# CAT 2023 Slot 1 QA Solution

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**Ques 1. If x and y are real numbers such that $x^2+(x-2y-1)^2=-4y(x+y)$, then the value x−2y is**

**Solu.** Given, $x^2 + (x - 2y - 1)^2 = - 4y(x + y)$
⇒ $x^2 + 4xy + 4y^2 + (x - 2y - 1)^2 = 0$
⇒ $(x + 2y)^2 + (x - 2y - 1)^2 = 0$
For the L.H.S. of the equation to be 0, each of the square terms should be 0 (as squares cannot be negative)
⇒ x-2y-1=0 ⇒ x - 2y = 1

**Ques 2. Let n be the least positive integer such that 168 is a factor of $1134^n$. If m is the least positive integer such that $1134^n$ is a factor of $168^m$, then m + n equals**

**Solu.** Prime Factorising 1134, we get $1134 = 2 * 3^4 * 7$ and $168= 2^3*3* 7$
$1134^n$ is a factor of 168
=> the factor of 2 should be atleast 3, for 168 to be a factor => n = 3.
Now, [ 134 ^ n = 1134 ^ 3 = 2 ^ 3 * 3 ^ 12 * 7 ^ 3 is a factor of 168 ^ m = (2 ^ 3 * 3 * 7) ^ m ⇒ m=12 as power of 3 should be atleast 12.
=> So ,m+n=15

**Ques 3. If √(5x + 9) + √(5x - 9) = 3(2 + √(2)) then √(10x + 9) is equal to**

**Solu.** Given, √(5x + 9) + √(5x - 9) = 3(2 + √(2))
⇒ √(5x + 9) + √(5x - 9) = 6 + 3√(2) ⇒ √(5x + 9) + √(5x - 9) = √(36) + √(18)
Comparing the L.H.S. and R.H.S.
⇒ 5x + 9 =36 ⇒5x=27 ⇒ x= 27/5 ; (can be verified second term as well).
using the ⇒ √(10x + 9) = √((10 * 27/5) + 9) = √(63) = 3√(7)

**Ques 4. If x and y are positive real numbers such that log_x($x^2$ + 12) = 4 and 3 log_y(x) = 1 then x + y equals**

**Solu.** Given, log_x(x ^ 2 + 12) = 4
=> x ^ 2 + 12 = x ^ 4 ⇒ x ^ 4 - x ^ 2 - 12 = 0 = x ^ 4 - 4x ^ 2 + 3x ^ 2 - 12 = 0 = x ^ 2 * (x ^ 2 - 4) + 3(x ^ 2 - 4) = 0 ⇒ (x ^ 2 - 4)(x ^ 2 + 3) =0 ⇒ since, x is a positive real number (given)=> x = 2 . ⇒ log_y(x) = 1/3
Now, Given 3 * log_y(x) = 1
=> x = y ^ (1/3)
=> y = x ^ 3 =>y=8.
=> x + y = 2 + 8 = 10 .

**Ques 5. The number of integer solutions of equation 2|x| * (x ^ 2 + 1) = 5x ^ 2 is**

**Solu.** 1) x = 0 This is a solution, as both L.H.S and R.H.S will be equal (0) when x = 0 (1 solution)
2) x > 0
⇒ 2x(x ^ 2 + 1) = 5x ^ 2
= 2(x ^ 2 + 1) = 5x
⇒2x^ 2 -5x+2=0 ⇒ 2x ^ 2 - 4x - x - 2 = 0
⇒ 2x(x - 2) - (x - 2) = 0
=> (x-2) (2x-1) = 0 => x = 2 or 1/2 => (1 integer solution)
3) x < 0
= - 2x(x ^ 2 + 1) = 5x ^ 2
=> 2x ^ 2 + 5x + 2 = 0
⇒ 2x ^ 2 + 4x + x + 2 = 0
⇒ 2x(x + 2) + 1(x + 2) = 0 => (x+2) (2x + 1) = 0 => x = -2 or -1/2 => (1 integer solution) So, the total number of integer solutions are 0, 2 ,-2=>3,

**Ques 6. The equation x ^ 3 + (2r + 1) * x ^ 2 + (4r - 1) * x + 2 = 0 has -2 as one of the roots. If the other two roots are real, then the minimum possible non-negative integer value of r is**

