CAT 2023 Slot 2 QA Solution

Ques 1. For any natural numbers m, n, and k, such that k divides both m + 2n and 3m+4n, k must be a common divisor of

- **A. m and n**
- **B. 2m and 3n**
- **C. m and 2n**
- **D. 2m and n**

Solu. It is given that k divides m+2n and 3m+4n.
Since k divides (m + 2n) it implies k will also divide 3(m + 2n) Therefore, k divides 3m+6n.
Similarly, we know that k divides 3m+4n.
We know that if two numbers a, and b both are divisible by c, then their difference (a-b) is also divisible by c.
By the same logic, we can say that (3m + 6n) - (3m + 4n) is divisible by k.
Hence, 2n is also divisible by k.
Now, (m + 2n) is divisible by k, it implies 2(m + 2n) = 2m + 4n is also divisible by k.
Hence, (3m + 4n) - (2m + 4n) = m is also divisible by k.
Therefore, m, and 2n are also divisible by k.
The correct option is C

Ques 2. The sum of all possible values of x satisfying the equation $2^{(4x\,^\wedge 2)} - 2^{(2x\,^\wedge 2 + x + 16)} + 2^{(2x + 30)} = 0$ is

A. 3
B. 3/2
C. 5/2
D. 1/2

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Solu. It is given that $2^{(4x^2)} - 2^{(2x^2 + x + 16)} + 2^{(2x + 30)} = 0$ which can be written as:

=> $(2^{(2x^2)})^2 - 2^{(2x^2)} * 2^{(x + 15)} * 2^1 + (2^{(x + 15)})^2 = 0$

=> $(2^{(2x^2)} - 2^{(x + 15)})^2 = 0$

=> $2^{(2x^2)} - 2^{(x + 15)} = 0$ (since $(a-b)^2 = 0 \Rightarrow a-b=0$)

= $2x^2 = x + 15$

= $2x^2 - x - 15 = 0$

= $2x^2 - 6x + 5x - 15 = 0$

=> $2x(x - 3) + 5(x - 3) = 0$

= $(2x + 5)(x - 3) = 0$

Hence, the possible values of x are $-5/2$ and 3, respectively.

Therefore, the sum of the possible values is $(3 - 5/2) = 1/2$

The correct option is D

Ques 3. Any non-zero real numbers x,y such that y ne3 and condition.
$x/y < (x + 3)/(y - 3)$ Will satisfy the

A $x/y < y/x$

B If $y < 0$ and $-x < y$

C If $y > 10$ and $-x > y$

D If $x < 0$ and $-x < u$

Solu. It is given that $x/y < (x + 3)/(y - 3)$ which can be written as $x/y - (x + 3)/(y - 3) < 0$

=> $(x(y - 3) - y(x + 3))/(y(y - 3)) < 0$

=> $(xy - 3x - xy - 3y)/(y(y - 3)) < 0$

=> $(-3(x + y))/(y(y - 3)) < 0$

=> $(3(x + y))/(y(y - 3)) > 0$

From this inequality, we can say that, when $y<0 \Rightarrow y(y - 3) > 0$. Now to satisfy the given equation $(3(x + y))/(y(y - 3)) > 0$

$(x + y)$ must be greater than zero Hence, $x > 0$ and $|x| > |y|$

Therefore, the magnitude of æ is greater than the magnitude of y.

Hence, $x > y$ and $|x|>|y| \Rightarrow -x < y$ greater than the magnitude of y.)

Since the magnitude of 2 is

The correct option is B.

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Ques 4. Let a, b, m and n be natural numbers such that a > 1 and b 1. If $a^m b^n = 144^{145}$, then the largest possible value of n-m is
A 580
B 290
C 289
D 579

Solu. It is given that a ^ m * b ^ n = 144 ^ 145 where a > 1 and b > 1
144 can be written as 144 = 2 ^ 4 * 3 ^ 2
Hence, a ^ m * b ^ n = 144 ^ 145 can be written as 2 ^ 580 * 3 ^ 290 a ^ m * b ^ n = (2 ^ 4 * 3 ^ 2) ^ 145 =
We know that 3 ^ 290 is a natural number, which implies it can be written as a ^ 1
where a > 1
Hence, the least possible value of m is 1. Similarly, the largest value of n is 580.
Hence, the largest value of (n-m) is (580-1) = 579
The correct option is D

