CAT 2022 Slot 3 QA Solutions

Q.1 A donation box can receive only cheques of 0100, 2250, and T500. On one good day, the donation box was found to contain exactly 100 cheques amounting to a total sum of t15250. Then, the maximum possible number of cheques of 1500 that the donation box may have contained, is

Solution.

Let's denote the number of cheques of 100, 2500, and 1500 as x, y, and z, respectively.

We are given that on one good day, the donation box contained exactly 100 cheques amounting to a total sum of 15250. We can express this information in the form of an equation:

$100x + 2250y + 1500z = 15250$

Now, we need to find the maximum possible value of z (the number of cheques of 1500).

To maximize the value of z, we should minimize the values of x and y. This can be achieved by setting x and y to their minimum values, which are both zero, as there cannot be a negative number of cheques.

So, our equation becomes:

$100(0) + 2250(0) + 1500z = 15250$

Simplify:

0 + 0 + 1500z = 15250

Now, solve for z:

1500z = 15250

z = 15250 / 1500

z = 10.1667

Since the number of cheques must be a whole number, we cannot have a fraction of a cheque. Therefore, the maximum possible number of cheques of 1500 that the donation box may have contained is 10.

Q.4 Suppose the medians BD and CE of a triangle ABC intersect at a point 0. If area of triangle ABC is 108 sq. cm., then, the area of the triangle ECM, in sq. cm., is

Solution.
To find the area of triangle ECM, we can use the fact that the medians of a triangle divide it into six equal triangles of equal area.

Since the area of triangle ABC is 108 sq. cm., each of the six triangles (including triangle ECM) has an area of 108 sq. cm. / 6 = 18 sq. cm.

So, the area of triangle ECM is 18 sq. cm.

Q.5 Bob can finish a job in 40 days, if he works alone. Alex is twice as fast as Bob and thrice as fast as Cole in the same Job. Suppose Alex and Bob work together on the first day, Bob and Cole work together

on the second day, Cole and Alex work together on the third day, and then, they continue the work by repeating this three-day roster, with Alex and Bob working together on the fourth day, and so on. Then, the total number of days Alex would have worked when the job gets finished, is

Solution.

Let's first find out how quickly Alex, Bob, and Cole can each finish the job working alone:

1. Bob can finish the job in 40 days.
2. Alex is twice as fast as Bob, so Alex can finish the job in 40 / 2 = 20 days.
3. Alex is also thrice as fast as Cole, so Cole can finish the job in 20 * 3 = 60 days.

Now, let's look at their work schedule:

- On the first day, Alex and Bob work together. Together, they can complete 1/20 + 1/40 = 3/40 of the job in one day.
- On the second day, Bob and Cole work together. Together, they can complete 1/40 + 1/60 = 1/24 of the job in one day.
- On the third day, Cole and Alex work together. Together, they can complete 1/60 + 1/20 = 4/60 = 1/15 of the job in one day.

Now, let's calculate the total amount of work done in this three-day cycle:

(3/40) + (1/24) + (1/15) = (9/120) + (5/120) + (8/120) = 22/120 = 11/60 of the job.

So, in each three-day cycle, they complete 11/60 of the job. Now, we need to figure out how many three-day cycles it takes to complete the entire job.

The job consists of 60/11 three-day cycles (because each three-day cycle completes 11/60 of the job). However, since we can't have a fraction of a three-day cycle, they would need 6 three-day cycles to complete the job.

Now, let's calculate how many days Alex works:

- In each three-day cycle, Alex works for 1 day.
- There are 6 three-day cycles to complete the job.

So, Alex works for a total of 6 days to complete the job.

Q.6 A glass contains 500 cc of milk and a cup contains 500 cc of water. From the glass, 150 cc of milk is transferred to the cup and mixed thoroughly. Next, 150 cc of this mixture is transferred from the cup to the glass. Now, the amount of water in the glass and the amount of milk in the cup are in the ratio

Ans
1.1 : 1
2. 10 : 3
3. 10 : 13
4. 3 : 10

Solution.
Let's break down the steps of this process to determine the final ratio of water in the glass to milk in the cup:

1. Initially, the glass contains 500 cc of milk, and the cup contains 500 cc of water.

2. 150 cc of milk from the glass is transferred to the cup. Now, the glass contains 350 cc of milk, and the cup contains 500 cc of water and 150 cc of milk.

3. 150 cc of the mixture (water and milk) from the cup is transferred back to the glass. This means that 150 cc of water from the cup is now in the glass, and the cup contains 150 cc of milk and 350 cc of water.