**Solu.** Given that -2 is a root of the given cubic equation. => Dividing the given equation by $(x + 2)$ Using the Horners method of synthetic division: coefficient of $x ^ 2$ is 1, and coefficient of x is $(2r + 1) - 2 = 2r - 1$ and the constant term $= (4r - 1) - 2(2r - 1) = 1$ .
=> The quadratic obtained by dividing the cubic $= x ^ 2 + (2r - 1) x + 10$,
Since, this equation has 2 real roots => Discriminant should be greater than $0 \Rightarrow (2r - 1) ^ 2 >4 \Rightarrow 2r-1>2$ or $2r - 1 <-2 \Rightarrow r>3/2$ or $r < - 1/2$
=> Minimum possible non-negative integer value of r is 2.

**Ques 7. Let a and beta be the two distinct roots of the equation $2x ^ 2 - 6x + k = 0$ such that (alpha + beta) and alpha*beta are the distinct roots of the equation $x ^ 2 + px + p = 0$ Then, the value of $8(k - p)$ is**

**Solu.** Given a and b are the distinct roots of the equation $2x ^ 2 - 6x + k = 0$
=> $a + b = - (- 6/2) = 3$ (Sum of the roots)
=> $ab = k / 2$ Product of the roots)
Now, $(a + b)$ and ab are the roots of the quadratic equation $x ^ 2 + px + p = 0$ =>$a+b+ab=-p$
$\Rightarrow 3 + k / 2 = - p$ - (1) =>$(a+b)(ab)=p$
$\Rightarrow 3(k / 2) = p$ - (2)
$3+ k/2 = - 3k/2 \Rightarrow 2k=-3 \Rightarrow k = - 3/2$
$p = (3k)/2 = 3/2 * (- 3/2) = - 9/4$
=> $8(k - p) = 8(- 3/2 + 9/4) = - 12 + 18 = 6$

**Ques 8. In an examination, the average marks of 4 girls and 6 boys is 24. Each of the girls has the same marks while each of the boys has the same marks. If the marks of any girl is at most double the marks of any boy, but not less than the marks of any boy, then the number of possible distinct integer values of the total marks of 2 girls and 6 boys is**

**Solu.** Given that the average marks of 4 girls and 6 boys is 24.
Let us assume 'b' is the marks scored by a boy and 'g' is the marks scored by a girl.

=>4g+6b=10^ * 24 = 240 - (1)
Given that, b <= g <= 2b We need to find the distinct possible values of 2g + 6b = 2g + 240 - 4g = 240 2g. From (1)
when b = g =>10g=240 Rightarrow g=24
when b=g/2 Rightarrow7g=240 Rightarrow g = 240/7
=> 240 - 2g varies from 240 -2^ * prime 24 to 240-2^ * 240/7
Rightarrow 171.42 to 192 => Integer values of 172 to 192 Rightarrow>21 values.

**Ques 9. The salaries of three friends Sita, Gita and Mita are initially in the ratio 5 : 6 : 7, respectively. In the first year, they get salary hikes of 20%, 25% and 20%, respectively. In the second year, Sita and Mita get salary hikes of 40% and 25%, respectively, and the salary of Gita becomes equal to the mean salary of the three friends. The salary hike of Gita in the second year is**

**Solu.** Given, the salaries of Sita, Gita and Mita are initially in the ratio 5: 6:7, respectively, Let us assume their salaries are 5p, 6p and 7p.
They get salary hikes of 20%, 25% and 20%, respectively.
=> Their salaries are 6/5^ * 5D 5/4* 6p and 6/5^ * 7p Rightarrow 6p , 7.5p, 8.4p
Now, Sita and Mita get salary hikes of 40% and 25%, respectively => Sita's salary =1.4^ * 6p = 8.4p and Mita's salary = 1.25 ^ * 8.4 4p = 10.5p Let Gita's salary be 'g' after hike
Rightarrow3g= 8.4p + g +10.5p Rightarrow2g=18.9p Rightarrow g=9.45p
=> Hike %= (9.45 - 7.5)/7.5 * 100 = 26%

**Ques 10. The minor angle between the hours hand and minutes hand of a clock was observed at 8:48 am. The minimum duration, in minutes, after 8.48 am when this angle increases by 50% is**

**Solu.** The given time is 8:48 AM.
Angle made by hours hand w.r.t 12 is 8 ^ * 30 (30 degrees in 1 hour) + 0.5 ^ * 48 (0.5 degree in 1 minute) = 240 + 24 = 264 degrees.
Angle made by minutes hands w.r.t 12 is 48^ * 6 = 288 degrees.