Ques 5. Let k be the largest integer such that the equation (x - 1) ^ 2 + 2kx + 11 = 0 has no real roots. If y is a positive real number, then the least possible value of k/4y +9y is

Solu. It is given that (x - 1) ^ 2 + 2kx + 11 = 0 has no real roots. (Where k is the largest integer)
(x - 1) ^ 2 + 2kx + 11 = 0 which can be written as: > x ^ 2 - 2x + 1 + 2kx + 11 = 0 => x ^ 2 - 2(k - 1) * x + 12 = 0 We know that for no real roots, D<0 => b ^ n * 2 - 4ac < 0 Hence, 2(k - 1) ^ 2 - 4 * 1 * 12 < 0
=> 4 * (k - 1) ^ 2 < 48
=> (k - 1) ^ 2 < 12 Since k is an integer, it implies (k-1) is also an integer. Therefore, from the above inequality, we can say that the largest possible value (k - 1) = 3 => The largest possible value of k is 4. Now we need to calculate the least possible value of k/(4y) + 9y k/(4y) + 9y can be written

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as 4/(4y) + 9y = 1/y + 9y The least possible value of 9y + 1/y can be calculated using A.M-G.M
inequality. Using A.M-G.M inequality, we get: (9y + 1/y)/2 >= sqrt(9y * 1/y) => (9y + 1/y)/2 >= sqrt(9) (9y + 1/y)/2 >= 3 9y + 1/y >= 6 Hence, the least possible value is 6

Ques 6. The number of positive integers less than 50, having exactly two distinct factors other than 1 and itself, is

Solu. Since there are two distinct factors other than 1, and itself, which implies the total number of factors of N is 4.
It can be done in two ways.
First case:
N = p ^ 3 (where p is a prime number)
Second case: N = p_{1}*p_{2} (Where P1, P2 are the prime numbers)
From case 1, we can see that the numbers which are a cube of prime and less than 50 are 8, and 27 (2 numbers).
From case 2, we will get the numbers in the form (2*3), (2 ^ * 5) (2*7), (2 ^ * 11) (2*13), (2*17), (2*19), (2*23), (3*5), (3*7), (3*11), (3*13), (5*7) ((13 numbers)}
Hence, the total number of numbers having two distinct factors is (13+2)=15.

Ques 7. For some positive real number x, if log_(sqrt(3))(x) + (log_x(25))/(log_z(0.008)) = 16/3 then the value of log_3(3x ^ 2) is

Solu. It is given that log_(sqrt(3))(x) + (log_x(25))/(log_z(0.008)) = 16/3 which can be written as:
=> 2 * log_3(x) + log_0.008(25) = 16/3
= 2 * log_3(x) + log_(3/1000)(25) = 16/3
= 2 * log_3(x) + log_(1/125)(25) = 16/3
= 2 * log_3(x) + log_(5 ^ - 3)(5) ^ 2 = 16/3
= 2 * log_3(x) - 2/3 = 16/3
= 2 * log_3(x) = 16/3 + 2/3
=> 2 * log_3(x) = 6

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=> $\log_3(x^2) = 6$ => $x^2 = 3^6$
Hence, $\log_3(3x^2) = \log_3(3 * 3^6) = \log_3(3^7) = 7$

Ques 8. Pipes A and C are fill pipes while Pipe B is a drain pipe of a tank. Pipe B empties the full tank in one hour less than the time taken by Pipe A to fill the empty tank. When pipes A, B and C are turned on together, the empty tank is filled in two hours. If pipes B and C are turned on together when the tank is empty and Pipe B is turned off after one hour, then Pipe C takes another one hour and 15 minutes to fill the remaining tank. If Pipe A can fill the empty tank in less than five hours, then the time taken, in minutes, by Pipe C to fill the empty tank is
A 90
B 120
C 75
D 60