Now, let's calculate the final amounts:

- The glass contains 350 cc of milk and 150 cc of water.
- The cup contains 150 cc of milk and 350 cc of water.

Now, let's find the ratio of water in the glass to milk in the cup:

Water in the glass / Milk in the cup = (150 cc / 350 cc) / (150 cc / 150 cc) = (150 / 350) / (150 / 150) = (3/7) / 1 = 3/7

So, the ratio of water in the glass to milk in the cup is 3:7.

Q.7 Consider six distinct natural numbers such that the average of the two smallest numbers is 14, and the average of the two largest numbers is 28. Then, the maximum possible value of the average of these six numbers is

Ans
1. 22.5
2. 23
3. 23.5
4. 24

Solution.
Let's denote the six distinct natural numbers as A, B, C, D, E, and F. We're given two conditions:
1. The average of the two smallest numbers is 14.
2. The average of the two largest numbers is 28.

From condition 1, we can write the equation:

$(A + B) / 2 = 14$

This implies:

$A + B = 28$

From condition 2, we can write the equation:

$(E + F) / 2 = 28$

This implies:

$E + F = 56$

Now, we need to find the maximum possible value of the average of these six numbers, which is the sum of all six numbers divided by 6.

Average = $(A + B + C + D + E + F) / 6$

To maximize the average, we should maximize the sum of the numbers A, B, C, D, E, and F. To do this, we can use the values from conditions 1 and 2:

$A + B + C + D + E + F = (A + B) + (C + D) + (E + F) = 28 + (C + D) + 56$

Now, we need to maximize the sum of C and D. Since A and B are the two smallest numbers, C and D should be as large as possible. To maximize their sum, we can make them consecutive natural numbers:

C = 29 (next natural number after 28)
D = 30 (next natural number after 29)

Now, we can calculate the sum of all six numbers:

A + B + C + D + E + F = 28 + (29 + 30) + 56 = 28 + 59 + 56 = 143

Now, we can find the maximum possible average:

Average = (A + B + C + D + E + F) / 6 = 143 / 6 = 23.83 (approximately)

Since the numbers are natural numbers, we should round down to the nearest natural number, which is 23.

So, the maximum possible value of the average of these six numbers is 23.

Q.9 Two ships are approaching a port along straight routes at constant speeds. Initially, the two ships and the port formed an equilateral triangle with sides of length 24 km. When the slower ship traveled 8 km, the triangle formed by the new positions of the two ships and the port became right-angled. When the faster ship reaches the port, the distance, in km, between the other ship and the port will be

Ans
1. 4
2. 6
3. 12
4. 8

Solution.
Let's denote the initial positions of the two ships as A and B, and the position of the port as P, such that triangle APB is initially equilateral with sides of length 24 km.

When the slower ship travels 8 km, let's denote its new position as A'. The triangle formed by the new positions of the two ships (A'PB) becomes right-angled. Since triangle APB is equilateral, it means that angle APB is 60 degrees (1/6th of 360 degrees).

Now, let's consider triangle A'PB, where A'P represents the distance traveled by the slower ship (8 km), and AB is the initial distance between the two ships (24 km).

We know that in a right-angled triangle, the sine of an angle is defined as the length of the side opposite that angle divided by the length of the hypotenuse. In our case, sin(60 degrees) = A'P / AB.

sin(60 degrees) = $\sqrt{3}/2$

So, we have:

$\sqrt{3}/2$ = 8 km / AB

Now, we can solve for AB:

AB = (8 km) / ($\sqrt{3}/2$) = (8 km) * ($2/\sqrt{3}$) = ($16/\sqrt{3}$) km

To rationalize the denominator, we multiply both the numerator and denominator by $\sqrt{3}$:

AB = ($16\sqrt{3}/3$) km

Now, we need to find the distance between the other ship (B) and the port (P) when the faster ship reaches the port.

Since the sides of triangle APB are in a 1:2:$\sqrt{3}$ ratio (because it's a 30-60-90 triangle), and AB represents the shortest side, BP represents the middle side, and AP represents the longest side. Therefore:

BP = 2 * AB = 2 * ($16\sqrt{3}/3$) km = ($32\sqrt{3}/3$) km

So, when the faster ship reaches the port, the distance between the other ship (B) and the port (P) is (32√3/3) km.

The answer is not one of the provided options, but it's approximately 18.49 km.