=> The angle between them is 288-264 = 24 degrees.
This should further increase by 12 degrees (50% of 24)
After m minutes, the further increase in angle = 24/11 =(6-0.5)^ * m = 11/2 * m =12 Rightarrow m

**Ques 11. Brishti went on an 8-hour trip in a car. Before the trip, the car had travelled a total of$x$x km till then, where$x$x is a whole number and is palindromic, i.e.,$x$x remains unchanged when its digits are reversed. At the end of the trip, the car had travelled a total of 26862 km till then, this number again being palindromic. If Brishti never drove at more than 110 km/h, then the greatest possible average speed at which she drove during the trip, in km/h, was**

**Solu.** Given the total number of kilometres travelled, including the trip = is 26862 Km, and the duration of the trip is 8 hrs.
If avg. speed of the car during the trip is 's' => the km travelled till just before the trip is 26862 - 8s, which should also be a palindrome.
=> From the options if s = 110 => The reading will be 26862 - 110*8 = 25982 (Not a palindrome)
=> If s = 100 => The reading will be 26862 - 100*8 = 26062 => It is a palindrome.
=> s = 100 is the correct option.

**Ques 12. Gita sells two objects A and B at the same price such that she makes a profit of 20% on object A and a loss of 10% on object B. If she increases the selling price such that objects A and B are still sold at an equal price and a profit of 10% is made on object B, then the profit made on object A will be nearest to**

**Solu.** Let us assume the initial selling prices of A and B is p.
Given, she made profit of 5 of A is P 20% on A => 1.2*c = p => c = 5p/6 => cost
Given, she made a loss of 10% on B => 0.9 *c = p => c = 10p/9 => cost of B is 10 9P

Now, she sold them at a price such that a 10% profit is made on B

=> Selling price = s 11/10*10/9p => 11/9 p
=> Profit % on A = (11/9 - 5/6)/ (5/6) × 100 = 46.66% = nearly 47%

**Ques 13. A mixture P is formed by removing a certain amount of coffee from a coffee jar and replacing the same amount with cocoa powder. The same amount is again removed from mixture P and replaced with same amount of cocoa powder to form a new mixture Q. If the ratio of coffee and cocoa in the mixture Q is 16 : 9, then the ratio of cocoa in mixture P to that in mixture Q is**

**Solu.** Given that in the final mixture, the ratio of coffee and cocoa is 16:9
Let us assume coffee is 16 units and cocoa is 9 units.
=> Initially, there are 25 units of coffee and 0 units of cocoa
Let's say x units of the mixture is removed and replaced with cocoa
=> Now, we have (25-x) coffee and x units of cocoa.
=> Mixture P Now, if x units of the mixture is removed: Amount of coffee present = (25 - x) * (25 - x)/25 * x
Rightarrow (25 - x)(1 - x/25) = 16 = (25 - x) ^ 2 = 16 * 25
=> 25 - x = 20 =>x=5.
In mixture P, cocoa = x = 5
In mixture Q, cocoa = 9 units.
=> Required ratio = 5:9

**Ques 14. Anil invests Rs. 22000 for 6 years in a certain scheme with 4% interest per annum, compounded half-yearly. Sunil invests in the same scheme for 5 years, and then reinvests the entire amount received at the end of 5 years for one year at 10% simple interest. If the amounts received by both at the end of 6 years are same, then the initial investment made by Sunil, in rupees, is**

**Solu.** Anil invested 22000 for 6 years at 4% interest compounded half-yearly
=> Amount = 22000 (1.02)

Let Sunil invest 'S' rupees for 5 years at 4% C.I. half-yearly and 10% S.I. for 1 additional year

=> Amount = S (1.02) 10 (1.1)

Given that the both amounts are equal

Rightarrow 22000 * (1.02) ^ 12 = S * (1.02) ^ 10 (1.1)

Rightarrow S = (22000 * (1.02) ^ 2)/1.1 = 20808

**Ques 15. The amount of job that Amal, Sunil and Kamal can individually do in a day, are in harmonic progression. Kamal takes twice as much time as Amal to do the same amount of job. If Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job. Then the number of days Sunil will take to finish the job working alone, is**

**Solu.** Let us assume the efficiencies of Amal, Sunil, and Kamal are a, s, and k, respectively.

Given that they are in H.P.

