CAT 2023 Slot 3 QA Solution

Ques 1. For a real number x, if 1/2, ($log_3(2^x - 9)$)/($log_3(4)$) and ($log_5(2^x + 17/2)$)/($log_5(4)$) are in an arithmetic progression, then the common difference is
A. $log_4(3/2)$
B. $log_4(7)$
C. $log_4(23/2)$
D. $log_4(7/2)$

Solu. $log_4(7/2)$
It is given that 1/2, (log_3(2 ^ x - 9))/(log_3(4)) (log_3(2 ^ x - 9))/(log_3(4)) and (log_5(2 ^ x + 17/2))/(log_5(4)) can be written as log_4(2 ^ x - 9) and log_4(2 ^ x + 17/2) are in an arithmetic progression. (log_5(2 ^ x + 17/2))/(log_5(4)) can be written as
Hence, 2 * log_4(2 ^ x - 9) = 1/2 + log_4(2 ^ x + 17/2)
1/2 can be written as log_4(2) . Therefore, = > 2 * log_4(2 ^ x - 9) = log_4(2) + log_4(2 ^ x + 17/2) = log_4(2 ^ x - 9) ^ 2 = log_4(2) * (2 ^ x + 17/2) = > (2 ^ x - 9) ^ 2 = 2(2 ^ x + 17/2) > 2 ^ (2x) - 18 * 2 ^ x + 81 = 2 * 2 ^ x + 17 => 2 ^ (2x) - 20 * 2 ^ x + 64 = 0 = 2 ^ (2x) - 16 * 2 ^ x - 4 * 2 ^ x + 64 = 0 => 2 ^ x * (2 ^ x - 16) - 4(2 ^ x - 16) = 0 => (2 ^ x - 4)(2 ^ x - 16) = 0
The values of $$2^$$ can't be 4 (log will be undefined), which implies The value 2 ^ x is 16.
Therefore, the common difference is log_4(2 ^ x - 9) - log_4(2) > log_4(7) - log_4(2) = log_4(7/2) The correct option is D

Ques 2. Let n and m be two positive integers such that there are exactly 41 integers greater than 8^m and less than 8^m, which can be expressed as powers of 2. Then, the smallest possible value of n + m is
A 42
B 44

C 14
D 16

Solu. 16
It is given that there are exactly 41 numbers, which can be expressed as the power of two, and exist between 8 ^ m and 8 ^ n (where m, and n are positive integers, and m < n
Hence, 2^{3m} < 41 numbers 2^{3n}
Since, m is a positive integer, the least value of m is 1. Therefore, $2^{(3m)} = 2^3$
Hence, the 41 numbers between them are 2^4, 2^5, 2^6 ,...,2^{44}.
Then the lowest possible value of 8 ^ n is 2 ^ 45 Hence, the smallest value of n is 2^ 45 =8^ n => 2 ^ (3n) = 2 ^ 45 =>n=15 Hence, the smallest value of m + n is (15+1) = 16
The correct option is D

Ques 3. For some real numbers a and b, the system of equations x + y = 4 and (a + 5) x + (b ^ 2 - 15) * y = 8b has infinitely many solutions for x and y. Then, the maximum possible value of ab is

Solu. 33
It is given that for some real numbers a and b, the system of equations x + y = 4 and (a + 5) * x + (b ^ 2 - 15) * y = 8b has infinitely many solutions for x and y.
Hence, we can say that
=> (a + 5)/1 = (b ^ 2 - 15)/1 = (8b)/4
This equation can be used to find the value of a, and b.
Firstly, we will determine the value of b.
=> b^ 2 -15 1 = 8b 4 => b ^ 2 - 2b - 15 = 0 => (b - 5)(b + 3) = 0 Hence, the values of b are 5, and -3, respectively.
The value of a can be expressed in terms of b, which is a + 5 = b ^ 2 - 15 = a = b ^ 2 - 20
When b = 5, a = 5 ^ 2 - 20 = 5
When b = - 3, a = 3 ^ 2 - 20 = - 11
The maximum value of ab = (- 3)(- 11) = 33
The correct option is A