Q. 10 Nitu has an initial capital of 120,000.Out of this, she invests *,8,000 at 5.5%in bank A, 15,000 at 5.6% in bank B and the remaining amount at x% in bank C, each rate being simple interest per annum. Her combined annual interest income from these investments is equal to 5% of the initial capital. If she had invested her entire initial capital in bank C alone, then her annual interest income, in rupees, would have been

Ans
1. 900
2. 800
3. 1000
4. 700

Solution.
Let's first calculate the total annual interest income Nitu receives from her investments in banks A, B, and C.

1. Amount invested in Bank A = 8,000
   Interest rate in Bank A = 5.5%
   So, the annual interest income from Bank A = (8,000 * 5.5%) = 440 rupees.

2. Amount invested in Bank B = 15,000
   Interest rate in Bank B = 5.6%
   So, the annual interest income from Bank B = (15,000 * 5.6%) = 840 rupees.

Now, let's find the amount invested in Bank C. We know that the remaining amount after investing in Banks A and B is invested in Bank C.

Amount invested in Bank C = Initial capital - (Amount in Bank A + Amount in Bank B)
Amount invested in Bank C = 120,000 - (8,000 + 15,000) = 120,000 - 23,000 = 97,000 rupees.

Now, let's use the information that her combined annual interest income from these investments is equal to 5% of the initial capital to find the total interest income:

Total annual interest income = 5% of initial capital
Total annual interest income = (5/100) * 120,000 = 6,000 rupees.

Now, we already found the interest income from Banks A and B, which is 440 + 840 = 1,280 rupees.

So, the remaining interest income must come from Bank C:

Interest income from Bank C = Total annual interest income - Interest income from Banks A and B
Interest income from Bank C = 6,000 - 1,280 = 4,720 rupees.

Now, if she had invested her entire initial capital of 120,000 rupees in Bank C alone at the interest rate of x%, her annual interest income from Bank C would have been:

Interest income from Bank C = (120,000 * x%) / 100

We want to find the value of x such that this interest income is equal to 4,720 rupees:

(120,000 * x%) / 100 = 4,720

Now, let's solve for x:

(120,000 * x%) = 4,720 * 100

120,000x = 472,000

x = 472,000 / 120,000
x = 3.9333 (approximately)

Now, if she had invested her entire initial capital in Bank C alone at an interest rate of approximately 3.9333%, her annual interest income would have been:

Interest income from Bank C = (120,000 * 3.9333%) / 100
Interest income from Bank C ≈ 4,720 rupees.

So, her annual interest income, if she had invested her entire initial capital in Bank C alone, would have been approximately 4,720 rupees.

The closest option to this value is option 3: 1,000 rupees, but it doesn't match the calculated value. There might be a typo or error in the answer options provided.

Q.12 In an examination, the average marks of students in sections A and B are 32 and 60, respectively. The number of students in section A is 10 less than that in section B. If the average marks of all the students across both the sections combined is an integer, then the difference between the maximum and minimum possible number of students in section A is

Solution.

Let's denote the number of students in section A as "x" and the number of students in section B as "y."

Given:
1. The average marks in section A is 32.
2. The average marks in section B is 60.
3. The number of students in section A is 10 less than that in section B.

From the average formula, we know that the sum of marks is equal to the average marks multiplied by the number of students. So, we can write equations for the sum of marks in each section:

For section A: Sum of marks in A = 32x
For section B: Sum of marks in B = 60y

Now, we are given that the average marks of all the students across both sections combined is an integer. The average marks across both sections is calculated as:

Total average = (Sum of marks in A + Sum of marks in B) / (Total number of students)

We can represent this equation as:

Total average = (32x + 60y) / (x + y)

Since the average is an integer, the numerator must be a multiple of the denominator. In other words, (32x + 60y) must be a multiple of (x + y).

Now, let's find the difference between the maximum and minimum possible number of students in section A:

1. Minimum possible number of students in section A:

To minimize the number of students in section A while keeping it 10 less than section B, let's assume x = 1. Then, y (number of students in section B) would be 11.

2. Maximum possible number of students in section A:

To maximize the number of students in section A while keeping it 10 less than section B, let's assume y = 20. Then, x (number of students in section A) would be 10.

Now, we can calculate the minimum and maximum differences:

Minimum difference = Number of students in section B - Number of students in section A = 11 - 1 = 10
Maximum difference = Number of students in section B - Number of students in section A = 20 - 10 = 10

The difference between the maximum and minimum possible number of students in section A is 10.

So, the answer is 10.

Q.14 A group of N people worked on a project. They finished 35% of the project by working 7 hours a day for 10 days. Thereafter, 10 people left the group and the remaining people finished the rest of the project in 14 days by working 10 hours a day. Then the value of N is

Ans
1.150
2.36
3.140
4.23

Solution.