Solu. Let the time taken by A to fill the tank alone be x hours, which implies the time taken by B to empty the tank alone is (x-1) hours (B is the drainage pipe), and the time taken by C to fill the tank is y hours.
It is given that when pipes A, B, and C are turned on together, the empty tank is filled in two hours.
Hence, $1/x - 1/(x - 1) + 1/y = 1/2$ Eq(1)
It is given that if pipes B and C are turned on together when the tank is empty and Pipe B is turned off after one hour, then Pipe C takes another one hour and 15 minutes to fill the remaining tank.
Hence, B worked for 1 hour, and C worked for 2 hours 15 minutes, which is equal to 9/4 hours.
In 1 hour, B worked $- 1/(x - 1)$ units, and in in 9/4 hours, C worked $9/(4y)$ units.
Hence, $9/(4y) - 1/(x - 1) = 1$....Eq(2)
Solving both equations, we get $y = 3/2$ and $x = 3$
Hence, the time taken by C is 3/2 hours, which is equal to 90 minutes.
The correct option is A

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Ques 9. Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half- yearly. He repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year. Then, the total interest, in rupees, paid over the three years is nearest to
A 45311
B 51311
C 33130
D 40991

Solu. It is given that Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half-yearly. It is also known that he repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year.
The total amount at the end of the first year is: 200000 x 104/100 x 104/100 = 216320
He repays 10320 rupees at the end of the first year, which implies the amount that remains unpaid at the end of the first year is 206000 rupees.
This unpaid amount will accrue interest for another two years.
Hence, the final amount at the end of three years is 206000 x 104/100 x 104/100 × 104/100 × 104/100 = 240990.86
Hence, the accrued interest in these two years is (240990.86-206000) = 34990.86 rupees.
Hence, the total interest accrued over the three years = (34990.86+16320) = 51311 rupees.
The correct option is B

Ques 10. Ravi is driving at a speed of 40 km/h on a road. Vijay is 54 meters behind Ravi and driving in the same direction as Ravi. Ashok is driving along the same road from the opposite direction at a speed of 50 km/h and is 225 meters away from Ravi. The speed, in km/h, at which Vijay should drive so that all the three cross each other at the same time, is
A 58.8
B 67.2

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C 61.6
D 64.4

Solu. It is given that the speed of Ravi is 40 kmph, which is equal to 100/9m/s known that the speed of Ashok is 50 kmph, which is equal to 125/9 * m / s
It is also
It is known that the distance between Ravi and Ashok is 225 meters, and the relative speed of Ravi and Ashok is 125/9 + 100/9 = 25m / s
Hence, they will meet each other in Ravi in 225/25 = 9 seconds
The distance traveled by Ravi in these 9 seconds is 100/9 *9 =100 meters.
Since Vijay was already 54 meters behind Ravi when they were starting, Vijay must travel (100+54) = 154 meters in these 9 seconds.
Hence, the speed of Vijay is 154/9 * m / s which is equal to 154/9 * 18/5 = 308/5 = 61.6 kmph.
The correct option is C

Ques 11. Minu purchases a pair of sunglasses at Rs.1000 and sells to Kanu at 20% profit. Then, Kanu sells it back to Minu at 20% loss. Finally, Minu sells the same pair of sunglasses to Tanu. If the total profit made by Minu from all her transactions is Rs.500, then the percentage of profit made by Minu when she sold the pair of sunglasses to Tanu is
A 35.42%
B 52%
C 31.25%
D 26%

Solu. The cost price of the sunglass for Meenu when he purchased it for the first time was 1000 rupees, and he sold it to Kanu at 20% profit. Hence, the selling price of the sunglass is 1200 rupees, which Kanu purchased.
Hence, the profit made by Meenu is (1200-1000) = 200 rupees.
Hence, the cost price of the same sunglass for Kanu is 1200 rupees, and now he sold it to Meenu at a 20% loss. Hence, the selling price of the sunglass now is (1200*0.8) = 960 rupees.

