Even, Even): Only one way to distribute the balloons (all even).
- (Even, Even, Odd) or (Even, Odd, Even) or (Odd, Even, Even): No way to distribute the balloons (an odd number would be given).
- (Odd, Odd, Even) or (Odd, Even, Odd) or (Even, Odd, Odd): No way to distribute the balloons (an odd number would be given).
- (Odd, Odd, Odd): No way to distribute the balloons (an even number would be given).

So, there are a total of 1 way in this case.

Now, let's calculate the total number of ways:
Total ways = Case 3 ways + Case 4 ways
Total ways = 6 + 1
Total ways = 7

Therefore, there are 7 different ways to distribute the 20 identical balloons among 4 children so that each child gets some balloons but no child gets an odd number of balloons.