Let's break down the problem step by step.

First, we need to find out how much work is completed by the group of N people in the initial 10 days when they work 7 hours a day.

Work completed in 1 day by the group of N people = 7 hours/day * (1/10) = 7/10 of the project.

So, in 10 days, the group of N people complete:
(7/10) * 10 = 7 projects.

Now, we are told that 35% of the project is completed in these 10 days, which means:

0.35 * Total Project = 7 projects

Let's solve for the Total Project:

Total Project = 7 projects / 0.35 = 20 projects.

Now, we know the total project size is 20 projects.

Next, 10 people leave the group, so the remaining number of people is N - 10.

Now, the remaining people finish the rest of the project in 14 days by working 10 hours a day.

Work completed in 1 day by the remaining group = 10 hours/day * (1/14) = 10/14 = 5/7 of the project.

Now, let's set up an equation for the remaining work:

Remaining Work = Total Project - Work Completed Initially
Remaining Work = 20 projects - 7 projects
Remaining Work = 13 projects.

Now, we can calculate the number of days needed by the remaining group to complete the remaining work:

Number of days needed = Remaining Work / Work completed in 1 day by the remaining group
Number of days needed = 13 projects / (5/7) projects/day
Number of days needed = (13 * 7) / 5
Number of days needed = 91 / 5 = 18.2 days.

Since the remaining group cannot work for a fraction of a day, they would need 19 days to complete the remaining work.

Now, we know that in the initial 10 days, the group of N people completed 7 projects. So, the remaining 13 projects were completed in 19 days by the remaining group.

We can set up an equation to find the value of N:

(7 projects) / (10 days) = (13 projects) / (19 days) * (N - 10 people)

Now, let's solve for N:

7/10 = 13/19 * (N - 10)

Now, we can cross-multiply:

7 * 19 = 10 * 13 * (N - 10)

133 = 130 * (N - 10)

Now, divide by 130:

(N - 10) = 133 / 130
(N - 10) = 1.0231

Now, add 10 to both sides:

N = 10 + 1.0231
N ≈ 11.0231

Since the number of people must be a whole number, we can round N to the nearest whole number:

N ≈ 11

So, the value of N is approximately 11.

Q.15 Moody takes 30 seconds to finish riding an escalator if he walks on it at his normal speed in the same direction. He takes 20 seconds to finish riding the escalator if he walks at twice his normal speed in the same direction. If Moody decides to stand still on the escalator, then the time, in seconds, needed to finish riding the escalator is

Solution.
Let's denote the speed of the escalator as "E" (in units of Moody's normal walking speed) and Moody's normal walking speed as "M."

When Moody walks on the escalator at his normal speed (M), he effectively has a relative speed of (1 + E) since he's walking in the same direction as the escalator's movement. We can represent this as a fraction of the escalator's speed relative to his own speed:

Relative Speed = (Speed of Moody + Speed of Escalator) / Speed of Moody
Relative Speed = (M + E) / M

We are given that it takes 30 seconds for him to finish riding the escalator at his normal speed, so we can set up an equation:

Distance (D) = Speed (Relative Speed) × Time
D = (M + E) / M × 30

Now, when Moody walks at twice his normal speed (2M), his relative speed is (2M + E) / 2M, and it takes him 20 seconds to finish riding the escalator:

D = (2M + E) / 2M × 20

Since the distance D remains the same in both cases (the length of the escalator doesn't change), we can set these two equations equal to each other:

(M + E) / M × 30 = (2M + E) / 2M × 20

Now, let's solve for E, the speed of the escalator:

(M + E) / M × 30 = (2M + E) / 2M × 20

Cross-multiply:

30(M + E) = 20(2M + E)

30M + 30E = 40M + 20E

Subtract 20E from both sides:

30M + 10E = 40M

Subtract 30M from both sides:

10E = 10M

Now, divide both sides by 10:

E = M

So, the speed of the escalator (E) is equal to Moody's normal walking speed (M).

Now, let's calculate the time needed for Moody to finish riding the escalator if he decides to stand still. When he stands still, his relative speed is just the speed of the escalator:

Relative Speed = E / M = 1

Now, we can use the formula for time:

Time = Distance / Relative Speed

Since the distance is the same as before (D) and Relative Speed is 1, we have:

Time = D / 1 = D

So, the time needed to finish riding the escalator when Moody stands still is equal to the length of the escalator, which is D.

Therefore, the time needed to finish riding the escalator when he stands still is D seconds.