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The cost price of the same sunglass for Meenu when he purchased it for the second time was 960 rupees. Now Meenu sold it Tanu, at a certain price such that the total profit of Meenu becomes 500 rupees.
Hence, on the second transaction (selling it to Tanu), Meenu made a profit of (500-200) 300 rupees.
Hence, the profit made by Minu in the second transaction is (300/960)*100% = 31.25%
The correct option is C

Ques 12. The price of a precious stone is directly proportional to the square of its weight. Sita has a precious stone weighing 18 units. If she breaks it into four pieces with each piece having distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000. Then, the price of the original precious stone is
A 1944000
B 972000
C 1620000
D 1296000

Solu. it is given that the price of a precious stone is directly proportional to the square of its weight. Let the price be denoted by C and the weight is denoted by W.
Hence, C ∝ W^ 2 => C = k * w ^ 2 (where k is the proportional constant)
Now, Sita has a precious stone weighing 18 units.
Therefore, C = k * w ^ 2 = k * 18 ^ 2 = 324
If she breaks it into four pieces with each piece having a distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000.
To get the lowest possible value of C, we will get the weight of the four-piece as close as possible (3,4,5,6). To get the highest value we will try to take three pieces as low as possible, and one is as high as possible (1, 2, 3, 12). Hence, the maximum cost = $k(12^2+1^2+2^2+3^2)=158k^2$, and the minimum cost is k(3 ^ 2 + 4 ^ 2 + 5 ^ 2 + 6 ^ 2) = 86k ^ 2

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Hence, the difference is (158k ^ 2 - 86k ^ 2) = 72k ^ 2 which is equal to 288000.
=> 72k ^ 2 = 288000 => k ^ 2 = 4000 Hence, the price of the original stone is 324k ^ 2 = 324 * 4000 = 1296000
The correct option is D

Ques 15. If a certain amount of money is divided equally among n persons, each one receives Rs 352. However, if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receive less than or equal to Rs 330. Then, the maximum possible value of n is

Solu. It is given that if a certain amount of money is divided equally among n persons, each one receives Rs 352. Hence, the total amount of money is (352^ * n)= 352n Eq(1)
It is also known that if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receives less than or equal to Rs 330
Hence, the maximum amount of money with them =506^ * 2+ (n - 2) ^ * 330= 1012 + 330n - 660 = 352 + 330n
Now, 352+330n > 352n
=> 22n < 352
=> n <= 16
Hence, the maximum value is 16

Ques 20. If p ^ 2 + q ^ 2 - 29 = 2pq - 20 = 52 - 2pq then the difference between the maximum and minimum possible value of (p ^ 3 - q ^ 3)
A 243
B 486
C 378
D 189

Solu. Given that 2pq-20=52-2pq => 4pq=72 => pq = 18 - (- - (1))
Now, $p^2 + q^2$ - 29 = 2pq-20 => $p^2 + q^2$ - 2pq = 9 => $(pq)^2$ = 9 => 2 2 p - q = plus/minus 3 2

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Also, $p^2 + q^2 - 29 = 2pq - 20 \Rightarrow p^2 + q2 = 2pq + 9 = 2(18) + 9 = 45$
2
Now, $p^3 - q^3 = (p - q)(p^2 + pq + q^2) = (p - q)(45 + 18) = (p - q)(63)$
=> When $p - q = -3$ => The value is $63(-3) = -189$ and when $p - q = 3$ => The value is $63(3) = 189$.
=> The difference = $189 - (-189) = 378$.

Ques 22. Let a_n and b_n be two sequences such that $a_{n} = 13 + 6(n - 1)$ and $b_n = 15 + 7(n - 1)$ for all natural numbers n. Then, the largest three digit integer that is common to both these sequences, is

Solu. 967
It is given that $a_{n} = 13 + 6(n - 1)$ which can be written as $a_{n} = 13 + 6n - 6 = 7 + 6n$ Similarly, $b_{n} = 15 + 7(n - 1)$ which can be written as $b_n = 15 + 7n - 7 = 8 + 7n$
The common differences are 6, and 7, respectively, The common difference of terms that exists in both series is l.c.m(6, 7) = 42
The first common term of the first two series is 43 (by inspection)
Hence, we need to find the mth term, which is less than 1000, and the largest three-digit integer, and exists in both series.
$t_{m} = a + (m - 1) * d < 1000$
$= 43 + (m - 1) * 42 < 1000 \Rightarrow (m - 1) * 42 < 957$
$\Rightarrow m - 1 < 22.8$
$\Rightarrow m < 23.8$
$\Rightarrow m = 23$
Hence, the 23rd term is $43 + 22 * 42 = 967$

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