Ques 4. If x is a positive real number such that $x^8 + (1/x)^8 = 47$ then the value of $x^9 + (1/x)^9$ is
A $40\sqrt{5}$
B $30\sqrt{5}$
C $36\sqrt{5}$
D $34\sqrt{5}$

Solu. $34\sqrt{5}$
It is given that $x^8 + (1/x)^8 = 47$ which can be written as:
=> $(x^4)^2 + (1/(x^4))^2 = 47$
=> $(x^4 + 1/(x^4))^2 - 2x^4 * 1/(x^4) = 47$
=> $(x^4 + 1/(x^4))^2 = 49$
=> $x^4 + 1/(x^4) = 7$
Similarly, $x^4 + 1/(x^4) = 7$ can be expressed as:
=> $(x^2)^2 + (1/(x^2))^2 = 7$
=> $(x^2 + 1/(x^2))^2 - 2x^2 * 1/(x^2) = 7$
=> $(x^2 + 1/(x^2))^2 = 9$
=> $x^2 + 1/(x^2) = 3$
By the same logic, we get $x + 1/x = \sqrt{5}$
Now, $x^3 + 1/(x^3) = (x + 1/x)^3 - 3x * 1/x * (x + 1/x)$
=> $x^3 + 1/(x^3) = (\sqrt{5})^3 - 3\sqrt{5} = 2\sqrt{5}$
By the same logic, we can say that
=> $x^9 + 1/(x^9) = (x^3 + 1/(x^3))^3 - 3x^3 * 1/(x^3) * (x^3 + 1/(x^3))$
=> $x^9 + 1/(x^9) = (2\sqrt{5})^3 - 3(2\sqrt{5}) = x^9 + 1/(x^9) = 40\sqrt{5} - 6\sqrt{5} = 34\sqrt{5}$
The correct option is D

Ques 5. A quadratic equation $x^2 + bx + c = 0$ has two real roots. If the difference between the reciprocals of the roots is, and the sum of the reciprocals of the squares of the roots is, then the largest possible value of (b + c) is

Solu. 9

It is given that $x ^ 2 + bx + c = 0$ has two real roots. Let the roots of the equation be α, β. $(\alpha > \beta)$
Then, we can say that $1/\alpha - 1/\beta = 1/3$ Eq(1)
Similarly, $1/(\alpha ^ 2) + 1/(\beta ^ 2) = 5\ 9$ ....Eq(2)
Eq(2) can be written as
$(1/\alpha - 1/\beta)^2 + 2 * 1/\alpha * 1/\beta = 5/9$
$\Rightarrow (1/3) ^ 2 + 2 * 1/\alpha * 1/\beta = 5/9$
$\Rightarrow 2/ \alpha^* \beta = 4/9 \Rightarrow 1/(\alpha^*\beta) = 2/9 \Rightarrow \alpha^*\beta = 9/2$
We know that the product of the roots is equal to c, which implies $c=9/2$
We also know that the sum of the roots is equal to -b.
$\Rightarrow 1/(\alpha ^ 2) + 1/(\beta ^ 2) = (1/\alpha + 1/\beta) ^ 2 - 2/(\alpha^*\beta) = 5/9$
$\Rightarrow ((\alpha + \beta)/(\alpha^*\beta)) ^ 2 - 4/9 = 5/9$
$\Rightarrow ((\alpha + \beta)/(\alpha^*\beta)) ^ 2 = (1)^2$
$\Rightarrow \alpha + \beta = -+ \alpha * \beta$
Hence, the maximum value of b is 9/2 .
Hence, the maximum value of (b + c) is 9

Ques 6. Let n be any natural number such that $5 ^ {(n - 1)} < 3 ^ {(n + 1)}$ Then, the least integer value of m that satisfies $3 ^ {(n + 1)} < 2 ^ {(n + m)}$ for each such n, is

Solu. 5
It is given that $5 ^ {(n - 1)} < 3 ^ {(n + 1)}$ where n is a natural number. By inspection, we can say that the inequality holds when n = 1, 2 ,3 4 and 5.
Now, we need to find the least integer value of m that satisfies $3 ^ {(n + 1)} < 2 ^ {(n + m)}$
For, n = 1 the least integer value of m is 2.
For, n = 2 the least integer value of m is 3
For, n = 3 the least integer value of m is 4.
For, n = 4, the least integer value of m is 4.
For, n= 5, the least integer value of m is 5.
Hence, the least integer value of m such that for all the values of n, the equation holds is 5.