Rightarrow 2/s = 1/a + 1/k - (1)

Also, given that Kamal takes twice as much time as Amal to do the same amount of job => a = 2k

Given that when Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job.

=> If W is the total work

=> 4a + 9s + 16k = W.

from (1) 2/s = 1/a + 2/a

Rightarrow a = 3/2s and k = 3/4s

Rightarrow 4((3s)/2) + 9s + 16((3s)/4) = W => 6s + 9s + 12s = W

Rightarrow27s=W

Rightarrow s = W/27

=> Sunil will take 27 days to finish the work when working alone.

**Ques 16. Arvind travels from town A to town B, and Surbhi from town B to town A, both starting at the same time along the same route. After meeting each other, Arvind takes 6 hours to reach town B while**

**Surbhi takes 24 hours to reach town A. If Arvind travelled at a speed of 54 km/h, then the distance, in km, between town A and town B is**

**Solu.** Let us assume the speeds of Arvind and Surbhi are 'a' and 's', respectively.
Let us say they meet after 't' hours
=> Arvind travelled s*t distance in 6 hrs and Surbhi travelled a*t in 24 hrs
s ^ * t=a^ * 6 and a^ * t=s^ * 24
Rightarrow t ^ 2 =6*24
Rightarrow t=12
Given a=54
Rightarrow s^ * 12=54^ * 6
Rightarrow s = 27
=> Total distance between A and B is (s + a) ^ * t=(54+27)^ * 12=81^ * 12=972 Kms.

**Ques 17. A quadrilateral ABCD is inscribed in a circle such that AB : CD = 2 : 1 and BC : AD = 5 : 4. If AC and BD intersect at the point E, then AE : CE equals**

**Solu.** Given ABCD is a cyclic quadrilateral.
Angle ADB = Angle ACB (Angle subtended by chord on the same side of arc)
Angle DAC = Angle DBC (Angle subtended by chord on the same side of arc)
=> Triangles AED and BEC are similar triangles
Similarly triangles AEB and DEC are also similar using AA similarity property.
Now, given that AB : CD = 2 : 1 and BC : AD = 5 : 4
AE/BE = AD/BC = 4/5 (Similar Triangles AED and BEC)
BE/CE = AB/CD = 2/1 (Similar Triangles AEB and DEC)
Multiplying both, we get AE/CE = 8/5.

**Ques 21. The number of all natural numbers up to 1000 with non-repeating digits is**

**Solu.** 1-digit numbers => We have 1 to 9 => 9
2-digit numbers => x y, we have 9 ways to choose x from 1 to 9 => 9 ways and 9 ways to choose y (0 to 9 except x) => 9*9 = 81
3-digit numbers => x y z, we have 9 ways to choose x, 9 ways to choose y and 8 ways to choose z => 9*9*8 = 648.
Total numbers till 1000 without digits repeated in them is 9 + 81 + 648 = 738.

**Ques 22. A lab experiment measures the number of organisms at 8 am every day. Starting with 2 organisms on the first day, the number of organisms on any day is equal to 3 more than twice the number on the previous day. If the number of organisms on the nth day exceeds one million, then the lowest possible value of n is**

**Solu.** Given on day-1, there are 2 organisms.
On day-2, there are 2^ * 2 + 3 = 7 and on day-3, there are 2^ * 7 + 3 = 17
Let us try to form a pattern:
2 = 2 + 0(n = 1)
7 = 4 + 3(n = 2)
17=8+9[8+3*3] (n = 3)
37=16+21[16+3* 7] n = 4
$T(n) = 2^n + 3(2^{(n-1)} - 1)$
We know that 2 ^ 20 = 2 ^ 10 * 2 ^ 10 = 1024 * 1024 which is more than 1 million.
Let us check for n = 19
$2^{19} + 3(2^{18} - 1) = 2^{19} + 3 * 2^{18} - 3 = 2 * 2^{19} + 2^{18} - 3 = 2^{20} + 2^{18} - 3$ which is more than 1 million.
Let us check for n = 18
Rightarrow $2^{18} + 3(2^{17} - 1) = 2^{18} + 3 * 2^{17} - 3 = 2 * 2^{18} + 2^{17} - 3 = 2^{19} + 2^{17} - 3$ which is not more than a million.
=> n = 19