Ques 7. The sum of the first two natural numbers, each having 15 factors (including 1 and the number itself), is

Solu. 468
We know that the number of factors of these two numbers is 15. We know that the factors of 15 are 1, 3, 5, and 15.
The number of factors of N is (p +1) q + 1) Where, N = a ^ p * b ^ q and a, b are prime numbers).
Hence, the value of N will be least when (p + 1) and (q + 1) are as close as possible and a, and b are the least distinct prime numbers.
Hence, p + 1 = 3 =>p=2, and q + 1 = 5 =>q=4, and the prime numbers a, and b are 2, and 3, respectively.
Hence, the lowest value of N is N = 2 ^ 4 * 3 ^ 2 = 144 value of N is N = 2 ^ 2 * 3 ^ 4 = 324 and the second lowest
Hence, the sum is (144+324) = 468

Ques 8. A merchant purchases a cloth at a rate of Rs.100 per meter and receives 5 cm length of cloth free for every 100 cm length of cloth purchased by him. He sells the same cloth at a rate of Rs.110 per meter but cheats his customers by giving 95 cm length of cloth for every 100 cm length of cloth purchased by the customers. If the merchant provides a 5% discount, the resulting profit earned by him is
A 4.2%
B 9.7%
C 15.5%
D 16%

Solu. C
It is given that a merchant purchases a cloth at a rate of Rs.100 per meter and receives 5 cm length of cloth free for every 100 cm length of cloth purchased by him.
Hence, the cost price of 105 cm clothes is 100 rupees.
It is also known that he marked the price of 100 cm clothes as 110 rupees, and gave a 5% discount, and he cheated his customers by giving 95 cm

length of cloth for every 100 cm length of cloth purchased by the customers.
Hence, the selling price of 95 cm clothes is 110*(19/20) rupees.
Therefore, the selling price of 105 cm clothes is 115.5 rupees.
Hence, the profit is 15.5%
The correct option is C

Ques 9. A boat takes 2 hours to travel downstream a river from port A to port B, and 3 hours to return to port A. Another boat takes a total of 6 hours to travel from port B to port A and return to port B. If the speeds of the boats and the river are constant, then the time, in hours, taken by the slower boat to travel from port A to port B is
A 12(√(5) - 2)
B 3(3 + √(5))
C 3(√(5) - 1)
D 3(3 - √(5))

Solu. Let us assume the speed of the 1st boat is b, the 2nd boat is s, and the river's speed is r.
Let 'd' be the distance between A and B.
=> d = 2(b + r) and d = 3(b - r)
=> b + r = d / 2 and b - r = d / 3 =>r=d/12 (subtracting both equations).
Now, it is given that d/(s + r) + d/(s - r) = 6
=> d/(s + d/12) + d/(s - d/12) = 6
2ds = 6(s ^ 2 - (d ^ 2)/144) => 144s ^ 2 - 48ds - d ^ 2 = 0
Solving the quadratic equation, we get:
s = d((48 + √(48 ^ 2 + 4(144)))/(2 * 144))
s = d(1/6 + (√(5))/12)
=> Required value of d/(s + r)
= d/(d/6 + (√(5) * d)/12 + d/12) = 12/(3 + √(5)) = ((12)(3 - √(5)))/4
=3(3-√5)

Ques 10. There are three persons A, B and C in a room. If a person D joins the room, the average weight of the persons in the room reduces by x kg. Instead of D, if person E joins the room, the average

weight of the persons in the room increases by 2x kg. If the weight of E is 12 kg more than that of D, then the value of x is
A 2
B 0.5
C 1
D 1.5

Solu. 1
Let us assume that A, B, C, D, and E weights are a, b, c, d, and e.
1st condition
(a + b + c)/3 - (a + b + c + d)/4 = x
2nd condition
(a + b + c + e)/4 - (a + b + c)/3 = 2x
Adding both the equations, we get:
(e - d)/4 = 3x
=> (e-d)/4 => 3x => e - d = 12x
Given that 12x = 12 =>x=1

Ques 11. The population of a town in 2020 was 100000. The population decreased by y% from the year 2020 to 2021, and increased by x% from the year 2021 to 2022, where x and y are two natural numbers. If population in 2022 was greater than the population in 2020 and the difference between x and y is 10, then the lowest possible population of the town in 2021 was
A 72000
B 74000
C 73000
D 75000

Solu. It is given that the population of the town in 2020 was 100000. The population decreased by y% from the year 2020 to 2021 and increased by x% from the year 2021 to 2022, where x and y are two natural numbers.
Hence, the population in 2021 is 100000 ((100 - y)/100)
The population in 2022 is 100000((100 - y)/100)((100 + x)/100)

It is also given that the population in 2022 was greater than the population in 2020 and the difference between x and y is 10.
Hence,
100000((100 - y)/100)((100 + x)/100) > 10000 and (x - y) = 10
=> 100000((100 - y)/100)((110 + y)/100) > 10000
=> (100 - y)/100 * ((110 + y)/100) > 1
To get the minimum possible value of 2021, we need to increase the value of y as much as possible.
Hence, (100 y) {(100+ y) + 10} > 10000
=> 10000 - y ^ 2 + 1000 - 10y > 10000
=> y ^ 2 + 10y < 1000
=> y ^ 2 + 10y + 25 < 1025
=> (y + 5) ^ 2 = 1024 < 1025
=> (y + 5) ^ 2 = 32 ^ 2 = y = 27
Hence, the population in 2021 is 10000^ * (100 - 27) = 73000
The correct option is C

Ques 15. Gautam and Suhani, working together, can finish a job in 20 days. If Gautam does only 60% of his usual work on a day, Suhani must do 150% of her usual work on that day to exactly make up for it. Then, the number of days required by the faster worker to complete the job working alone is

Solu. Let 'g' and 's' be the efficiencies of Gautam and Suhani. Let W is the total amount of work.
=> g + s = W / 20 (1 day work) -(1)
Also Gautam doing only 60\%=> 3g / 5 and Suhani doing 150%=> 3s / 2
=> 3g / 5 + 3s / 2 = W / 20 (1 day work)
=> g + s = (3g)/5 + (3s)/2
=> s/g = 4/5
=> Gautam is the more efficient person.
Now, from the 1st equation => g + (4g)/5 = W/20
9/5 * g = W/20
g = W/36 => Gautam takes 36 days to finish the complete work.

Ques 19. In a regular polygon, any interior angle exceeds the exterior angle by 120 degrees. Then, the number of diagonals of this polygon is

Solu. 54

The sum of the interior angles of a polygon of 'n' sides is given by (2n - 4) * 90, and the sum of the exterior angles of a polygon is 360 degrees.
So, the difference between them will be 120 * n
= (2n - 4) * 90 - 360 = 120n
=> 60n =720 => n=12.
We know that the number of diagonals of a regular polygon is $^{n}C_{2\text{-}n}$=$^{12}C_{2}$ - 12 = 66 - 12 = 54

Ques 22. Suppose f(x, y) is a real-valued function such that f(3x + 2y, 2x - 5y) = 19x for all real numbers x and y. The value of x for which f(x, 2x) = 27 is

Solu. 3

Given that f(3x + 2y, 2x - 5y) = 19x
Let us assume the function f(a, b) is a linear combination of a and b.
=> f(3x + 2y, 2x - 5y) = m(3x + 2y) + n(2x - 5y) = 19x = 3m + 2n = 19 and 2m - 5n = 0
Solving we get m = 5 and n = 2
=> f(a, b) = 5a + 2b => f(x, 2x) = 5x + 2(2x) = 9x = 27 =>x=3